

Universal Multiple-Octet Coded Character Set
International Organization for Standardization
Internationale Standardisierungs-Organisation
Organisation Internationale de Normalisation
Διεθνής Οργανισμός Τυποποίησης
Международная организация по стандартизации

Doc Type: Working Group Document

Title: Proposal to encode Leibnizian ambiguous signs

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Related: L-2402n (section c); L-2404

Version: 4th revision

Previous version: N5329R3 (L-2608)

Status: final

Action: for SEWG and UTC review

Date: May 26, 2026

Requester's reference: LUCPL-2618

Background

For background information about the Philiumm project (headed by Prof. David Rabouin, Paris) and the related research work, please visit the [Philiumm website](#).

Leibnizian ambiguous signs

This proposal requests the encoding of (new:) **59** ambiguous operator signs (testified in works of Gottfried Wilhelm Leibniz) of which 5 characters are proposed as variation sequences. This document includes an in-depth explanation about the systematics and semantics of these signs.

The ambiguous characters are related to the well-known $\pm$ and $\mp$ characters (00B1, 2213), both by their graphical structure and historically. For editorial work the ambiguous signs are important for e.g. ascribing dates to manuscript sources which lack an original *datum*. The signs also inform about Leibniz's way of systematic thinking about how to notate certain logical concepts.

We propose to encode these signs in a dedicated block named **Leibnizian Ambiguous Signs**.

1st revision of proposal

Upon the first version N5329 (L-2504) we received the recommendation from the UTC, to introduce a different naming scheme for the ambiguous signs. Following that advice, we now propose character names which identify each character's semantical content with possible accuracy.

The initial nomenclature with A-xx, B-yy, C-zz numbering is, however, retained as *GWLB-ID* numbers, for sake of brevity and practical reasons. This nomenclature has been established at Gottfried-Wilhelm-Leibniz-Bibliothek Hannover (GWLB), for research and editorial usage.

2nd revision of proposal

Subgroup B has seen a few changes. The wording “component” in the character names has been replaced by the more appropriate term “partial”. Some characters formerly named “variant” are now proposed as variation sequences. Despite these changes we kept the order of listing as it was, for ease of orientation.

3rd revision of proposal: two additional characters

In the course of continued manuscript research at GWLB Hanover, two more ambiguous signs have been found recently. They belong to system 5 (group C) and fit into the logical structure inherent in that system (see p. 10, 11). This proposal has been extended by these two new symbols. The sorting and numbering of the characters has been re-ordered. The proposed code block scheme (see p. 15) has been updated and improved, according to recommendations we received from SEWG and UTC experts.

4th revision of proposal: updated character names

Upon a request forwarded by SEWG all figures in character names (1, 2, 3 ...) have been changed into ONE, TWO &c^a. A typo (B-13 1DC09 **FB**00) has been corrected into “FE00”.

Basic descriptions have been added for the proposed variation sequences.

1. Introductory remarks on Leibnizian Ambiguity signs

Early in his career, Leibniz wrote several texts in which he designed and systematically examined systems of symbols for analytical calculations. Complex systems of ambiguity signs, with which more than two cases are distinguished, represent an elementary and novel component of this *Méthode de l'universalité*. As part of the *Ars Characteristica*, the treatment of this method belongs to that branch of philosophy that is “the art of forming and arranging characters so that they agree with thoughts” (Mugnai 2018, abstract).

However, Leibniz’s interest is not only theoretical. Rather, the design of higher ambiguity signs is closely linked to his occupation with the mathematics of conic sections. There he has to consider sub-cases of cases, but would like to write only one equation to treat them all together, since often the equations do not differ except for the signs of the terms. The use of double signs, which allows to represent two cases simultaneously, is already part of common practice in mathematics. The characters $\pm$ and $\mp$, which are still in use today, are used for this purpose.

According to current knowledge, Leibniz designed six different systems over the course of time—as long as transitional forms and preliminary considerations are ignored.

One reason why a system of ambiguity signs is abandoned by Leibniz is the consideration that a large number of specific new printing types are required if a system does not rely on the traditional set of printing types. Leibniz’s further penetration of the topic also

led to improved, simpler or, in some cases, even more complex characters. The draft of the first system, for example, provides for special characters to express the product of the two double signs A-01 $\neq$ and A-02 $\neq$. For these, Leibniz envisages the ligature A-07 $\neq$ and A-08 $\neq$ of these two symbols with the LEIBNIZIAN PRODUCT SIGN he typically uses. Only later does he take advantage of the fact that the mathematical meaning can also be expressed using existing symbols.

Some systems also take into account the relationship between several ambiguity signs in the same expression. Ambiguity signs can be *homogeneous* or *corresponding* and therefore dependent on one another, as well as *heterogeneous* and therefore independent of one another.

Likewise, it was only in his 5th system that Leibniz gave up structuring ambiguity signs according to the distinction between cases and sub-cases as they arise in the calculation process. Even though, from the perspective of modern mathematics, it makes no difference with regard to calculations whether the ambiguity sign $(mp)m$ (i. e. a sign which has the sub-cases mp in the first case and m in the second case, with p as abbreviation for plus and m for minus) or $m(pm)$ (i. e. a sign which has m in the first case and the sub-cases pm in the second case) is used, they do refer to two fundamentally different conceptions of the mathematical situation.

Design questions also play a role when considering the layout of systems of ambiguity signs, which lead Leibniz to the discussion of different positioning of lines and thus to variants that are compared to the systems ultimately favored.

Particularly in Leibniz's drafts, the ambiguity signs that occur can contribute to the dating of the texts, as a sequence of systems can be observed.

2. Overview of systems and character names for the UCS

Some systems of ambiguity signs are designed in such a way that they can be extended to distinguish any number of cases. In systems that use specific new characters and do not use the existing character set of a typesetting box, the surviving texts only contain characters that distinguish a maximum of four different cases. Usually, not all possible combinations of p and m occur in the texts. However, the systematics described or reconstructed on the basis of the surviving texts often allows to reconstruct the full set of ambiguity signs that belong to a system.

The overview in the appendix therefore contains only systems that use special new characters. For them, a list of all possible cases is provided. A representation of their glyphs is given, provided their use is documented in the texts written by Leibniz. In the overview, the meaning of the ambiguity signs is also stated in an abbreviated form.

For the encoding of Leibniz's ambiguity signs in the Unicode standard, we propose a name consisting of the components "AMBIGUITY SIGN", an identifier for the system ("A" for system 1, "B" for system 2, and "C" for system 5) which is followed by a hyphen, and a sequential number, with a leading zero being added to single-digit numbers. The character-specific parts of the proposed names are also included in the overview.

The characteristics of the systems are briefly described below with references to the overview.

2.1 System 1

System 1 is based on the signs $+$ for p and $-$ for m that are still in use today, with A-01 $\neq$ being understood as a combination of these signs. The additional bar in A-02 $\neq$ represents negation, so the sign stands for mp .

When looking at the layout of the triple signs A-03 $\neq$, A-04 $\neq$, A-05 $\neq$ and A-06 $\neq$, it can be seen that Leibniz takes the structure of possible distinctions of cases and sub-cases

into account. In the texts, only signs for which a distinction between two sub-cases arises in the second case are described and documented. This second case with both sub-cases is represented in the right part of the sign in analogy to the associated double signs A-01 $\neq$ and A-02 $\neq$: the two upper crossbars suspended from the vertical bar again refer to the combination of p and m , while the third, lower bar appearing in A-04 $\neq$ and A-06 $\neq$ represents the negation of this part of the ambiguity sign. The value of the first case is on the left side (p or m). This part is connected to the second crossbar from the top in the right part of the ambiguity sign.

A special feature of the 1st system are the ambiguity signs A-07 $\neq$ and A-08 $\neq$ that stand for products of the double signs of the system.

2. 2 System 2

System 2 develops from the same combination of $+$ and $-$ that forms the sign A-01 $\neq$. Unlike in the first system, the negation of the sign is not expressed by a third crossbar with the same width, but by a longer, horizontal bar that is placed at the bottom of the vertical bar (e. g. in B-01 $\neq$). Such negations of the complete ambiguity sign can be applied to all of them, with B-05 $\neq$, B-06 $\neq$, B-07 $\neq$, and B-10 $\neq$ being examples.

As before, triple signs are composed of signs and double signs, with the first case on the left (subdivided or not) and the second case (not subdivided or subdivided) on the right. During this transition from double sign to triple sign, the negation bar of B-01 $\neq$ slides upwards, so to speak, so that the vertical bar in the partial sign that is given in one of Leibniz's texts, which rather coincidentally has the same design as A-02 $\neq$, now protrudes at the bottom, and the width of the crossbar is adjusted to that of others in the sign. In contrast to the first system, both parts of the triple sign are composed by connecting the horizontal bar of the single sign part to the top bar of the double sign part. The ambiguity signs B-02 $\neq$, B-03 $\neq$, B-04 $\neq$, B-05 $\neq$, B-06 $\neq$, B-07 $\neq$, B-08 $\neq$, B-09 $\neq$ and B-10 $\neq$ are of this kind.

The principle of composition is meant to be continued for distinguishing further ambiguities (i. e. the type of combination of cases that are distinguished). B-11 $\neq$ and B-12 $\neq$ which represent the negation of $p(mp)$ and $m(mp)$ make it clear that the negation bar is "moved up" again in these partial signs, and that its size corresponds to the size of all other crossbars.

As Leibniz discussed questions about the suitability of different positions of the crossbars when designing this system, the six ambiguity signs B-13 $\neq$, B-14 $\neq$, B-15 $\neq$, B-16 $\neq$, B-17 $\neq$, and B-18 $\neq$ have come down to us. They represent variants of ambiguity signs of the standard form.

2. 3 System 3

Leibniz builds system 3 from the ambiguity signs A-01 $\neq$ and B-01 $\neq$, which are used for pm and mp in the 2nd system. This means a reduction of the number of characters required, while still any complex ambiguity as well as all dependencies between ambiguity signs (i. e. whether they are *homogeneous*, *corresponding* or *heterogeneous*) can be expressed. To do this, numbers are added to the left and right of A-01 $\neq$ and B-01 $\neq$ according to certain given rules. Levels of case distinctions can also be expressed by building nested expressions according to rules. The entire expression is marked by a bracket with *vinculum* (i. e. they are connected by an overline).

As a ligature of the brackets "((" and "))" with the *vinculum* is needed, LEFT VIRGULA PARANTHESIS and RIGHT VIRGULA PARANTHESIS are included in the proposal to encode these expressions. The overview at the end of the document does not

contain any characters that are specifically assigned to this system since the glyphs of the ambiguity signs used in this system match with those of signs from the first two systems.

2. 4 System 4

System 4 has no new characters at all and instead uses lowercase letters of the Greek alphabet. Ambiguity is expressed by strings of certain pairs of letters such as α and ω , β and ψ , or γ and χ , where the two letters are equidistant from the beginning or end of the alphabet, and the letter from the beginning stands for p and the other for m . As in the 3rd system, the letters are written one after the other, following the order of the cases, and marked by brackets and a *vinculum*.

By using several pairs of letters it is possible to express the relationship between the ambiguity signs that occur in an expression, because the same pair of letters is used for interdependent ambiguity signs and different pairs of letters for independent ones. The level of cases can be represented by structuring these sequences with commas.

2. 5 System 5

The principles of the standard form of system 5 and all its predecessors were reconstructed on the basis of ambiguity signs that can be found in Leibniz's manuscripts.

In its final version (see 3. 5 subgroup "Standard" in the overview), the 5th system probably has the simplest structure in the design of the glyphs. The set of n -fold ambiguity signs consists of almost all possible n -combinations of p and m . All the signs with the meaning $pp...p$ (string with n characters) and $mm...m$ (string with n characters) are omitted as they have the same meaning as $+$ and $-$. The level of cases is not represented in this system. Likewise, dependencies that occur between several ambiguity signs of the same expression are not represented.

The double signs C-16 $\neq$ and C-17 $\neq$, the triple signs C-18 $\neq$, C-19 $\neq$, C-20 $\neq$, C-21 $\neq$, C-22 $\neq$, and C-23 $\neq$ as well as the quadruple signs C-24 $\neq$, C-25 $\neq$, C-26 $\neq$, C-27 $\neq$, C-28 $\neq$, C-29 $\neq$, C-30 $\neq$, and C-31 $\neq$ belong to this group.

Their design follows a uniform principle. On a vertical bar, horizontal bars of equal length are positioned at equal distances depending on the cases distinguished in the sign. If a bar represents p , it is bisected by the vertical bar. If it stands for m , it starts on the left at the same distance from the vertical bar as the p -bars, but already ends at the vertical bar.

This approach was derived from the 2nd system, and there is a total of four stages in the development of the standard version of system 5 of which only few examples of ambiguity signs have been preserved. The representation of the level of case distinction is a common feature of all these systems.

- The "Transition Form 2 $\rightarrow$ 5" (subgroup 3. 1) preserves the division into left and right part from the 2nd system as a means to represent two cases on the top level of case distinction. What is new, however, is the reduction of the representation of the cases on the second level, where, among other things, a crossbar shortened to the half width appears for the first time. The ambiguity sign C-01 $\neq$ has been handed down in this group.
- In the ambiguity signs of the group "prae-pro-*proto*-5" (subgroup 3. 2), instead of being divided into left and right halves to distinguish the two cases, there is a subdivision of the sign into an upper and lower section which is arranged along a vertical bar. In the case of the two known ambiguity signs C-02 $\neq$ and C-03 $\neq$, a subordinate case distinction occurs in the first (upper) case, where p and m are expressed by a long and a short horizontal bar, respectively, which are positioned in the middle of the vertical bar and to its left, respectively. A connection of these two

horizontal bars by a short vertical bar at their left which ends above the upper horizontal bar illustrates that they belong to the same group of sub-cases.

- From the group “pro-proto-5” (subgroup 3. 3), only the ambiguity signs C-04 $\ddagger$ and C-05 $\ddagger$ have come down to us. For both, signs that represent the same type of ambiguity in the previous group “prae-pro-proto-5” (subgroup 3. 2) are also known. Compared to them, the upper halves of the short vertical bars which illustrate in the previous variant that the horizontal bars that are connected by them belong to the same group of sub-cases are omitted, so that the resulting glyphs are further reduced compared to their predecessors.
- The group “proto-5” (subgroup 3. 4) already shows very close similarities to the standard form. However, Leibniz continues to distinguish the levels of case distinction in the triple signs, with the first case or its two sub-cases being shown in the upper part of the signs, the second case or its sub-cases in the lower section. C-09 $\ddagger$ and C-10 $\ddagger$, which stand for $p(mp)$ and $(pm)p$ respectively, differ only in the positions of the short crossbar that represents m . They have the same meaning as long as the levels of case distinction are ignored. The triple signs C-08 $\ddagger$ and C-11 $\ddagger$ also belong to this group, as well as the double signs C-06 $\ddagger$ and C-07 $\ddagger$.

In this system there are also composed forms: C-12 $\ddagger$ represents a composition of mp and $p(mp)$ according to a rule, the negation is C-13 $\ddagger$. There are also combinations based on this: in C-14 $\ddagger$, C-13 $\ddagger$ occurs as the second case of an ambiguity sign whose other case is p , while in C-15 $\ddagger$ it is the first case, again in combination with p .

Thus, it is giving up the representation of the level of case distinctions and the choice of equal distances that ultimately constitute the final step towards the standard form. At the same time, this reduces the number of ambiguity signs to be taken into account. While in its standard form the continuation of the system for distinguishing more cases is known, it is not clear from the surviving texts for the transitional and preliminary forms.

2. 6 System 6

System 6 which can be derived from several manuscripts shares its basic idea with system 4. In all examples known from these texts, p is expressed by 1. For m , Leibniz uses 3 in one text and 2 in all others.

By relying entirely on character types which are included in the usual typesetting box, this system does not have to be taken into account in the proposal.

References and additional literature

Mugnai 2018 Massimo Mugnai, *Ars Characteristica, Logical Calculus, and Natural Languages*, in: Maria Rosa Antognazza (ed.): *The Oxford Handbook of Leibniz*, Oxford University Press. Oxford 2018, p. 177-207 (published online as <https://doi.org/10.1093/oxfordhb/9780199744725.013.20>).

Probst & Trunk 2019 Siegmund Probst and Achim Trunk: *Einleitung*, in: Gottfried Wilhelm Leibniz: *Sämtliche Schriften und Briefe*, Vol. VII, 7.

Trunk 2016-2017 Achim Trunk, *Sechs Systeme: Leibniz und seine signa ambigua*, in: Wenchao Li (ed.): *Für unser Glück oder das Glück anderer, Vorträge des X. Internationalen Leibniz-Kongresses*. Hildesheim 2016-2017, vol. 4.

Overview of Leibniz's systems of ambiguity signs

The columns in this overview contain the following information:

- 1 number within the system
- 2 mathematical meaning
- 3 specific part of proposed character name (if it is part of the proposal)
- 4 representative glyph
- 5 ID in the proto standard as defined by the Leibniz-Edition (LE)
- 6 number and glyph in the current font of the Leibniz-Edition (if existing). The current glyphs can deviate from their actual layout.

The following signs are used to express the meaning of the signs:

- p plus
- m minus
- (...) group of sub-cases
- non[...] negation
- multiplication of signs
- composition of signs

1 System 1

1.1 Double Signs

<i>nr.</i>	<i>meaning</i>	<i>character name ID</i>	<i>repr. glyph</i>	<i>ID in proto standard of LE</i>	<i>glyphs used in the Leibniz Edition (LE)</i>
1	Sys. 1 pm (= sys. 2 pm)	A-01	≠	T-01 = T-19	12: ≠
2	Sys. 1 mp (= sys. 2 (parts) mp)	A-02	≡	T-20 = T-02	13: ≡

1.2 Triple Signs

1	Sys. 1 p(pm)	A-03	≠	T-03	—
2	Sys. 1 p(mp)	A-04	≡	T-04	—
3	Sys. 1 m(pm)	A-05	≠	T-05	—
4	Sys. 1 m(mp)	A-06	≡	T-06	—

1.3 Multiplication Forms

1	Sys. 1 pm · pm	A-07	≠	T-07	—
2	Sys. 1 mp · pm	A-08	≡	T-08	—

2 System 2

2.1 Double Signs

2.1.1 Standard Forms

1	Sys. 2 pm (= sys. 1 pm)	(A-01)	≠	T-01 = T-19	6: ≠
2	Sys. 2 mp	B-01	≡	T-12	7: ≡

2.1.2 Standard Forms (Parts)

1	Sys. 2 (parts) pm	—	—	—	—
2	Sys. 2 (parts) mp (= sys. 1 mp)	(A-02)	≡	T-20 = T-02	230: ≡

2.2 Triple Signs

2.2.1 Standard Forms

2.2.1 a) Type _ (_ _)

1	Sys. 2 p(pm)	B-02	≠	R-118	118: ≠
2	Sys. 2 p(mp)	B-03	≠	T-10	120: ≠
3	Sys. 2 m(pm)	—	—	—	—
4	Sys. 2 m(mp)	B-04	≠	T-09	—
5	Sys. 2 <i>non</i> [p(pm)]	B-05	≡	T-15	222: ≡
6	Sys. 2 <i>non</i> [p(mp)]	B-06	≡	R-119	119: ≡
7	Sys. 2 <i>non</i> [m(pm)]	B-07	≡	R-84	84: ≡
8	Sys. 2 <i>non</i> [m(mp)]	—	—	—	—

2.2.1 b) Type (_ _) _

9	Sys. 2 (pm)p	B-08	≠	T-17	231: ≠
10	Sys. 2 (mp)p	—	—	—	—
11	Sys. 2 (pm)m	B-09	≠	R-233	233: ≠

12	Sys. 2 (mp)m	—	—	—	—
13	Sys. 2 <i>non</i> [(pm)p]	B-10	‡ ⁺	R-234	234: ‡ ⁺
14	Sys. 2 <i>non</i> [(mp)p]	—	—	—	—
15	Sys. 2 <i>non</i> [(pm)m]	—	—	—	—
16	Sys. 2 <i>non</i> [(mp)m]	—	—	—	—

2. 2. 2 Standard Forms (Parts)

2. 2. 2 a) Type _ (_ _)

1	Sys. 2 (parts) p(pm)	—	—	—	—
2	Sys. 2 (parts) p(mp)	—	—	—	—
3	Sys. 2 (parts) m(pm)	—	—	—	—
4	Sys. 2 (parts) m(mp)	—	—	—	—
5	Sys. 2 (parts) <i>non</i> [p(pm)]	—	—	—	—
6	Sys. 2 (parts) <i>non</i> [p(mp)]	B-11	‡ ₃ ⁺	T-25	224: ‡ ₃ ⁺
7	Sys. 2 (parts) <i>non</i> [m(pm)]	—	—	—	—
8	Sys. 2 (parts) <i>non</i> [m(mp)]	B-12	‡ ₃	R-226	226: ‡ ₃

2. 2. 2 b) Type (_ _) _

9	Sys. 2 (parts) (pm)p	—	—	—	—
10	Sys. 2 (parts) (mp)p	—	—	—	—
11	Sys. 2 (parts) (pm)m	—	—	—	—
12	Sys. 2 (parts) (mp)m	—	—	—	—
13	Sys. 2 (parts) <i>non</i> [(pm)p]	—	—	—	—
14	Sys. 2 (parts) <i>non</i> [(mp)p]	—	—	—	—
15	Sys. 2 (parts) <i>non</i> [(pm)m]	—	—	—	—
16	Sys. 2 (parts) <i>non</i> [(mp)m]	—	—	—	—

2. 2. 3 Variants

1	Sys. 2 p(pm)	B-13	𐀓	T-13	228: 𐀓
2	Sys. 2 p(mp)	B-14	𐀔	T-14	223: 𐀔
6	Sys. 2 <i>non</i> [p(mp)]	B-15	𐀕	T-16	225: 𐀕
10	Sys. 2 (mp)p	B-16	𐀖	T-18	229: 𐀖
6	Sys. 2 (part) <i>non</i> [p(mp)]	B-17	𐀗	R-232	232: 𐀗
6	Sys. 2 (part) <i>non</i> [p(mp)]	B-18	𐀘	R-227	227: 𐀘

3 System 5

3. 1 Subgroup “Transition Form 2 → 5”

—	Sys. Transition Form 2 to 5 p(mp)	C-01	𐀓	T-75	—
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3. 2 Subgroup “prae-pro-proto-5”

—	Sys. prae-pro-proto-5 (pm)p	C-02	𐀓	T-72	—
—	Sys. prae-pro-proto-5(mp)p	C-03	𐀔	T-73	—

3. 3 Subgroup “pro-proto-5”

—	Sys. pro-proto-5 (pm)p	C-04	𐀓	R-220	220: 𐀓
—	Sys. pro-proto-5 (mp)p	C-05	𐀔	T-74	221: 𐀔

3. 4 Subgroup “proto-5”

3. 4. 1 Double Signs

1	Sys. proto-5 pm	C-06	𐀓	T-46	—
2	Sys. proto-5 mp	C-07	𐀔	T-71	—

3. 4. 2 Triple Signs

1	Sys. proto-5 p(pm)	C-08	𐀓	T-69	—
2	Sys. proto-5 p(mp)	C-09	𐀔	T-47	—
3	Sys. proto-5 m(pm)	C-32	𐀓	<i>new</i>	—

4	Sys. proto-5 m(mp)	C-33	𐌚	<i>new</i>	—
5	Sys. proto-5 (pm)p	C-10	𐌛	T-42	—
6	Sys. proto-5 (mp)p	C-11	𐌜	T-70	—
7	Sys. proto-5 (pm)m	—	—	—	—
8	Sys. proto-5 (mp)m	—	—	—	—

3. 4. 3 Composed Forms

—	Sys. proto-5 mp ◦ p(mp)	C-12	𐌚	T-41	—
—	Sys. proto-5 <i>non</i> [mp ◦ p(mp)]	C-13	𐌛	T-48	—
—	Sys. proto-5 <i>pnon</i> [mp ◦ p(mp)]	C-14	𐌜	T-44	—
—	Sys. proto-5 <i>non</i> [mp ◦ p(mp)]p	C-15	𐌛	T-43	—

3. 5 Subgroup “Standard”

3. 5. 1 Double Signs

1	Sys. 5 pm	C-16	𐌛	T-55	8: 𐌛
2	Sys. 5 mp	C-17	𐌜	T-56	38: 𐌜

3. 5. 2 Triple Signs

1	Sys. 5 ppm	C-18	𐌛	T-57	99: 𐌛
2	Sys. 5 pmp	C-19	𐌜	T-58	39: 𐌜
3	Sys. 5 mpp	C-20	𐌛	T-45	85: 𐌛
4	Sys. 5 pmm	C-21	𐌜	T-59	40: 𐌜
5	Sys. 5 mpm	C-22	𐌛	T-60	42: 𐌛
6	Sys. 5 mmp	C-23	𐌜	T-61	41: 𐌜

3. 5. 3 Quadruple Signs

1	Sys. 5 pppm	C-24	𐌛	T-66	46: 𐌛
2	Sys. 5 ppmp	C-25	𐌜	T-65	45: 𐌜
3	Sys. 5 pmpp	C-26	𐌛	T-64	44: 𐌛
4	Sys. 5 mppp	C-27	𐌜	T-63	43: 𐌜
5	Sys. 5 ppmm	C-28	𐌛	T-67	47: 𐌛

6	Sys. 5 pmpm	—	—	—	—
7	Sys. 5 mppm	C-29	≡	R-49	—
8	Sys. 5 pmmp	—	—	—	—
9	Sys. 5 mpmp	—	—	—	—
10	Sys. 5 mmpm	C-30	≡	T-68	—
11	Sys. 5 pmmm	—	—	—	—
12	Sys. 5 mpmm	C-31	≡	T-62	48: ≡
13	Sys. 5 mmppm	—	—	—	—
14	Sys. 5 mmmp	—	—	—	—

Introduction and tables by Elisabeth Rinner



Leibniz-Akademie-Ausgabe (LAA, general edition of Leibniz's writings)
LAA series VII (mathematical manuscripts, volumes 3 to 7 available online)
LAA series VII volume 3
LAA series VII volume 5
LAA series VII volume 7

3.a Proposed block name

We propose to create a new block of four columns or 64 slots, for these characters. As block name we propose: **Leibnizian Ambiguous Signs**.

The proposed codepoint range is **1DC00 to 1DC3F**.

3.b Proposed explanatory text:

Used by Leibniz for analytical calculations, when equations for several cases and sub-cases do not differ except for the signs of the terms, with these signs two or more cases are distinguished.

3.c Characters proposed for encoding

If this proposal gets accepted, the following characters will exist:

Glyph	N°	Code	GWL	
			B	Unicode Character Names
⌘	1	1DC00	A-01	LEIBNIZIAN SYSTEM ONE AND TWO PLUS-MINUS AMBIGUOUS SIGN
⌘	2	1DC01	A-02	LEIBNIZIAN SYSTEM ONE MINUS-PLUS AMBIGUOUS SIGN
⌘	3	1DC02	A-03	LEIBNIZIAN SYSTEM ONE PLUS OR PLUS-MINUS AMBIGUOUS SIGN
⌘	4	1DC03	A-04	LEIBNIZIAN SYSTEM ONE PLUS OR MINUS-PLUS AMBIGUOUS SIGN
⌘	5	1DC04	A-05	LEIBNIZIAN SYSTEM ONE MINUS OR PLUS-MINUS AMBIGUOUS SIGN
⌘	6	1DC05	A-06	LEIBNIZIAN SYSTEM ONE MINUS OR MINUS-PLUS AMBIGUOUS SIGN
⌘	7	1DC06	A-07	LEIBNIZIAN SYSTEM ONE PLUS-MINUS TIMES PLUS-MINUS AMBIGUOUS SIGN
⌘	8	1DC07	A-08	LEIBNIZIAN SYSTEM ONE MINUS-PLUS TIMES PLUS-MINUS AMBIGUOUS SIGN
⌘	9	1DC08	B-01	LEIBNIZIAN SYSTEM TWO MINUS-PLUS AMBIGUOUS SIGN
⌘	10	1DC09	B-02	LEIBNIZIAN SYSTEM TWO PLUS OR PLUS-MINUS AMBIGUOUS SIGN
⌘	11	1DC0A	B-03	LEIBNIZIAN SYSTEM TWO PLUS OR MINUS-PLUS AMBIGUOUS SIGN
⌘	12	1DC0B	B-04	LEIBNIZIAN SYSTEM TWO MINUS OR MINUS-PLUS AMBIGUOUS SIGN
⌘	13	1DC0C	B-05	LEIBNIZIAN SYSTEM TWO NEGATED PLUS OR PLUS-MINUS AMBIGUOUS SIGN
⌘	14	1DC0D	B-06	LEIBNIZIAN SYSTEM TWO NEGATED PLUS OR MINUS-PLUS AMBIGUOUS SIGN
⌘	15	1DC0E	B-07	LEIBNIZIAN SYSTEM TWO NEGATED MINUS OR PLUS-MINUS AMBIGUOUS SIGN
⌘	16	1DC0F	B-08	LEIBNIZIAN SYSTEM TWO PLUS-MINUS OR PLUS AMBIGUOUS SIGN
⌘	17	1DC10	B-09	LEIBNIZIAN SYSTEM TWO PLUS-MINUS OR MINUS AMBIGUOUS SIGN
⌘	18	1DC11	B-10	LEIBNIZIAN SYSTEM TWO NEGATED PLUS-MINUS OR PLUS AMBIGUOUS SIGN
⌘	19	1DC12	B-11	LEIBNIZIAN SYSTEM TWO NEGATED PLUS OR MINUS-PLUS PARTIAL AMBIGUOUS SIGN
⌘	20	1DC13	B-12	LEIBNIZIAN SYSTEM TWO NEGATED MINUS OR MINUS-PLUS PARTIAL AMBIGUOUS SIGN
⌘	21	1DC14	B-16	LEIBNIZIAN SYSTEM TWO NEGATED PLUS-MINUS OR PLUS PARTIAL AMBIGUOUS SIGN
⌘	22	1DC15	C-01	LEIBNIZIAN SYSTEM FIVE-ONE PLUS OR PLUS-MINUS AMBIGUOUS SIGN
⌘	23	1DC16	C-02	LEIBNIZIAN SYSTEM FIVE-TWO PLUS-MINUS OR PLUS AMBIGUOUS SIGN
⌘	24	1DC17	C-03	LEIBNIZIAN SYSTEM FIVE-TWO MINUS-PLUS OR PLUS AMBIGUOUS SIGN
⌘	25	1DC18	C-04	LEIBNIZIAN SYSTEM FIVE-THREE PLUS-MINUS OR PLUS AMBIGUOUS SIGN
⌘	26	1DC19	C-05	LEIBNIZIAN SYSTEM FIVE-THREE MINUS-PLUS OR PLUS AMBIGUOUS SIGN
⌘	27	1DC1A	C-06	LEIBNIZIAN SYSTEM FIVE-FOUR PLUS OR MINUS AMBIGUOUS SIGN
⌘	28	1DC1B	C-07	LEIBNIZIAN SYSTEM FIVE-FOUR MINUS OR PLUS AMBIGUOUS SIGN
⌘	29	1DC1C	C-08	LEIBNIZIAN SYSTEM FIVE-FOUR PLUS OR PLUS-MINUS AMBIGUOUS SIGN
⌘	30	1DC1D	C-09	LEIBNIZIAN SYSTEM FIVE-FOUR PLUS OR MINUS-PLUS AMBIGUOUS SIGN
⌘	31	1DC1E	C-10	LEIBNIZIAN SYSTEM FIVE-FOUR PLUS-MINUS OR PLUS AMBIGUOUS SIGN
⌘	32	1DC1F	C-11	LEIBNIZIAN SYSTEM FIVE-FOUR MINUS-PLUS OR PLUS AMBIGUOUS SIGN
⌘	33	1DC20	C-12	LEIBNIZIAN SYSTEM FIVE-FIVE INVERTED MINUS-PLUS AMBIGUOUS SIGN
⌘	34	1DC21	C-13	LEIBNIZIAN SYSTEM FIVE-FIVE INVERTED PLUS-MINUS AMBIGUOUS SIGN
⌘	35	1DC22	C-14	LEIBNIZIAN SYSTEM FIVE-FIVE INVERTED PLUS OR PLUS-MINUS AMBIGUOUS SIGN
⌘	36	1DC23	C-15	LEIBNIZIAN SYSTEM FIVE-FIVE INVERTED PLUS-MINUS OR PLUS AMBIGUOUS SIGN
⌘	37	1DC24	C-16	LEIBNIZIAN SYSTEM FIVE PLUS-MINUS AMBIGUOUS SIGN
⌘	38	1DC25	C-17	LEIBNIZIAN SYSTEM FIVE MINUS-PLUS AMBIGUOUS SIGN
⌘	39	1DC26	C-18	LEIBNIZIAN SYSTEM FIVE PLUS-PLUS-MINUS AMBIGUOUS SIGN
⌘	40	1DC27	C-19	LEIBNIZIAN SYSTEM FIVE PLUS-MINUS-PLUS AMBIGUOUS SIGN
⌘	41	1DC28	C-20	LEIBNIZIAN SYSTEM FIVE MINUS-PLUS-PLUS AMBIGUOUS SIGN
⌘	42	1DC29	C-21	LEIBNIZIAN SYSTEM FIVE PLUS-MINUS-MINUS AMBIGUOUS SIGN

	‡	43	1DC2A	C-22	LEIBNIZIAN SYSTEM FIVE MINUS-PLUS-MINUS AMBIGUOUS SIGN
	‡	44	1DC2B	C-23	LEIBNIZIAN SYSTEM FIVE MINUS-MINUS-PLUS AMBIGUOUS SIGN
	‡	45	1DC2C	C-24	LEIBNIZIAN SYSTEM FIVE PLUS-PLUS-PLUS-MINUS AMBIGUOUS SIGN
	‡	46	1DC2D	C-25	LEIBNIZIAN SYSTEM FIVE PLUS-PLUS-MINUS-PLUS AMBIGUOUS SIGN
	‡	47	1DC2E	C-26	LEIBNIZIAN SYSTEM FIVE PLUS-MINUS-PLUS-PLUS AMBIGUOUS SIGN
	‡	48	1DC2F	C-27	LEIBNIZIAN SYSTEM FIVE MINUS-PLUS-PLUS-PLUS AMBIGUOUS SIGN
	‡	49	1DC30	C-28	LEIBNIZIAN SYSTEM FIVE PLUS-PLUS-MINUS-MINUS AMBIGUOUS SIGN
	‡	50	1DC31	C-29	LEIBNIZIAN SYSTEM FIVE MINUS-PLUS-PLUS-MINUS AMBIGUOUS SIGN
	‡	51	1DC32	C-30	LEIBNIZIAN SYSTEM FIVE MINUS-MINUS-PLUS-PLUS AMBIGUOUS SIGN
	‡	52	1DC33	C-31	LEIBNIZIAN SYSTEM FIVE MINUS-PLUS-MINUS-MINUS AMBIGUOUS SIGN
new	‡	53	1DC34	C-32	LEIBNIZIAN SYSTEM FIVE-FOUR MINUS OR PLUS-MINUS AMBIGUOUS SIGN
new	‡	54	1DC35	C-33	LEIBNIZIAN SYSTEM FIVE-FOUR MINUS OR MINUS-PLUS AMBIGUOUS SIGN

3.d Characters proposed for variation sequence encoding

Additionally, five characters have been identified which are semantically based on other characters, so they get suitably encoded as variation sequences:

Glyph	N°	ID	char. sequence	description
‡	55	B-13	1DC09 FE00	alternate form
‡	56	B-14	1DC0A FE00	alternate form
‡	57	B-15	1DC0D FE00	alternate form
‡	58	B-17	1DC12 FE00	alternate form
‡	59	B-18	1DC12 FE01	alternate form

Standardized Variation Sequences			
1DC09 B-02		1DC09	LEIBNIZIAN SYSTEM TWO PLUS OR PLUS-MINUS AMBIGUOUS SIGN
B-13		1DC09 FE00	alternate form
1DC0A B-03		1DC0A	LEIBNIZIAN SYSTEM TWO PLUS OR MINUS-PLUS AMBIGUOUS SIGN
B-14		1DC0A FE00	alternate form
1DC0D B-06		1DC0D	LEIBNIZIAN SYSTEM TWO NEGATED PLUS OR MINUS-PLUS AMBIGUOUS SIGN
B-15		1DC0D FE00	alternate form
1DC12 B-11		1DC12	LEIBNIZIAN SYSTEM TWO NEGATED PLUS OR MINUS-PLUS PARTIAL AMBIGUOUS SIGN
B-17		1DC12 FE00	alternate form 1
B-18		1DC12 FE01	alternate form 2

We propose the following encoding scheme:

1DC00

Leibnizian Ambiguous Signs

1DC3F

	1DC0	1DC1	1DC2	1DC3
0	 1DC00	 1DC10	 1DC20	 1DC30
1	 1DC01	 1DC11	 1DC21	 1DC31
2	 1DC02	 1DC12	 1DC22	 1DC32
3	 1DC03	 1DC13	 1DC23	 1DC33
4	 1DC04	 1DC14	 1DC24	 1DC34
5	 1DC05	 1DC15	 1DC25	 1DC35
6	 1DC06	 1DC16	 1DC26	
7	 1DC07	 1DC17	 1DC27	
8	 1DC08	 1DC18	 1DC28	
9	 1DC09	 1DC19	 1DC29	
A	 1DC0A	 1DC1A	 1DC2A	
B	 1DC0B	 1DC1B	 1DC2B	
C	 1DC0C	 1DC1C	 1DC2C	
D	 1DC0D	 1DC1D	 1DC2D	
E	 1DC0E	 1DC1E	 1DC2E	
F	 1DC0F	 1DC1F	 1DC2F	

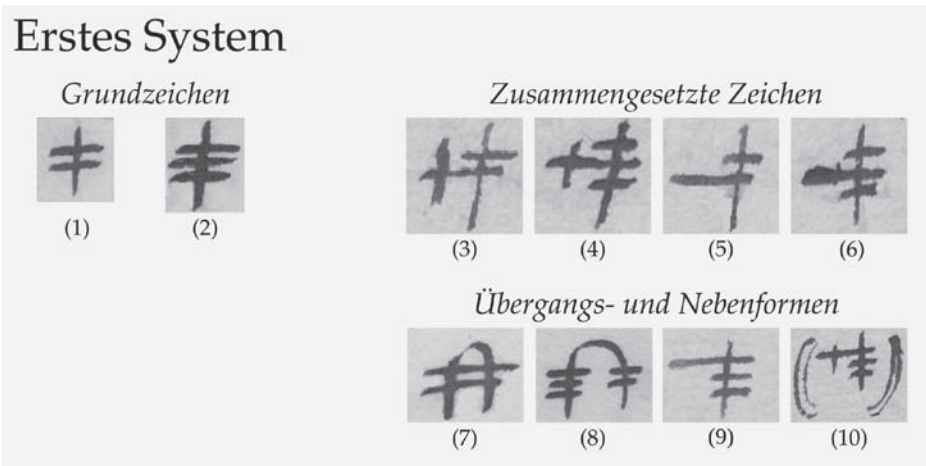
for reference:

			
A-01	B-09	C-12	C-28
			
A-02	B-10	C-13	C-29
			
A-03	B-11	C-14	C-30
			
A-04	B-12	C-15	C-31
			
A-05	B-16	C-16	C-32
			
A-06	C-01	C-17	C-33
			
A-07	C-02	C-18	
			
A-08	C-03	C-19	
			
B-01	C-04	C-20	
			
B-02	C-05	C-21	
			
B-03	C-06	C-22	
			
B-04	C-07	C-23	
			
B-05	C-08	C-24	
			
B-06	C-09	C-25	
			
B-07	C-10	C-26	
			
B-08	C-11	C-27	

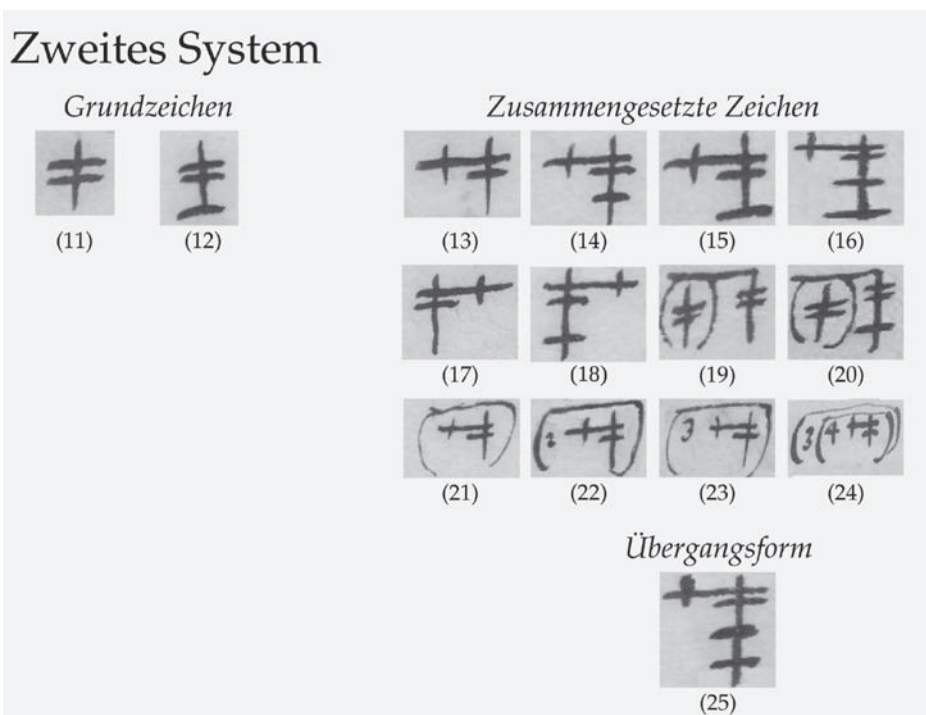
The yet unoccupied positions would be a strategic reserve for future additions. As the recent discovery of two more signs demonstrated, it is likely that even more signs will be found in the future. In that case the option of addition to this very block would be the favourable solution.

Achim Trunk (GWLH Hanover) described six different systems, invented by Leibniz. System 3 deploys the same characters as system 2, mostly. The fourth system employs Greek letters and the sixth system uses ordinary numbers, so basically three systems remain (1., 2. and 5.) which consist of special graphic symbols.

4. The ambiguous signs in samples from Leibniz's manuscripts



First system (*basic signs, compound signs, transitional and secondary forms*)



Second system (*basic signs, compound signs, transitional form*)

Drittes System

Grundzeichen

(11) (12)

Zusammengesetzte Zeichen

(26) (27) (28) (29)

(30) (31) (32) (33)

(34) (35)

Third system (*basic signs, compound signs*)

Fünftes System

Grundzeichen

(55) (56)

Zusammengesetzte Zeichen

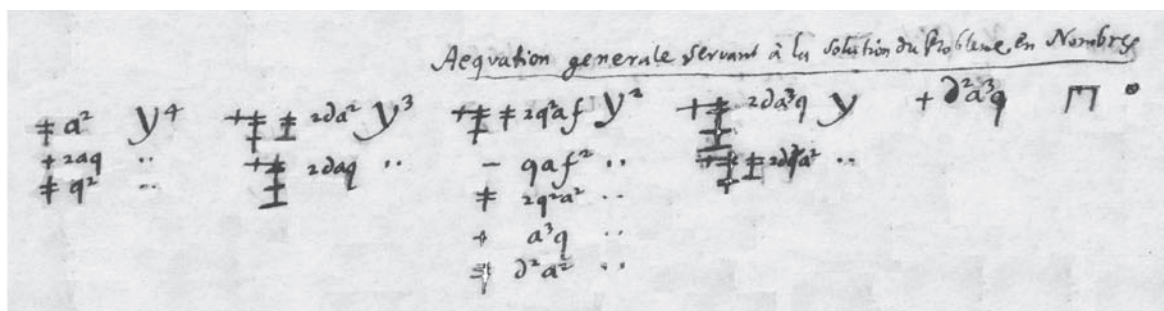
(57) (58) (59) (60)

(61) (62) (63) (64)

(65) (66) (67) (68)

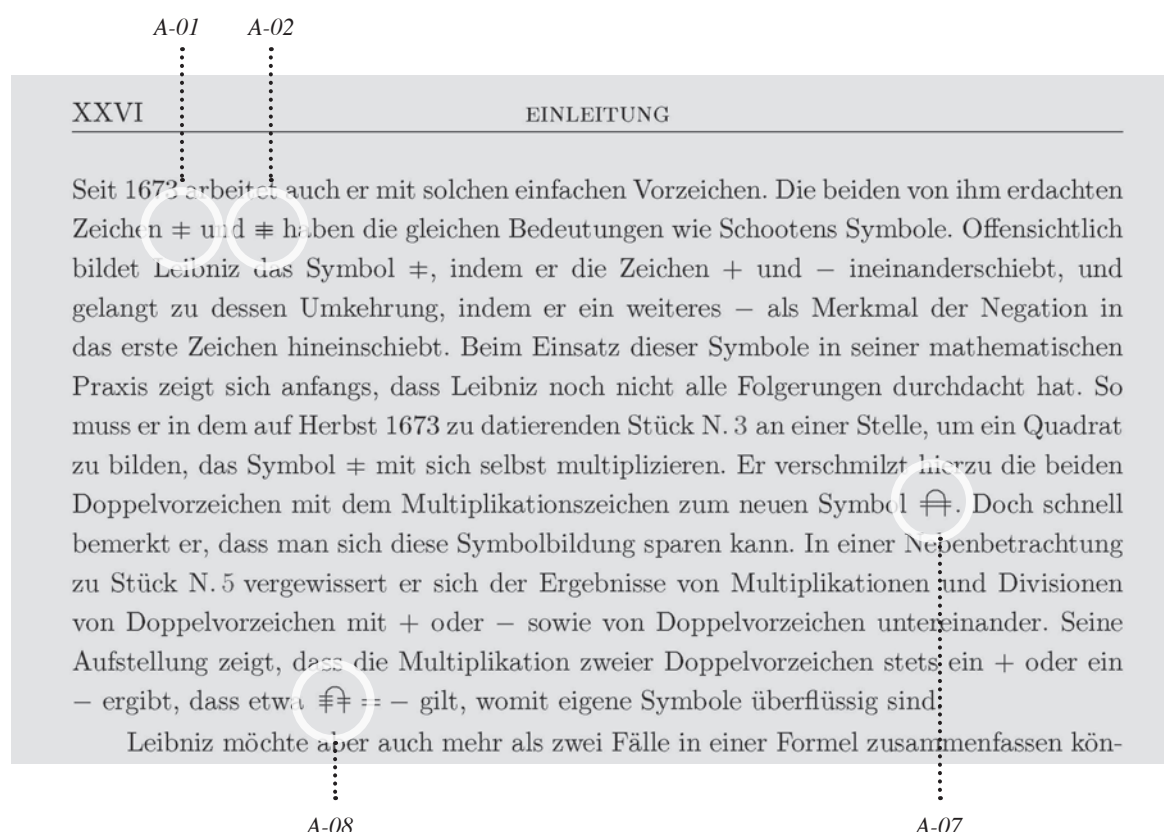
(69) (70) (71)

Fifth system (*basic signs, compound signs*)



A notation of an algebraic problem by Leibniz, using symbols of the 2nd system. (after A. Trunk)

5. Figures and explanations



Example of ambiguous signs, 1st system. LAA VII-7, p. XXVI

$$[2] xq \oplus 2x^2 \wedge \frac{x-f}{q \oplus 2x}.$$

$$d^2 \wedge a \oplus \frac{a}{q} x^2 = a^2 x^2 \oplus \frac{2a^2 x^2}{q} + 4x^2 q^2 \oplus 4x^2 q \wedge \frac{x^2 + f^2 - 2xf}{q^2 \oplus 4x^2 \oplus 4qx}$$

$$+ 2ax^2 q \oplus 3ax^2 \oplus \frac{a}{q} x^3$$

$$x \wedge x^2 + f^2 - 2xf \frac{+3xq^2}{\mathcal{D}} \wedge \frac{\odot}{4}$$

A-01 A-02 A-08 A-07

$\oplus$ AMBIGUITY SIGN A-07;

Example of ambiguous sign A-07, 1st system. LAA VII-7, p. 14



Leibniz-Akademie-Ausgabe (LAA, general edition of Leibniz's writings)
 LAA series VII (mathematical manuscripts, volumes 3 to 7 available online)
 LAA series VII volume 3
 LAA series VII volume 5
 LAA series VII volume 7

15

$$\left. \begin{array}{rrrr} +4g & +8ag & +4ag^2 & -2c^2g^2 \\ & -2c^2 & -4ae^2 & -2c^2e^2 \\ & +6g^2 & -4gc^2 & +g^4 \\ & +2e^2 & +4g^3 & +2g^2e^2 \\ & & +4ge^2 & +e^4 \end{array} \right\} = 0$$

Examinato ergo Canone, per exempla circuli, et parabolae, pergem(us) (cum) Calculo generali. Habuimus paulo ante valorem ipsius g . indagemus eum adhuc semel ope terminorum tertiorum, collatorum, seu ope multiplicantium secundae dimensionis incognitos. Fiet

$$20 \quad \frac{+2\frac{a}{q}g^2(\mp \mp)8ag\frac{+16a^{[2]}}{+2\frac{a}{q}+6} \wedge a^2}{+6} = \frac{+2\frac{a}{q}e^2 \mp 2\frac{a}{q}c^2 - 2e^2 + 2c^2 - 4a^2 \wedge \frac{a^2}{h^2}, \wedge \frac{a^2}{h^2}, \wedge \frac{a^2}{h^2}}{((\mp \mp \mp)) \mp 2q^2f - qf^2 \mp 2q^2a + a^2q \mp d^2a} = \frac{\mp a + 2q \mp \frac{q^2}{a}}{h^2}$$

$$20 \quad \text{Kontrollansatz zur quadratischen Ergänzung: } \sqrt{\mp 2\frac{a}{q} + 6} g \wedge \frac{(\mp \mp)4a}{\sqrt{\mp 2\frac{a}{q} + 6}}$$

15 f. } = 0 (1) Ponendo jam $x^2 = z^2 \frac{h}{a}$ (2) Examinato L

20 ... = ... : Die Koeffizienten, die Leibniz vergleicht, bezieht er wie oben aus den Gleichungen in N. 5 S. 35 sowie auf S. 48 Z. 3–12. Erneut vergisst er den Faktor $\frac{a^2}{q^2} \mp 2\frac{a}{q} + 1$. Zudem nimmt er die

⊞ AMBIGUITY SIGN B-04; a character belonging to the 2nd system. LAA VII-7, p. 52

N. 7 AEQUATIO EX INTERSECTIONE ORIENS, Ende Dezember 1673 – Juni 1674 53

seu extracta utrobique Radice

$$\frac{g\sqrt{\mp 2\frac{a}{q}+6}(\mp \mp)\frac{4a}{\sqrt{\mp 2\frac{a}{q}+6}}}{\frac{h}{a}} = \sqrt{\dots\dots\dots} \text{ sive}$$

$$g = \sqrt{\frac{\mp 2\frac{a}{q}e^2 \mp 2\frac{a}{q}c^2 - 2e^2 + 2c^2 - 4a^2 \wedge ((\mp \mp \mp)) \mp 2q^2f - qf^2 \mp 2q^2a + a^2q \mp d^2a \wedge \frac{h^2}{a^2}}{[\mp 2\frac{a}{q} + 6] \delta}} \wedge \frac{4 \wedge h}{\mp 2\frac{a}{q} + 6} \delta$$

Unde evanescit incognita g . valore ejus jam aliter supra dato. Ubi erat:

5

$$g = \frac{\frac{((\mp \mp)) \mp a((\mp \mp))q, \wedge 2d \wedge \frac{h^2}{a^2}(\theta) \wedge h^2 \mp 4\frac{a^2}{q} - 4a}{\mp a + 2q \mp \frac{q^2}{a}}}{\mp (\mp \mp) 4\frac{a}{q}(\mp \mp) 4}$$

Atque ita novam habemus aequationem inter hos duos valores, cujus aequationis ope

⊞ AMBIGUITY SIGN B-04; LAA VII-7, p. 53

er in der kurzen Notiz N. 8, die er vielleicht noch im Dezember 1673, vielleicht auch erst im Mai 1674 niederschreibt. Hier erläutert er vier neue Doppelvorzeichen, mit deren Hilfe sich jeweils drei Fälle unterscheiden lassen: das Symbol $+‡$, welches für „+ oder $\neq$ “ (sprich: „im einen Fall +, im anderen entweder + oder $-$ “) steht, $-‡$ als sein Gegenteil sowie die auf gleiche Weise durch Zusammenschieben eines $+‡$ oder $-‡$ mit einem einfachen Doppelvorzeichen gebildeten Symbole $+‡‡$ und $-‡‡$. Zusammen mit den beiden Grundzeichen bilden diese vier zusammengesetzten Doppelvorzeichen (oder *signes composés*, wie Leibniz solche Zeichen später nennt) ein erstes System aus einfachen und komplexen *signa ambigua*. Ein praktischer Einsatz der zusammengesetzten Zeichen dieses ersten Systems ist allerdings nicht bekannt. Zwar verwendet er in N. 7, das sich auf demselben Papierbogen wie N. 8 findet, tatsächlich zusammengesetzte Vorzeichen — womöglich zum ersten Mal überhaupt in seiner mathematischen Praxis (ein anderer Kandidat hierfür ist eine Nebenbetrachtung in N. 5). Und als deren Bausteine fungieren die einfachen Zeichen $\neq$ und $\neq$, die Grundzeichen des ersten Systems also. Die komplexen Zeichen werden jedoch nach geringfügig anderen Regeln gebildet, welche Leibniz erst

A-01

A-02

A-04

A-05

A-03

A-06

Example of ambiguous signs, 1st system. Introduction to LAA series VII volume 7, p. XXVI

A-02

B-13

B-01

Das zweite System, welches Leibniz in N. 10 darstellt, übernimmt zunächst die einfachen Doppelvorzeichen $\neq$ und $\neq$ aus dem ersten System und wendet für die Bildung zusammengesetzter Symbole wie $+‡$ aus $+$ und $\neq$ nur geringfügig abgewandelte Regeln an. Noch während der Arbeit am Konzept ersetzt Leibniz jedoch das negierte einfache Zeichen $\neq$ durch ein neues Zeichen, $\neq$, das sich aus dem Symbol $\neq$ ergibt, indem man an seinen Fuß einen (meist etwas länger gezogenen) Querbalken anfügt. Bereits in N. 7 negiert er zusammengesetzte Zeichen auf diese Weise; in der *Méthode* erhebt er sie zum allgemeinen Bildungsprinzip negierter Zeichen. Um aber das Zeichen $+‡$, die Negation von $+‡$, von dem aus $+$ und $\neq$ zusammengesetzten Doppelvorzeichen zu unterscheiden, wird bei letzterem der Längsstrich über den unteren Querbalken hinaus verlängert, so dass das Zeichen $+‡$ entsteht. Dessen Negation wiederum ist $+‡$. Dieses Symbol kann seinerseits zum Bestandteil eines noch weiter zusammengesetzten Zeichens werden; dies deutet Leibniz an, indem er den Längsbalken erneut verlängert und so den Baustein $+‡$ erzeugt. Ein entsprechendes Symbol schreibt er jedoch nicht einmal beispielshalber auf.

B-13

B-14

B-01

B-15

B-05

B-11

Example of ambiguous signs, 1st and 2nd system. LAA VII-7, p. XXVIII

sich aus $+$ und $\neq$ zusammen und bedeutet „im einen Fall +, im anderen Fall entweder + oder $-$ “. In seiner Praxis setzt Leibniz die zusammengesetzten Zeichen (*signes composés*) des ersten Systems allerdings niemals ein. Das Beispiel:

$+ \text{ vel } \neq \text{ esto } +‡$, et ejus contrarium seu $- \text{ vel } \neq \text{ erit } -‡$ et $- \text{ vel } \neq \text{ erit } -‡$ et ejus contrarium erit $+‡$. (N. 8)

A-04

A-03

A-02

A-06

A-01

A-05

Example of ambiguous signs, 1st system. LAA VII-7, p. XLI

hierfür hält er in der *Méthode de l'universalité* I (N. 10), verfasst wohl im Mai oder Juni 1674, fest. Aus + und + etwa bildet er das Symbol $\ddagger$, welches in Worten ausgedrückt bedeutet: „im einen Fall +, im anderen Fall entweder + oder –“. Auch hier gibt es also zwei Hierarchieebenen. Ist die Reihenfolge der beiden Fälle vertauscht, schreibt Leibniz dies als $\ddagger$. Das Symbol $\ddagger$ dagegen stellt die Negation von $\ddagger$ dar, bedeutet also „immer dann –, wenn $\ddagger$ für + steht, und immer dann +, wenn jenes Zeichen für –

B-08

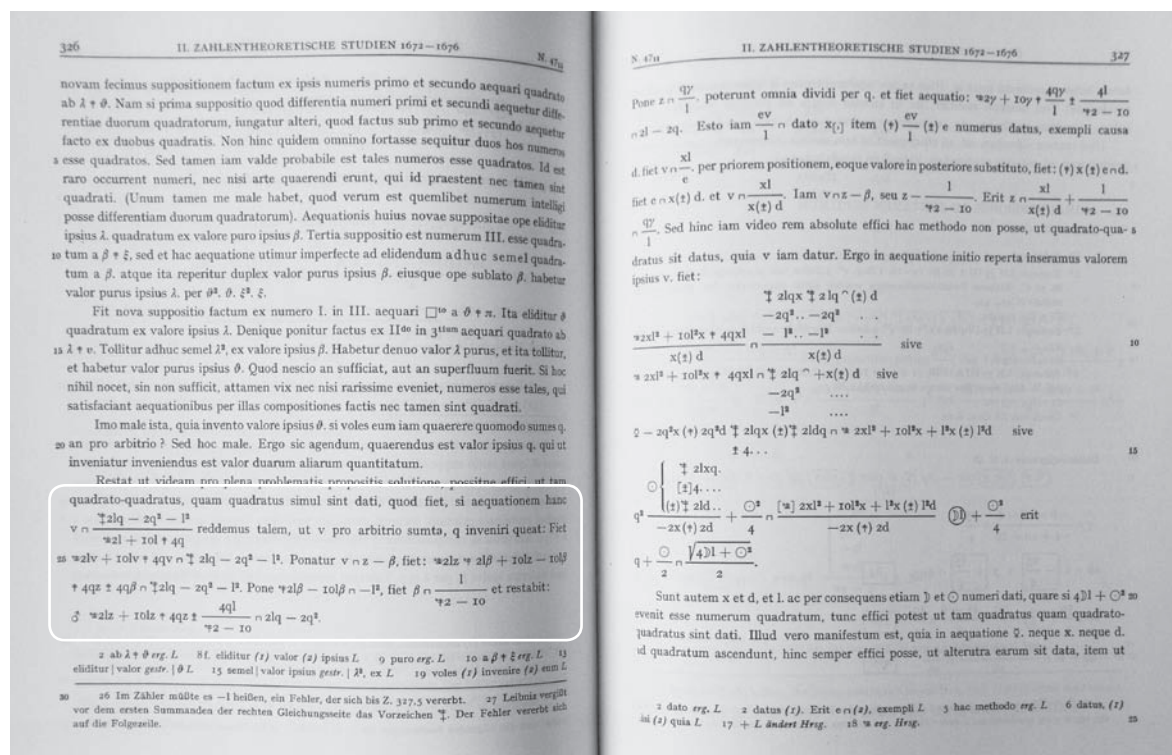
B-13

B-05

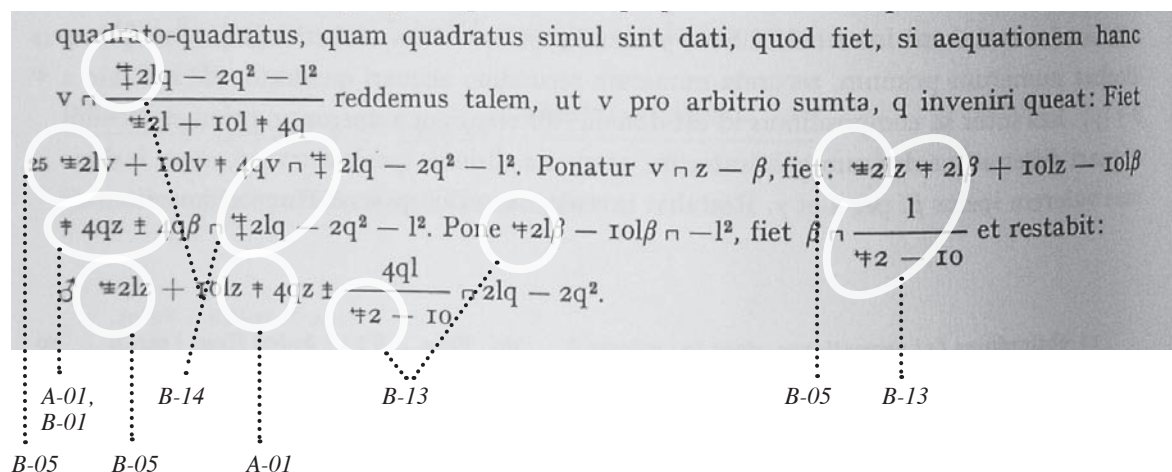
B-13

B-13

Example of ambiguous signs, 2nd system. LAA VII-7, p. XLI



Example of ambiguous signs, 2nd system. LAA VII-1, p. 326–327. See following figure for details.



A-01,
B-01

B-14

B-13

B-05

B-13

B-05 B-05 A-01

B-14 B-14 B-05 B-01

$$\begin{aligned} & \text{Sive} \\ & \left\{ \begin{array}{l} \text{2lxq.} \\ [\pm]4 \dots \end{array} \right. \\ & q^2 \frac{(\pm) 2ld \dots}{-2x(\pm) 2d} + \frac{\odot^2}{4} \frac{[\pm] 2xl^2 + 10l^2x + l^2x(\pm) l^2d}{-2x(\pm) 2d} \odot 1 + \frac{\odot^2}{4} \text{erit} \\ & q + \frac{\odot}{2} \cap \frac{\sqrt{4\odot 1 + \odot^2}}{2}. \end{aligned}$$

Sunt autem x et d, et l; ac per consequens etiam $\odot$ et $\odot$ numeri dati, quare si $4\odot 1 + \odot^2$ evenit esse numerum quadratum, tunc effici potest ut tam quadratus quam quadrato-quadratus sint dati. Illud vero manifestum est, quia in aequatione $\odot$. neque x. neque d. id quadratum ascendunt, hinc semper effici posse, ut alterutra earum sit data, item ut

B-05

B-01

B-14 B-13 B-14

$$\begin{aligned} & [L^3] \\ & e^2l \pm 2e^2v \mp 2e^2q (+ e^2v) (- f^2) \cap (\mp e^2v) (\mp e^2q) (+ e^2v) - g^2(\pm) 2gf (- f^2). \\ & \text{Ergo } f \cap \frac{e^2l \pm 2e^2v \mp e^2q + g^2}{(\pm) 2gl}. \text{ Pro } e^2, \text{ substitue potius } \frac{g^2h^2}{l^2}, \text{ fiet } f \cap \frac{h^2l \pm 2h^2v \mp h^2q + l^3}{(\pm) l^3} \\ & \text{sive } f \cap \frac{h^2r + l^3}{(\pm) l^3} g. \text{ Ergo } f^2 \cap \frac{h^4r^2 + 2h^2l^3r + l^6}{+l^6} g^2 \text{ fiet } \frac{l^3h^2r - h^4r^2 - 2h^2l^3r - l^6}{l^6} g^2 \\ & [Text bricht ab] \\ & \begin{array}{c} 9 \quad 6 \quad 2 \\ \pm \textcircled{2} l^2 + \textcircled{10} l^2 \mp \textcircled{4} ql, (\mp 2lq) \mp 10lq \pm \textcircled{4} q^2, - 2lq (\mp 2q^2) (\mp l^2), \end{array} \\ & \left\{ \begin{array}{l} \mp 6l \\ \mp 10l \\ - 2l \\ \mp 2l \end{array} \right. \\ & \mp 2lq - 2q^2 (- l^2) \cap 0 \quad \text{sive } q^2 \frac{\mp 2 - 2}{\mp 2 - 2} q + 36l^2 \\ & \text{II } \frac{e^2l \pm 2e^2v \mp e^2q + g^2}{(\pm) 2gl}. (I) \text{ fiet } (a) \text{ item } e^2v^2 (b) \text{ ergo } f (2) \text{ Porro } e^2 (3) \text{ Pro } L \end{aligned}$$

B-15

B-13

Example of ambiguous signs, 2nd system. LAA VII-1, p. 327 (top), 329

Soit maintenant une certaine grandeur affectée du signe $\neq$ par exemple $\neq a$, c'est à dire : $o \neq a$. car puisque $+$ aussi bien que $-$ signifie une Relation entre deux, et qu'il n'y a qu'une seule grandeur a , l'autre sera o ou rien : supposons donc que la dite grandeur $\neq a$ doit estre adjoutée à une autre b , le produit sera $b + \neq a <$ ou b plus $\neq a >$ c'est à dire $b \neq a$, car le signe $+$ ne change point les autres signes : mais à present supposons que la dite grandeur $\neq a$ doit estre soustraite d'une autre b , le produit sera $b - \neq a$, ou b moins $\neq a$, et | par ce que cela arrive bien souvent, je trouve à propos d'employer un seul signe, $\pm$ au lieu de ces deux $-$ et $\neq$ joints ensemble, et le produit susdit sera $b \pm a$, et $\pm$ vaudra $- \neq$ et generally j'observeray cette regle, qu'un signe ambigu insistant sur un $-$ aura une signification contraire à celle qu'il auroit sans cela, ou que le signe avec le $-$ $<$ au bas du caractere $>$ signifie moins le $<$ même $>$ signe sans $-$. Par exemple $\pm$ (que nous expliquerons cy après :) signifiera $-- \neq$. Par consequent si dans une meme formule ou Equation ces deux signes opposés se trouvent à la fois, comme par exemple $\neq a \pm b \sqcap c$, et que cette formule vienne à estre expliquée ou appliquée à un certain cas particulier, ou $\neq$ signifie par exemple $+$, alors $\pm$ s'expliquera aussi et signifiera $-$, et si $\neq$ signifie $-$ dans le cas particulier dont nous avons besoin, $\pm$ signifiera $+$

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B-15

Example of ambiguous signs, 2nd system.
Couturat 1903 (1961), p. 126

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DE LA MÉTHODE DE L'UNIVERSALITÉ II, Juni 1674

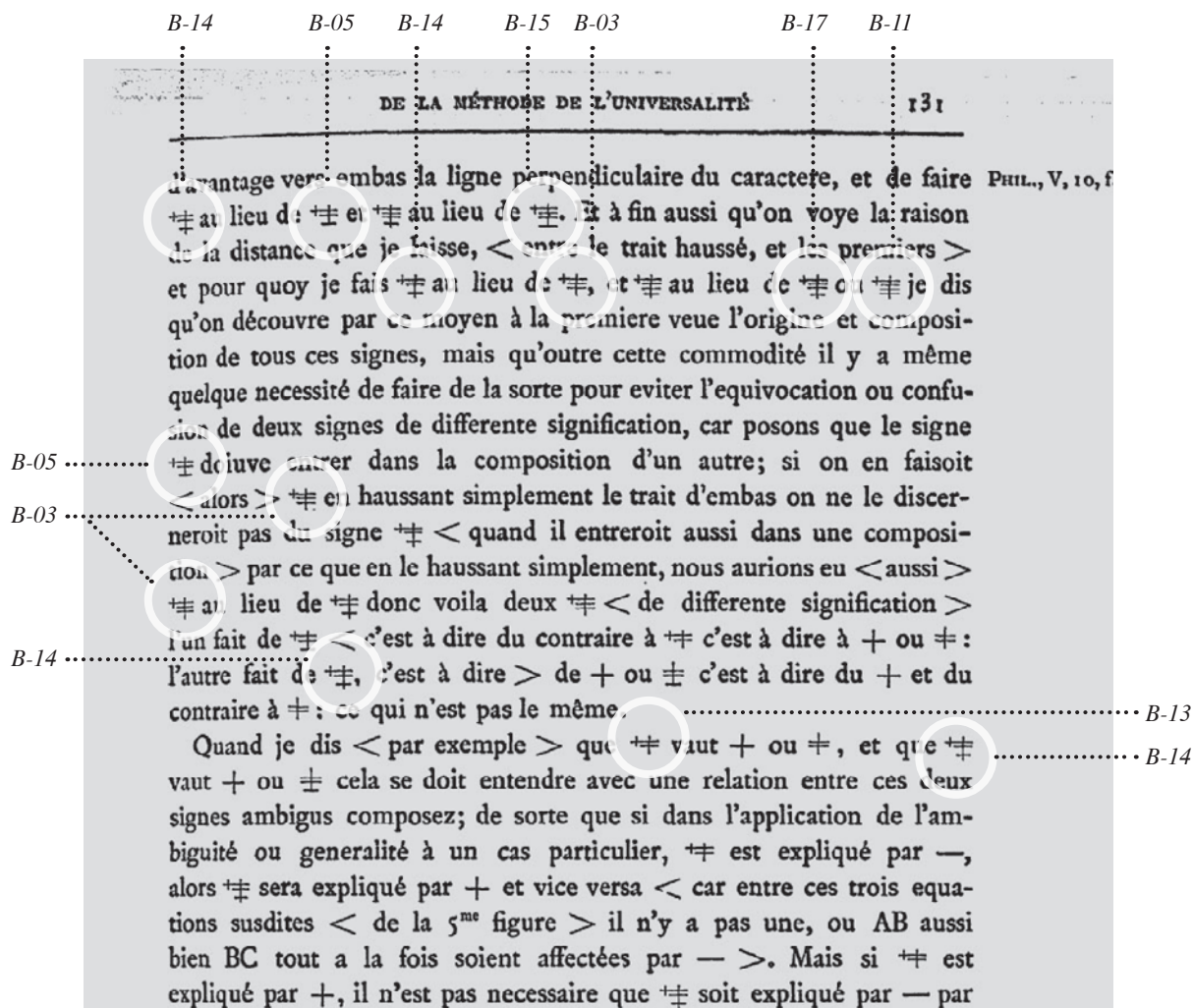
N. 11

fait voir que ces deux signes ambigus $\neq$ et $\pm$ signifient ou tous deux $+$, ou que l'un signifiant $\neq$, l'autre signifie $\pm$, je les exprime en mettant $+$ au devant, en tous deux $\neq$ et $\pm$, au lieu de $\neq$ et $\pm$ dont nous aurons besoin dans une autre rencontre.

On voit en fin par là; la grande difference qu'il y a entre le signe $\neq$, et tous les autres.

- 5 Car le signe simple $\neq$ peut subsister tout seul, sans changement, par ce qu'il ne dit point de relation à aucun autre; mais tous les autres contiennent quelque relation à un autre signe provenant d'une meme equation ambiguë, et pour cela je les appelle Correspondants. Par exemple si nous avons deux signes ambigus simples, $\neq$ et $\pm$ provenans de l'equation $\neq a \pm y \sqcap b$, et si dans la suite du calcul le signe $\neq$ evanouit, comme il arrive en cet exemple,
- 10 ou nous trouvons en fin cette equation, $y \sqcap \pm b + a$, alors si nous nous determinons à abandonner entierement la premiere equation, avec tout ce qui en est provenu, hormis cette nouvelle trouvée, dont nous pretendons nous servir à l'avenir dans le calcul qui reste à faire; nous pourrons sans scrupule changer le signe $\pm$ en $\neq$, et nous servir de cette

Example of ambiguous sign B-16, 2nd system. LAA VII-7, p. 126



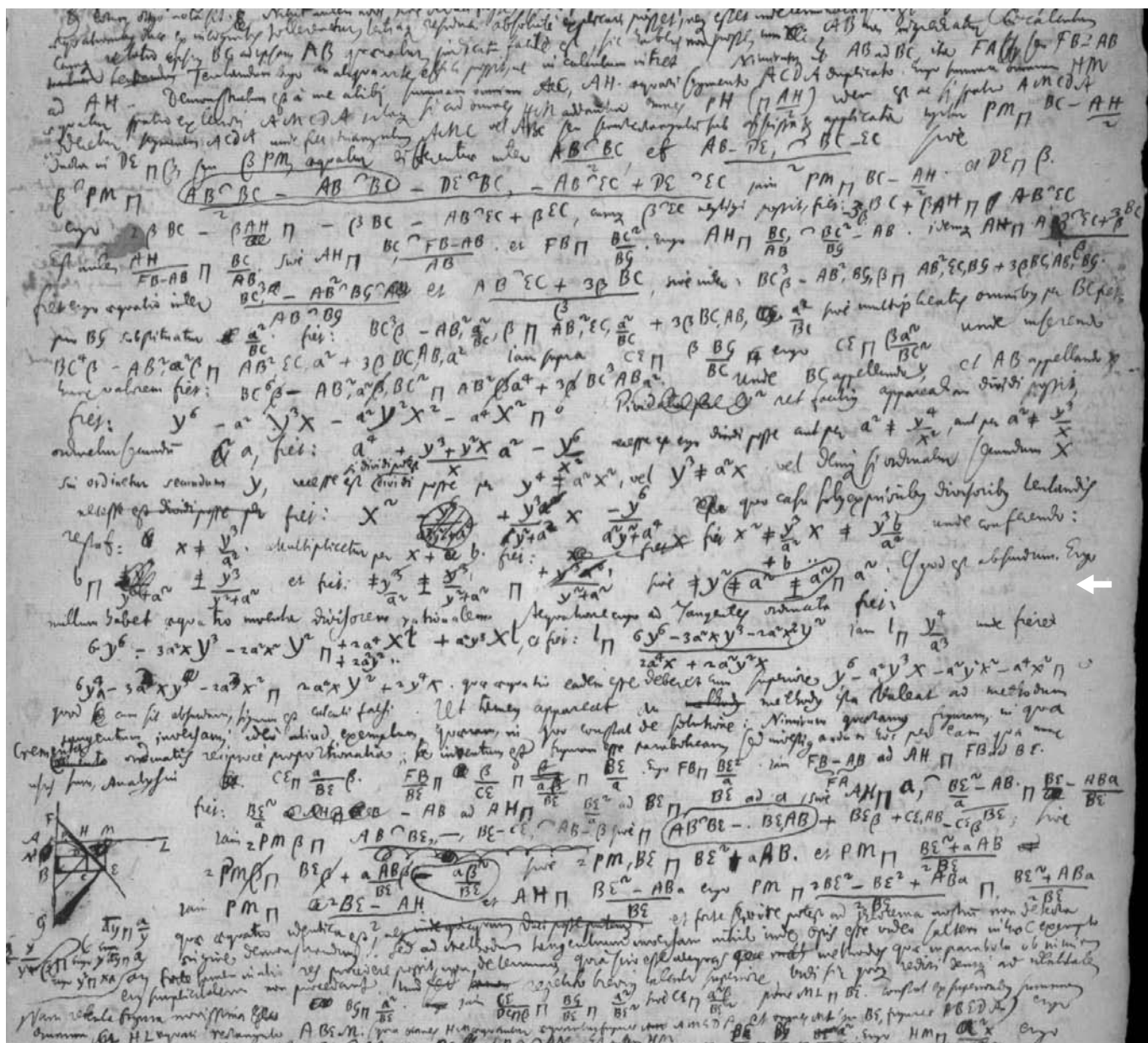
Example of ambiguous signs, 2nd system.
 Couturat 1903 (1961), p. 131

A-01

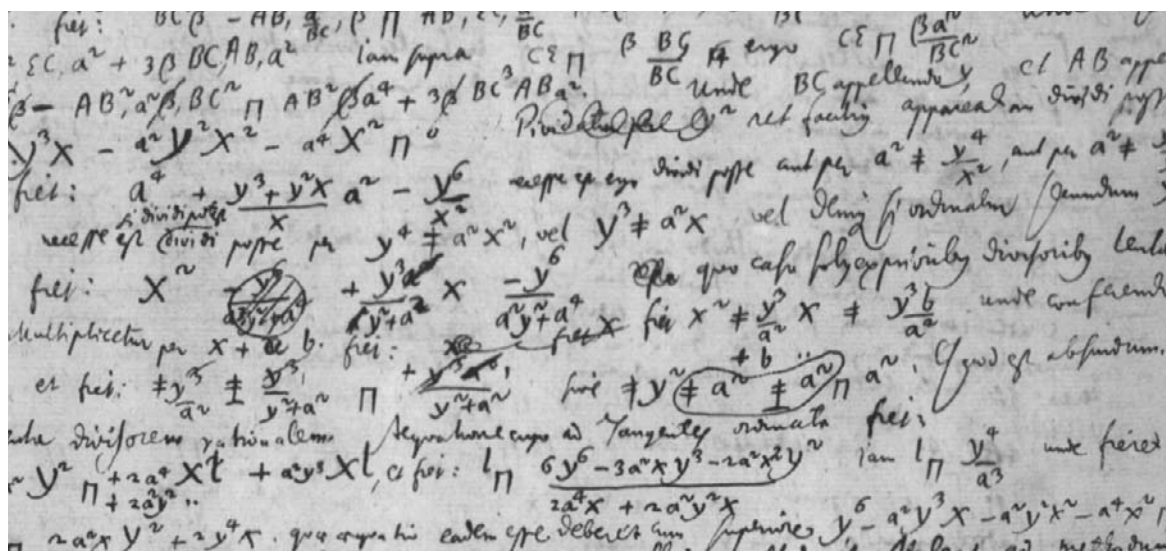
Necesse est ergo dividi posse aut per $a^2 \mp \frac{y^4}{x^2}$, aut per $a^2 \mp \frac{y^6}{x}$. Sin ordinetur secundum
 y , necesse est si dividi potest dividi posse per $y^4 \mp a^2 x^2$, vel $y^3 \mp a^2 x$ vel denique si ordina-
 tur secundum x , fiet: $x^2 \frac{+y^3 a^2}{a^2 y^2 + a^4} x \frac{-y^6}{a^2 y^2 + a^4}$ quo casu solus ex prioribus divisoribus
 tentandis restat: $x \mp \frac{y^3}{a^2}$. Multiplicetur per $x + b$. fiet: $x^2 \mp \frac{y^3}{a^2} x \mp \frac{y^3 b}{a^2}$. Unde conferendo:
 $b \mp \frac{y^3}{y^2 + a^2}$ et fiet: $\frac{\mp y^3}{a^2} \mp \frac{y^3}{y^2 + a^2} \mp \frac{y^3}{y^2 + a^2}$, sive $\mp y^3 \left(\frac{\mp a^2 \mp a^2}{y^2 + a^2} \right) \mp a^2$. Quod est
 absurdum. Ergo: nullum habet aequatio inventa divisorem rationalem. Aequatione ergo
 ad tangentes ordinata fiet:
 $6y^6 - 3a^2 xy^3 - 2a^2 x^2 y^2 \mp + 2a^4 xl + a^2 y^3 xl$, et fiet: $l \mp \frac{6y^6 - 3a^2 xy^3 - 2a^2 x^2 y^2}{2a^4 x + 2a^2 y^2 x}$.

B-01
B-01

Ambiguous signs, 2nd system. LAA VII-3, p. 567
 see also LH 35 V 2, f. 4v (next page)



Manuscript by G.W.Leibniz, LH 35 V 2, f. 4v



→
see
next
page

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DE LA MÉTHODE DE L'UNIVERSALITÉ II, Juni 1674
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au lieu de †‡ ; et †‡ au lieu de †‡ . Et à fin aussi qu'on voye la raison de la distance que je laisse entre le trait haussé, et les premiers, et pour quoy je fais †‡ au lieu de †‡ , et †‡ au lieu de †‡ ou †‡ je dis qu'on découvre par ce moyen à la première vue l'origine et composition de tous ces signes, mais qu'outre cette commodité il y a même quelque nécessité de faire de la sorte, pour éviter l'équivocation, ou confusion de deux signes de différente signification, car posons que le signe †‡ doive entrer dans la composition d'un autre, si on faisoit alors †‡ en haussant simplement le trait d'embas on ne le

B-11 B-17 B-18 B-05

ns une composition par ce que en le haussant simplement, nous aurions eu aussi †‡ au lieu de †‡ donc voila deux †‡ de différente signification l'un fait de †‡ , c'est à dire du contraire à †‡ c'est à dire à + ou †‡ : l'autre fait de †‡ , c'est a dire de + ou †‡ c'est à dire du + et du contraire à †‡ : ce qui n'est pas le même.

Quand je dis par exemple que †‡ vaut + ou †‡ , et que †‡ vaut + ou †‡ cela se doit entendre avec une relation entre ces deux signes ambigus composez; de sorte que si dans l'application de l'ambiguïté ou generalité à un cas particulier, †‡ est expliqué par —, alors †‡ sera expliqué par + et *vice versa* car entre ces trois equations susdites de la 5^{me} figure il n'y a pas une, ou AB aussi bien que BC , tout a la fois soient affectées par —. Mais si †‡ est expliqué par +, il n'est pas nécessaire que †‡ soit expliqué par — par ce que dans une de ces equations particulieres, AB , aussi bien que BC , sont affectées par +. Par consequent si l'un de ces deux signes composés est expliqué par + l'autre sera expliqué par †‡ et *vice versa* (: avec la caution pourtant, que nous y apporterons plus bas:) de sorte que l'ambiguïté decomposée qu'elle est, deviendra simple. Et par ce que la liste des Equations particulieres

$$\left. \begin{array}{l} AC \quad \cap \quad + \quad AB + \quad BC \\ - \quad \left. \right\} \text{†‡} \quad AB + \quad \left. \right\} \text{†‡} \quad BC \\ + \quad \left. \right\} \text{†‡} \quad AB - \quad \left. \right\} \text{†‡} \quad BC \end{array} \right\} \text{ qui peuvent estre entendues}$$

sous la Generale $\text{†‡} AB$ $\text{†‡} BC$,

B-12

2 entre ... premiers *erg.* L 3 de †‡ ou †‡ L ändert *Hrsg.* 8–10 signe †‡ | qvand ... composition *erg.* | par ce qve (1) si on haussoit le signe (2) en ... eu | aussi *erg.* | †‡ au ... deux †‡ | de differente signification *erg.* | l'un (a) faisoit de (aa) †‡ , l'autre de + ou †‡ (bb) †‡ , l'autre de †‡ , c'est à dire de + ou †‡ (b) fait de †‡ , c'est à dire (aa) de + ou †‡ (bb) du contraire L 13 (1) On voit par la, a (2) Qvand je dis | par exemple *erg.* | L 14f. dans (1) l'explication (2) l'application L 16f. car ... susdites | de la 5^{me} figure *erg.* | il ... bien | qve *erg.* *Hrsg.* | BC ... par — *erg.* L

Ambiguous signs, 2nd system. LAA VII-7, p. 125; *see also* LH 4 V 10, f. 31 (next page)

In this passage, Leibniz discusses how ambiguous signs of higher order should be derived from those of lower order. He suggests to slightly modify each ambiguity sign of order n (for example B-06) when it is used as a part of an ambiguity sign of order $n + 1$ (for example B-11 as a partial variant of B-06). Since the modification allows for freedom in design, Leibniz discusses different types as for example B-11, B-17, and B-18 for the partial ambiguity sign that corresponds to B-06. The lower part of the page shows the transition from ambiguity signs of degree 2 to those of degree 3.

[illegible]

LH 4 V 10, f. 31v-32r

prolonguer d'avantage vers embas la ligne
perpendiculaire du caractère, et de faire
++ au lieu de +; et ~~++~~ ++ au lieu de ++
Et à fin aussi qu'on voye la raison de la
distance que je laisse, et pour quoy je fais ++ entre le trait basillé et les premiers
++ au lieu de ++, et ++ au lieu de ++ ou ++
et je dis qu'on ne doit pas se déconner
par ce moyen à la première vue l'origine et composition
de tous ces signes, mais qu'on a cette commodité
il y a même quelque nécessité de faire de la sorte
pour éviter l'équivoque, ou confusion de deux
signes de différente signification, car posons
que le signe ++ doive entrer dans la composition
d'un autre, si on en faisoit ++ en baissant simplement
le trait d'embas on se le discerneroit pas.
du signe ++ quand il entrerait aussi dans une composition
par ce que on en baissant simplement
l'autre nous aurions ~~++~~ ++ au lieu de ++
donc, voilà deux ++ d'une même signification fait de
l'un ++ et de ++, c'est à dire de ++ ou ++; l'autre de ++ y ait
l'autre de ++ ou ++ c'est à dire du + et du
contraire à ++: ce qui n'est pas le même par exemple
Où voit par là, quand je dis que ++
vaut + ou ++, et que ++ vaut + ou ++
cela se doit entendre avec une relation entre de
ces deux signes ambigus composés, de sorte que
l'un n'ait pas la même signification de l'autre, ou

IV. Ambig.

$a \cap (\overline{3^+}) b (\overline{3^+}) c$ signifie $a \cap +b +c$, c'est à dire, $a \cap +b +c$
 $(\overline{3^+}) \left\{ \begin{matrix} +b \\ -b \end{matrix} \right. (\overline{3^+}) \left\{ \begin{matrix} -c \\ +c \end{matrix} \right.$ ou $(\overline{3^+}) b (\overline{3^+}) c$

a étant ou la somme, ou la difference de $b. c.$ cela fait voir clairement la raison de la
 fabrique des signes, et il faut remarquer seulement que de $+$ ou $(\overline{3^+})$, on a fait tout expres
 0 $(\overline{3^+})$ au lieu de $(\overline{3^+})$ par ce que $(\overline{3^+})$ signifie le signe opposé à $(\overline{3^+})$.

Si nous eussions eu

$$\begin{array}{rcl} f & \cap & +b +c \\ & (\overline{3^+}) \left\{ \begin{matrix} +b \\ -b \end{matrix} \right. & (\overline{3^+}) \left\{ \begin{matrix} -c \\ +c \end{matrix} \right. & (\overline{3^+}) \left\{ \begin{matrix} +e \\ -e \\ -e \end{matrix} \right. \end{array} \quad \text{cela auroit fait}$$

$$f \cap (\overline{3^+})b (\overline{3^+})c (\overline{3^+})e$$

LAA VII-7, p. 144

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traits, ou -, horsmis un qui se pourra placer ou l'on voudra, par exemple $\mp (\overline{3^+}) (\overline{2^+}) a$
 fait $\mp (\overline{3^+}) (\overline{2^+}) - a$ ou $\mp (\overline{3^+}) (\overline{2^+}) a$ et $\mp (\overline{3^+}) a$, fait $\mp (\overline{3^+}) a$.

Si les signes qui se multiplient, ou qui se divisent sont correspondants seulement:
 leur nature particuliere qui se reconnoit par la forme du Caractere, fera juger du produit.

5 Par exemple

$$\begin{array}{l} (\overline{2^+}) b \wedge (\overline{2^+}) a, \text{ fait } (\overline{2^+}) ab \\ (\overline{2^+}) a \wedge (\overline{2^+}) b, \text{ fait } (\overline{2^+}) ab \\ (\overline{2^+}) a \wedge (\overline{2^+}) b \wedge (\overline{2^+}) c, \text{ fait } +abc \\ (\overline{3^+}) a \wedge (\overline{3^+}) b, \text{ fait } (\overline{3^+}) ab \end{array}$$

10

Exemple d'une extraction d'une Racine Quarrée

Soit une Equation $2ax \mp \frac{a}{q}x^2, \cap y^2$, et la question est comment il faut exprimer la
 valeur de x , conformement à cette Equation, or je dis, que

$$+x \cap \mp \sqrt{\frac{aq^2 \mp y^2q}{a}} \pm q$$

dont voicy la preuve. En transposant nous aurons $+x \mp q \cap \mp \sqrt{\frac{aq^2 \mp y^2q}{a}}$, dont le quarré

15 sera, $+x^2 \mp 2xq + q^2 \cap \frac{aq^2 \mp y^2q}{a}$. Multipliant tout par a , ce sera: $+ax^2 \mp 2aqx + \cancel{aq^2} \cap$

$\cancel{aq^2} \mp y^2q$, ostons aq^2 de deux costez, et divisons tout, par $\mp q$, $\frac{+ax^2}{\mp q} \mp \frac{2aqx}{\mp q} \cap \frac{\mp y^2q}{\mp q}$, et
 nous aurons: $\mp \frac{a}{q}x^2 + 2ax \cap y^2$ qui est l'equation donnée.

La consideration de cette operation peut servir d'exemple à la plupart de nos pre-
 ceptes.

LAA VII-7, p. 146

Table des Signes de la Methode de l'Universalité.

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~~Table des Signes de la~~

~~Methode de l'Universalité.~~

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~~Methode de l'Universalité.~~

se trouve à propos de fermer par ententes
les parenthèses des signes, à fin de
les discerner d'autres parenthèses.

Signes Homogenes sont ou les mêmes, ou opposés
Signes Correspondants sont ou les mêmes, ou opposés
ou $\pm$ et $\mp$, ou $+$ et $-$

Signes ambigus sont ceux qui sont dans une
même ambiguïté, ou équation ambiguë
dans des cas particuliers, qui la composent
une même équation, comme $(\pm \mp)$ et $(\mp \pm)$

Les autres sont entièrement Heterogenes.

Un signe Homogene étant expliqué ou par l'applica-
tion de la formule générale, ou par l'appli-
cation d'un cas particulier, l'autre s'explique aussi
entièrement et toujours par le même.

Un signe correspondant est toujours expliqué, l'autre
parfois, pendant est toujours ambiguë, par
exemple, si $\pm \mp$ signifie $+$, alors
 $\mp \pm$ signifiera $-$; mais pas
ou $\pm \pm$ toujours avec entièrement, mais pas
quelques fois, comme dans le même exemple
 $\pm \mp$ signifiant $+$, on obtient $\mp \pm$ signifiant $-$

Addition et subtraction

Si les signes sont homogenes, on en peut faire
un seul par le moyen d'un vinculum.

$\pm a \pm b$ nous pouvons faire $\pm a - b$

Si une même grandeur se presente, alors
nous pouvons faire

$\pm 3y \pm y \pm 3y$ y, $\mp 3y \pm 1 \pm 3y$

Multiplication et Division

Deux signes Homogenes entièrement semblables font
un signe opposé.

$\pm a \pm b$ fait $-ab$

$\pm a, \pm a$ fait $+a^2$

$\pm a, \pm a$ fait $-a^2$

il n'y a point de différence entre une grandeur mul-
tipliée par un signe et divisée par
le même signe car

$\pm a, \pm a$ fait $\frac{a}{a}$

Un signe multiplié par + donne le même, par - donne l'opposé

c'est à dire, $\pm a \pm b \pm c$

ou $\pm a \pm b \pm c$

a étant ou la somme, ou la différence de b, c .

Cela fait voir clairement la raison de la fabrique
des signes, et il faut remarquer seulement
que de $+$ ou $(\pm \pm)$, on a fait tout expressément $(\pm \pm)$

au lieu de $(\pm \pm)$ parce que $(\pm \pm)$ signifie l'opposé de

le signe opposé à $(\pm \pm)$

Deux signes semblables mais différents par ce qu'ils
tirent leur origine de différentes ambiguïtés d'un
même calcul par seront distingués par des parenthèses

et s'il y en a beaucoup par des parenthèses marquées
de nombres. Comme s'il y avoit $a \pm b \pm c$,

$\pm$ et $\pm$ nous ferons $\pm (\pm) a (\pm) b (\pm) c$

et s'il y avoit deux une troisième, je la marquerai son $\pm$

ainsi : $\pm (\pm) a (\pm) b (\pm) c$

Si les signes qui se multiplient, ou qui se divisent
sont correspondants seulement: leur nature
particulière qui se reconnaît par la forme du
caractère, fera juger du produit. par exemple

$$(2 + \frac{1}{2})b \cdot (2 + \frac{1}{2})a, \text{ fait } (3 + \frac{1}{2})ab$$

$$(2 + \frac{1}{2})a \cdot (2 + \frac{1}{2})b \text{ fait } (3 + \frac{1}{2})ab$$

$$(2 + \frac{1}{2})a \cdot (2 + \frac{1}{2})b \cdot (2 + \frac{1}{2})c, \text{ fait } + abc.$$

$$(3 + \frac{1}{2})a \cdot (3 + \frac{1}{2})b, \text{ fait } (3 + \frac{1}{2})ab$$

Exemple d'une
d'une construction de Racine carrée

Soit une équation $2ax \pm \frac{a}{q}x^2 \pm y^2$, et la
question est comment il faut exprimer la valeur de x ,
conformément à cette équation, ou se dir, que

$$x \pm \frac{a}{q}x^2 \pm y^2 = 0$$

dont voici la preuve. En transposant nous aurons

$$x \pm \frac{a}{q}x^2 \pm \sqrt{aq^2 \pm y^2} = 0$$

dont le carré sera

$$x^2 \pm 2ax + \frac{a^2}{q^2}x^4 \pm \frac{a}{q}x^2 \pm y^2 = 0$$

Multipliant tout
par a , et divisant par a^2 , on a

$$ax^2 \pm 2ax + \frac{a}{q^2}x^4 \pm \frac{a}{q}x^2 \pm y^2 = 0$$

soient aq^2 de deux côtés, et divisons
tout, par $\pm q$, et nous aurons;

$$\frac{ax^2}{\pm q} \pm \frac{2ax}{\pm q} \pm \frac{ax^4}{q^2} \pm \frac{y^2}{\pm q} = 0$$

qui est l'équation donnée.

Considération de cette opération peut servir
d'exemple à la plupart de nos préceptes.

Opérations composées

Consistent dans la formation des Définitions, invention
des propriétés, et effectuation des Demandes.

Définitions de Equations simples et des Figures et
Equations propres à exprimer la relation de tous les
points des courbes, à une certaine droite ne sont
qu'une même chose. or trouver des définitions communes
à plusieurs figures, c'est les réduire en Harmonie
pour trouver des théorèmes, et des constructions com-
munes de quel problème propose.

Exemple de la Réduction des Figures
différentes en Harmonie, essayé dans
les Coniques.

Exemple d'une propriété commune
à toutes les Coniques, démontrée
généralement par la méthode de
l'universalité.

DE LA METHODE
DE L'UNIVERSALITE

Exemple d'un problème des plus
difficiles proposé dans les Coniques,
et résolu par deux constructions
universelles, l'une par le moyen
de la Conique donnée et de l'Hyperbole,
l'autre par le moyen de la Conique
donnée et du Cercle.

15 Hoc ut fiat, eam ita aequipollenter scribamus: $\vartheta^2 v^2 + \beta^2 v^2 + 2\vartheta\beta v^2 + 2\vartheta\beta v^2 \overset{(46)}{\cap} \vartheta^2 \beta^2 + 1$.
 Haec enim ipsi 45. coincidit. Eam porro ita divellemus in duas: $\vartheta^2 v^2 + \beta^2 v^2 + 2\vartheta\beta v^2 \overset{(47)}{\cap} 1$.

13f. Hinc exitus patet, nam scribentur: $\vartheta^2 v^2 - 1$ aequi. $\beta^2 \vartheta^2 - \beta^2 v^2$ et fiat 1, $\vartheta v + 1$ aequi.

$\vartheta (+)$ ergo $\vartheta v + 1$ aequi. $1\beta^2, \vartheta (+) v$. Ergo $2\vartheta v$ aequi. $\frac{\vartheta (+) v}{1}, +1\beta^2, \vartheta (+) v$. Pone ϑ aequi. fv .

fiet $2fv$ aequi. $\frac{f (+) v}{1}, +1\beta^2, \vartheta (+) v + 1$ adeoque datur v . et solutum est: problema universa-

C-16 C-17 C-16 C-17

Ambiguous signs, 5th system. LAA VII-1, p. 618

C-12 C-16 C-10 C-11 C-08

et $+ 2\vartheta\beta v^2 \overset{(48)}{\cap} \vartheta^2 \beta^2$. ex. 47. extrahendo fiet: $\vartheta \vartheta + \beta v \overset{(49)}{\cap} (+) 1$. Ex 48. dividendo fiet: $+ 2v^2$
 $\overset{(50)}{\cap} \vartheta\beta$. Ex 49. erit $\vartheta \overset{(50)}{\cap} \frac{(+)}{v} \vartheta 1 + \beta v$. et ex 50, erit $\vartheta \overset{(52)}{\cap} \frac{+ 2v^2}{\beta}$, quibus duobus ipsius ϑ valo-
 ribus aequatis, fiet: $(+) \vartheta \beta + \beta^2 v \overset{(53)}{\cap} 2v^3$. sive signo $(+)$ compendiose per novum $+$ ex-
 presso, fiet $+ f + \beta^2 v \overset{(54)}{\cap} 2v^3$. Sit $v \overset{(55)}{\cap} \gamma\beta$. fiet ex 54. haec: $+ 1 + \beta^2 \gamma \overset{(56)}{\cap} 2\beta^2 \gamma^3$ sive β^2
 $\overset{(57)}{\cap} \frac{+ 1}{2\gamma^3 - \gamma}$. Iamque nova suppositione faciendo $\gamma \overset{(58)}{\cap} + 1$ fiet $\beta \overset{(59)}{\cap} ((+)) 1$. 5

Ac proinde nisi forte in nihilo minores ita incidatur, erit problemati, particulariter quidem, satisfactum tamen. Et supererunt quatuor minimum casus, ob explicationes signorum $+$ $((+))$ a se invicem independentes, modo ut dixi nihilo minores non obstant, et error calculi abfuerit.

Nunc secundum inventos valores literas quaesitas retrogrado ordine explicemus: erit 10
 ex 55. $v \overset{(60)}{\cap} +((+)) 1$ et ex 52. erit $\vartheta \overset{(61)}{\cap} +((+)) 2$. Manentibusque e. s. n. pro arbitrio, erit ex
 41. $1 \overset{(62)}{\cap} +((+)) 1$ et ex 42. $p \overset{(63)}{\cap} ((+)) n$. $r \overset{(64)}{\cap} +((+)) 1$. Sed hinc iam absurdum orietur, in
 aequationibus 35, 36. aliisque fiet enim v. g. $\frac{1}{m} \cap 0$. adeoque suppositio 58. et quae ex

C-12 C-16 C-16

Ambiguous signs, 5th system. LAA VII-1, p. 619

Redeundum ergo ad aeq. 57. videndumque an non formula $+ 2\gamma^3 + \gamma$ aequari possit 15
 quadrato, hac enim ratione absolutum erit problema. Sit ergo $2\gamma^3 - \gamma \overset{(65)}{\cap} + \gamma^2 \lambda^2$. fiet: $2\gamma^3 - 1$
 $\overset{(66)}{\cap} + \gamma \lambda^2$. erit $\gamma^2 + \frac{\lambda^2}{2} \gamma + \frac{\lambda^4}{16} \overset{(67)}{\cap} \frac{\lambda^4}{16} + \frac{1}{2}$ sive $\vartheta \vartheta \overset{(68)}{\cap} \frac{\lambda^4 + 8}{4}$.

C-12 C-13 C-15 C-14

Ambiguous signs, 5th system. LAA VII-1, p. 619

	Einleitung	XXXIII
	sind sodann vier Fälle gemeinsam zu betrachten, und auch hier bedient sich Leibniz neuer Doppelvorzeichen, nämlich der Symbole $\equiv$, $\equiv$, $\equiv$ und $\equiv$. Er erläutert die neuen Symbole nicht; ihre Verwendung scheint für ihn entweder selbstverständlich oder selbst erklärend zu sein. Der Übergang zur Verwendung der neuen Symbole ist also, soweit es die zusammengesetzten Symbole anbelangt, Weihnachten 1674 offenkundig bereits vollzogen. Dass die älteren zusammengesetzten Symbole nach Dezember 1674 noch einmal eingesetzt werden, lässt sich nicht belegen.	
	Leibniz führt das fünfte System nicht in einer weiteren programmatischen Schrift ein. Doch liefern manche Stücke Hinweise auf seine Genese. So finden sich in dem auf Dezember 1674 datierten Stück <i>De descriptionibus curvarum</i> (N. 44) nicht nur die einfachen Doppelvorzeichen des fünften Systems, $\dagger$ und $\ddagger$, sondern mit den Symbolen $\ddot{\dagger}$ und $\ddot{\ddagger}$ auch Vorformen zusammengesetzter Zeichen. Diese unterscheiden sich von den bald darauf kanonisierten Formen des fünften Systems dadurch, dass sie jeweils zwei der Querbalken mit Hilfe einer weiteren Linie verbinden. Dieser Verbindungsstrich gliedert die Zeichen: Die durch ihn verbundenen beiden Querbalken bilden zusammen den (doppelseitigen) ersten Fall, der untere Querbalken den (eindeutigen) zweiten Fall. Das Symbol $\ddot{\dagger}$ bedeutet also „im ersten Unterfall des ersten Falles —, im zweiten Unterfall des ersten Falles +; im zweiten Fall +“. In seinen Exzerpten aus Mariottes <i>Du choc des corps</i> (VIII, 2 N. 50), die ebenfalls aus dem Dezember 1674 stammen dürften, kann sogar unmittelbar verfolgt werden, wie Leibniz das fünfte aus dem zweiten System ableitet. Er hält fest, die Notation müsse neu gestaltet werden, startet mit dem Zeichen $\ddagger$, ersetzt es zunächst durch die Übergangsform $\ddagger$ und gelangt schließlich zur Form $\ddot{\ddagger}$. Die Übergangsform $\ddagger$ findet sich ausschließlich in diesem Stück und an dieser Stelle, sie spiegelt Leibniz' Einfalt wider, ein Minus durch einen halben Querbalken auszudrücken. Diese Darstellungsweise wird — gemeinsam mit der geradlinigen Anordnung der Fälle an einem senkrechten Balken — für das fünfte System charakteristisch. Im selben Stück identifiziert Leibniz auch die Symbole des vierten Systems mit jenen des fünften: ($\alpha\omega$) setzt er mit $\ddot{\dagger}$ gleich, ($\omega\alpha$) mit $\ddot{\ddagger}$.	
	Die Form $\ddot{\dagger}$ entspricht dem später bevorzugten Symbol $\equiv$ bis auf eine Besonderheit: Bei ihr ist der untere Querbalken näher an den unteren als den oberen gerückt, wegen das Symbol $\equiv$ gleiche Abstände der Querbalken aufweist. Doch sind die Form $\ddot{\dagger}$ und analog gestaltete Zeichen, etwa Symbol $\ddot{\ddagger}$, nicht lediglich Ausdruck eines Übergangsstadiums, sondern Leibniz setzt sie bisweilen auch in der Praxis ein, etwa in VII, 1 N. 96 von April 1676. Tatsächlich lassen sich die beiden Symbole $\ddot{\dagger}$ und $\ddot{\ddagger}$ zwei unterschiedlichen	

Ambiguous signs, 2nd and 5th system. LAA VII-7, introduction, p. XXXIII















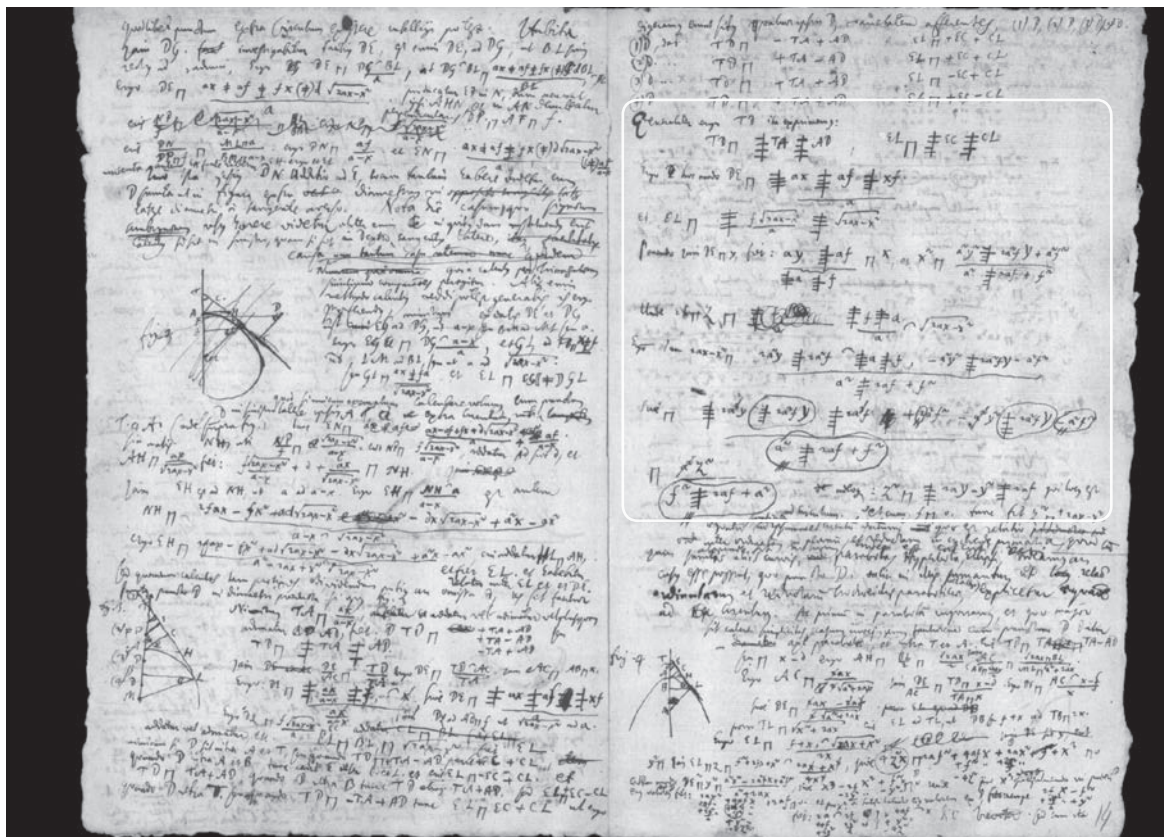

B-13 *C-01* *C-04* *C-05* *C-08* *C-11* *C-16* *C-17* *C-18* *C-19* *C-24* *C-25* *C-26* *C-27*

Quae theorematum vel ideo annotanda duxi, quod semel elapsa non facile rursus in mentem venirent, et non nisi per multas ambages deprehensa sint. Et haec quidem de Trianguli characteristici usu ad dimensiones curvilinearum nunc sufficiant.

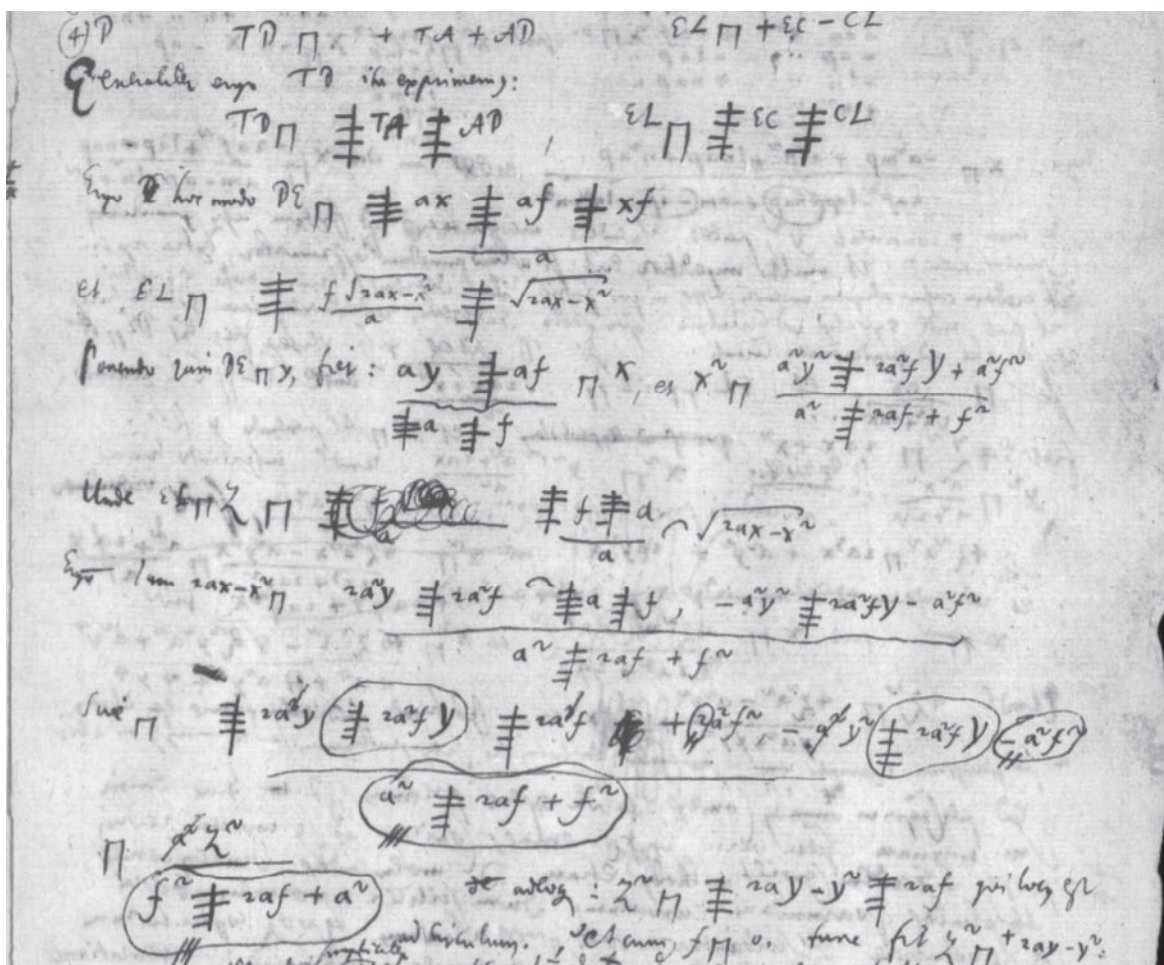
10

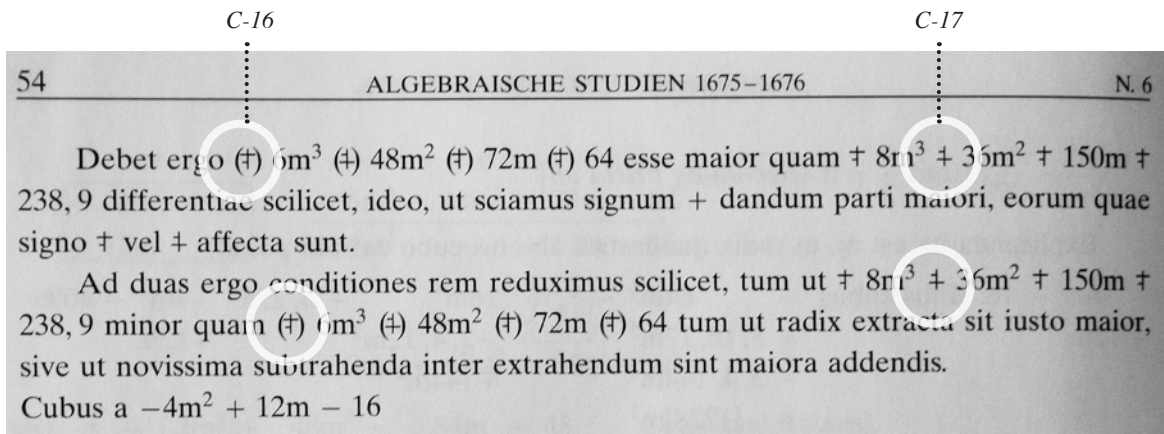
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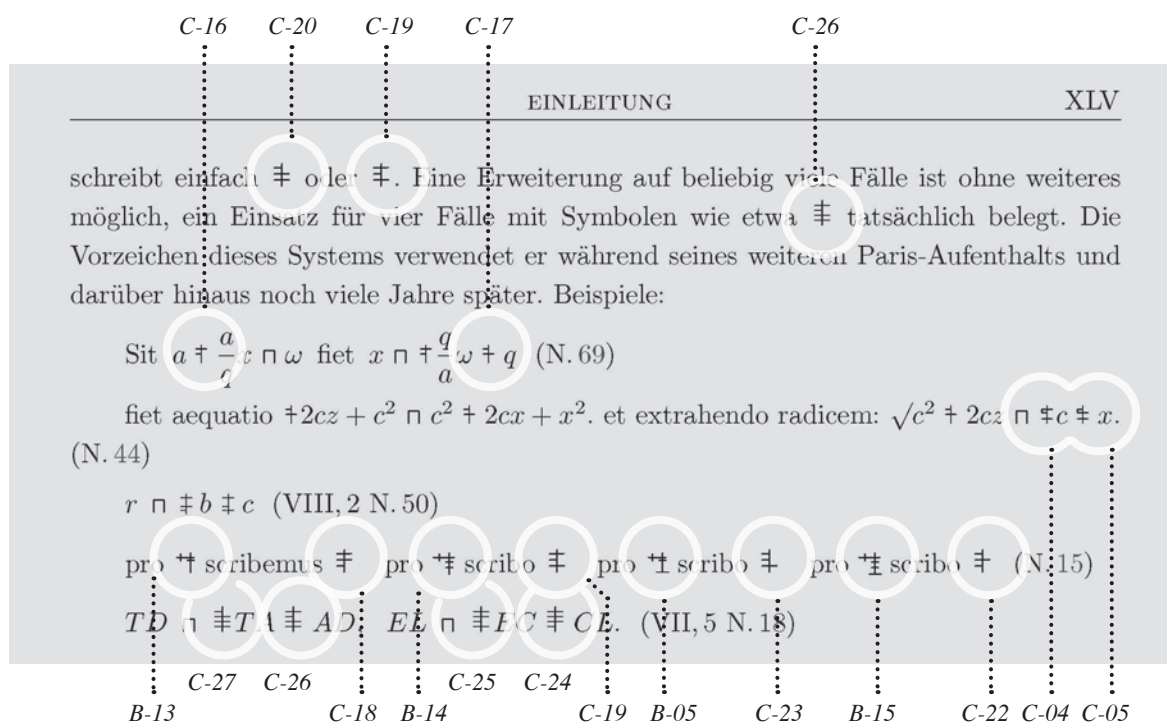


LH 35 V 3, f. 13v

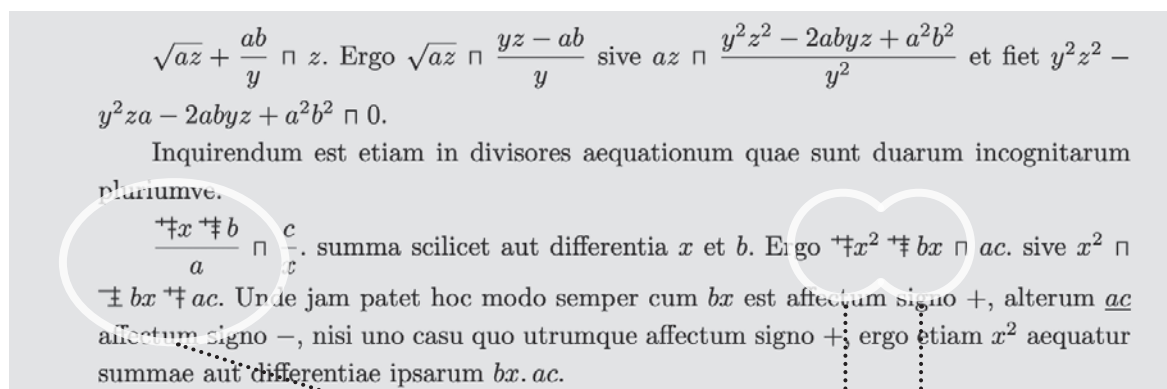




Ambiguous signs, 5th system. LAA VII-2, p. 54



Example of ambiguous signs, 2nd and 5th system. LAA VII-7, introduction p. XLV



Ambiguous signs, 2nd system. B-02, B-03, B-07
LAA VII-5, p. 233

C-18 B-01 C-19 B-13 C-19 B-13 C-01 C-08

N. 50 EXCERPTA EX LIBRO DU CHOC DES CORPS 439

vel quia $r \sqcap \pm b \pm c$, erit $b \sqcap \pm r \pm c$, sive $b \sqcap \pm r \pm c$; sed quia relatio apparere debet, erit potius $b \sqcap \pm r \pm c$.

Ac proinde reformanda nonnihil notatio est: nimirum pro $\pm$ faciemus $\pm$ vel i a $\pm$, vel etiam ita $\pm$ pro $(\alpha\alpha\omega)$ et pro $(\alpha\omega\alpha)$ fiet: $\pm$. Quae naturalissima omnium haud dubie notatio est. Itaque ponendo: $r \sqcap \pm b \pm c$, erit $b \sqcap \pm r \pm c$ et $c \sqcap \pm r \pm b$. Sed et utile forte erit summam differentiamque distinguere, et fiet: $r \sqcap \pm b \pm c$, unde $b \sqcap \pm r \pm c$, vel $b \sqcap \pm r \pm c$ C-11

C-09..... $\sqcap \pm r \pm [b]$. Quod si velimus totam formulam signo afficere, aut certam ejus partem, fiet, ... C-06

C-07..... v.g. $r \sqcap (sd) b \pm c$, sed signum ejusmodi cum sit instar signi radicalis incapax est, partium C-07

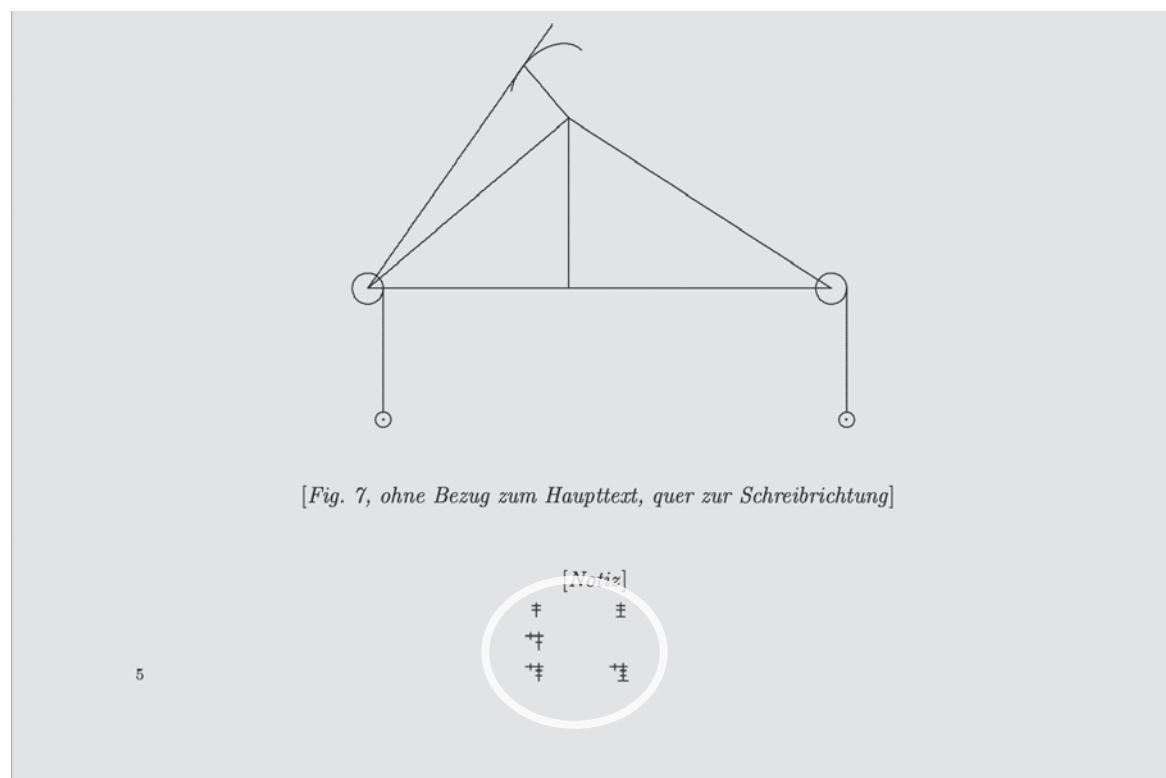
..... C-08

divulsionis: nisi aliunde ratiocineris. Nimirum perinde est ac si dicas esse $\sqrt{b^2 \pm 2bc + c^2}$, nam differentia seu $\overline{(d) b + c}$ est $\sqcap [\sqrt{b^2 - 2bc + c^2}]$. Caeterum posito $r \sqcap \overline{(sd) b + c}$ erit $b \sqcap \overline{[(ds)]r + c}$ et $c \sqcap \overline{(ds) r + b}$. Sed cum haec signa ut dixi intractabilia sint, nisi quatenus in alia resolvuntur, rectius ex Analysis ablegabuntur.

Redeamus ergo ad rem nostram, scilicet: $r \sqcap \pm b \pm c$, $b \sqcap \pm r \pm c$, $c \sqcap \pm r \pm b$. Quodsi compendii causa faciamus semper majorem celeritatem esse b , minorem semper esse c , fiet: $r \sqcap b \pm c$, adeoque $b \sqcap r \pm c$ et $c \sqcap r \pm b$.

C-18 C-19 C-18 C-20 C-19 C-20

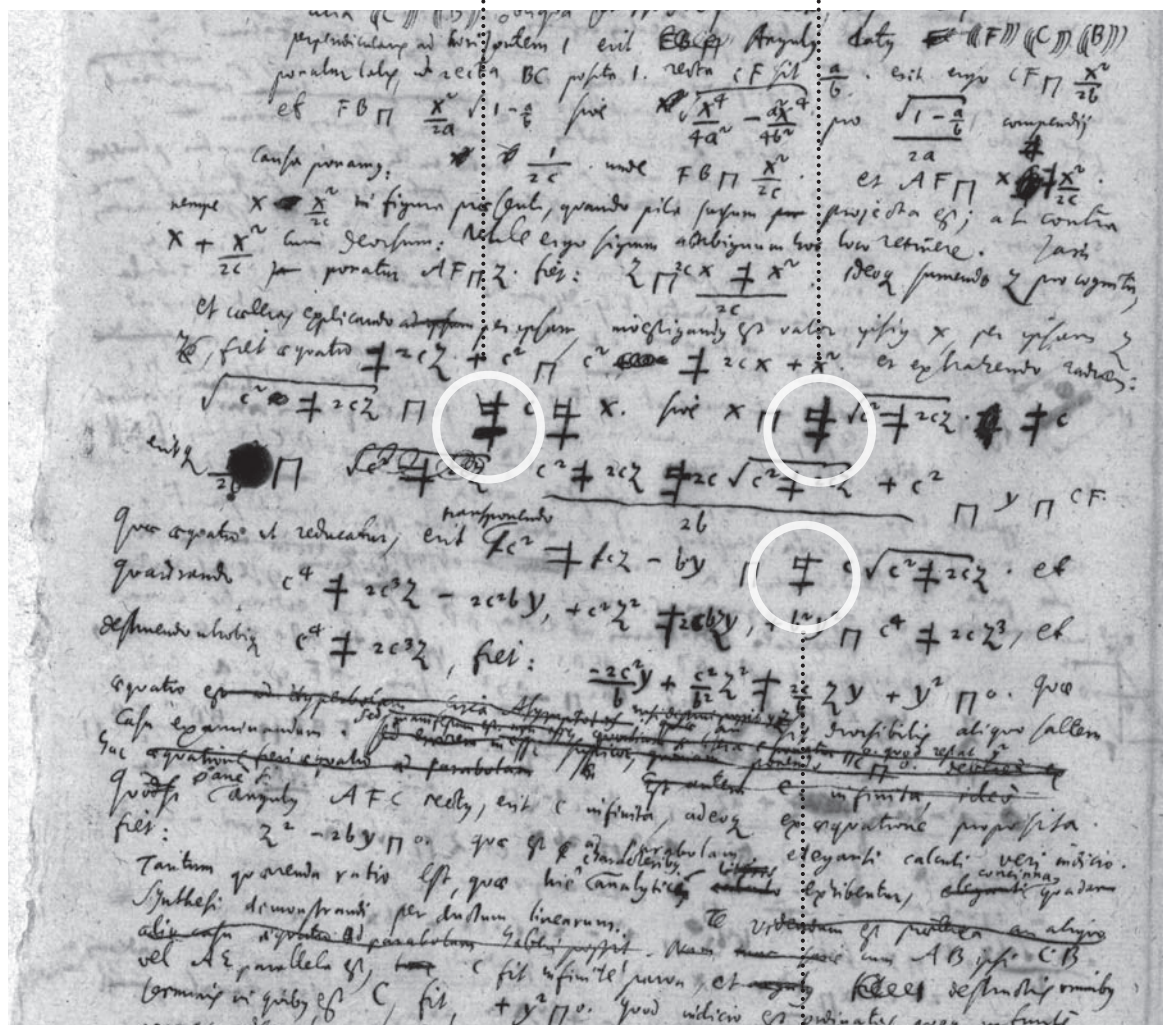
Example of ambiguous signs, 2nd and 5th system. LAA VIII-2, p. 439



B-02, B-06, B-03; LAA VII-3, p. 360

C-02

C-03



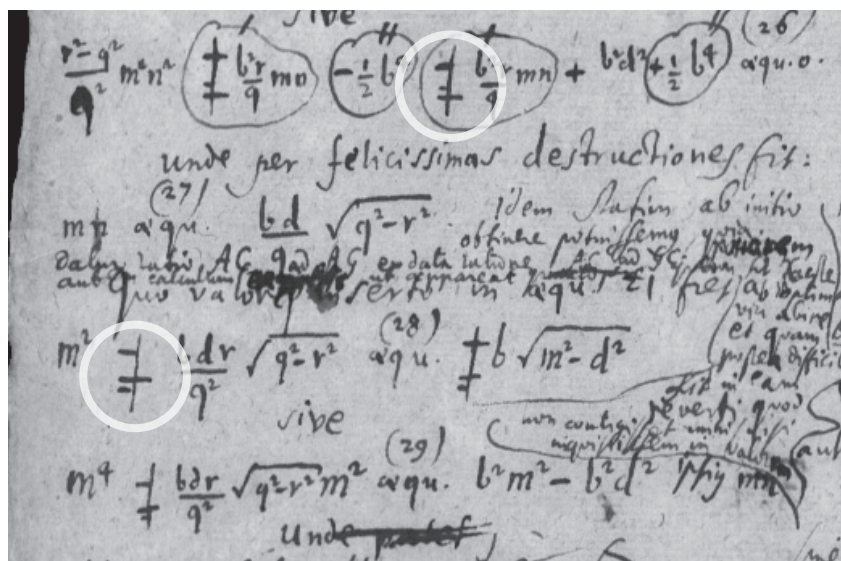
Ambiguous signs, 5th system.

LH 35 XII 1, fol. 227v

C-04

C-33

C-32



These two characters have been discovered recently, in manuscripts from Leibniz.
 Ambiguous signs of the 5th system. Manuscript LH 35 I 11, fol. 20v.

C-32

C-08

C-09

C-09

C-33

C-08

C-09

C-09

C-33

C-32

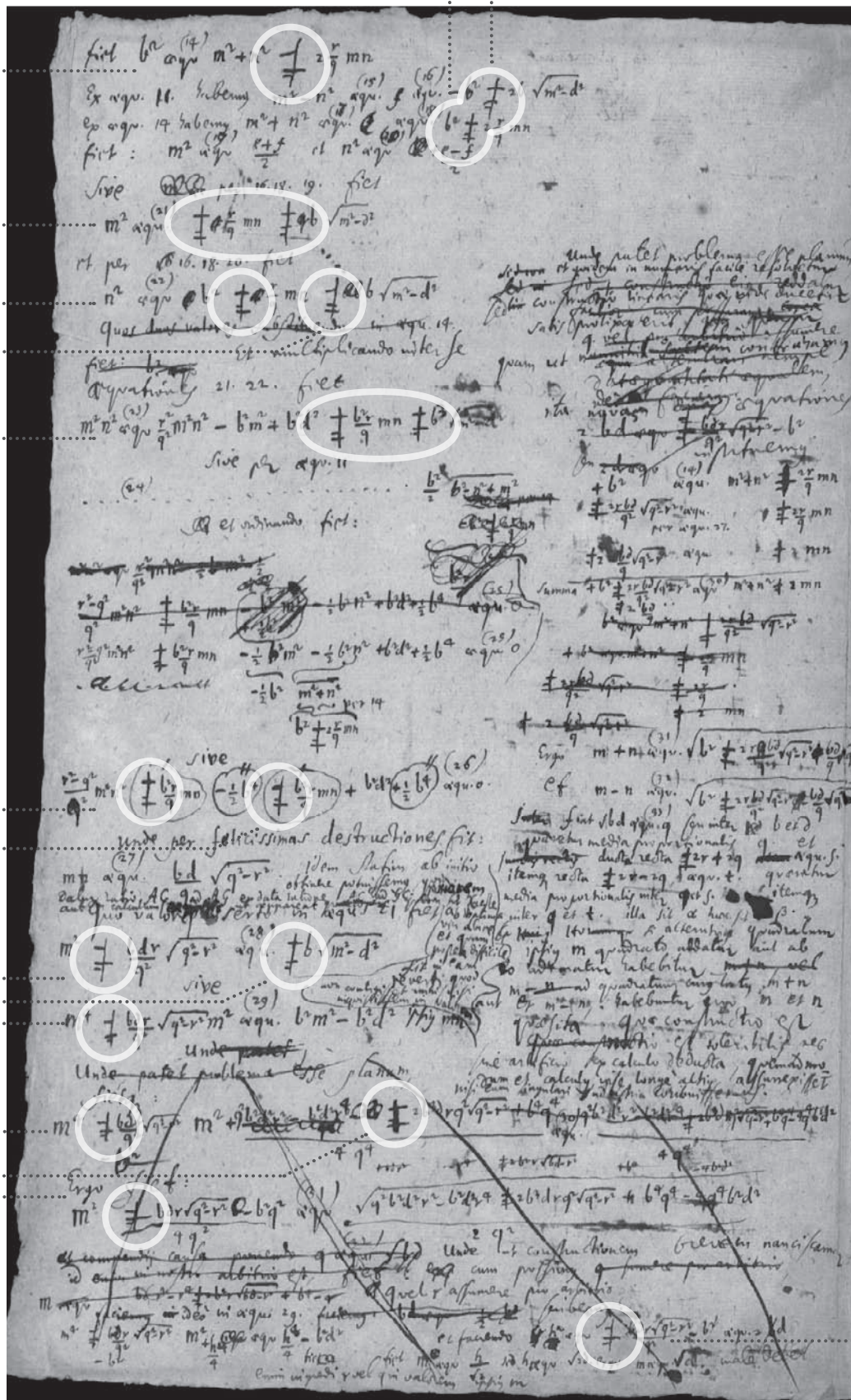
C-08

C-32

C-32

C-09

C-32



C-32

Ambiguous signs of the 5th system. **LH 35 I 11**, fol. 20v. To be published in LAA VII-9 in 2027.

6. Unicode Character Properties

A-01	1DC00;LEIBNIZIAN	SYSTEM ONE AND TWO PLUS-MINUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
A-02	1DC01;LEIBNIZIAN	SYSTEM ONE MINUS-PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
A-03	1DC02;LEIBNIZIAN	SYSTEM ONE PLUS OR PLUS-MINUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
A-04	1DC03;LEIBNIZIAN	SYSTEM ONE PLUS OR MINUS-PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
A-05	1DC04;LEIBNIZIAN	SYSTEM ONE MINUS OR PLUS-MINUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
A-06	1DC05;LEIBNIZIAN	SYSTEM ONE MINUS OR MINUS-PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
A-07	1DC06;LEIBNIZIAN	SYSTEM ONE PLUS-MINUS TIMES PLUS-MINUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
A-08	1DC07;LEIBNIZIAN	SYSTEM ONE MINUS-PLUS TIMES PLUS-MINUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
B-01	1DC08;LEIBNIZIAN	SYSTEM TWO MINUS-PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
B-02	1DC09;LEIBNIZIAN	SYSTEM TWO PLUS OR PLUS-MINUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
B-03	1DC0A;LEIBNIZIAN	SYSTEM TWO PLUS OR MINUS-PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
B-04	1DC0B;LEIBNIZIAN	SYSTEM TWO MINUS OR MINUS-PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
B-05	1DC0C;LEIBNIZIAN	SYSTEM TWO NEGATED PLUS OR PLUS-MINUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
B-06	1DC0D;LEIBNIZIAN	SYSTEM TWO NEGATED PLUS OR MINUS-PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
B-07	1DC0E;LEIBNIZIAN	SYSTEM TWO NEGATED MINUS OR PLUS-MINUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
B-08	1DC0F;LEIBNIZIAN	SYSTEM TWO PLUS-MINUS OR PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
B-09	1DC10;LEIBNIZIAN	SYSTEM TWO PLUS-MINUS OR MINUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
B-10	1DC11;LEIBNIZIAN	SYSTEM TWO NEGATED PLUS-MINUS OR PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
B-11	1DC12;LEIBNIZIAN	SYSTEM TWO NEGATED PLUS OR MINUS-PLUS PARTIAL AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
B-12	1DC13;LEIBNIZIAN	SYSTEM TWO NEGATED MINUS OR MINUS-PLUS PARTIAL AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
B-16	1DC14;LEIBNIZIAN	SYSTEM TWO NEGATED PLUS-MINUS OR PLUS PARTIAL AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
C-01	1DC15;LEIBNIZIAN	SYSTEM FIVE-ONE PLUS OR PLUS-MINUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
C-02	1DC16;LEIBNIZIAN	SYSTEM FIVE-TWO PLUS-MINUS OR PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
C-03	1DC17;LEIBNIZIAN	SYSTEM FIVE-TWO MINUS-PLUS OR PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
C-04	1DC18;LEIBNIZIAN	SYSTEM FIVE-THREE PLUS-MINUS OR PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
C-05	1DC19;LEIBNIZIAN	SYSTEM FIVE-THREE MINUS-PLUS OR PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
C-06	1DC1A;LEIBNIZIAN	SYSTEM FIVE-FOUR PLUS OR MINUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
C-07	1DC1B;LEIBNIZIAN	SYSTEM FIVE-FOUR MINUS OR PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
C-08	1DC1C;LEIBNIZIAN	SYSTEM FIVE-FOUR PLUS OR PLUS-MINUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
C-09	1DC1D;LEIBNIZIAN	SYSTEM FIVE-FOUR PLUS OR MINUS-PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
C-10	1DC1E;LEIBNIZIAN	SYSTEM FIVE-FOUR PLUS-MINUS OR PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
C-11	1DC1F;LEIBNIZIAN	SYSTEM FIVE-FOUR MINUS-PLUS OR PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
C-12	1DC20;LEIBNIZIAN	SYSTEM FIVE-FIVE INVERTED MINUS-PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
C-13	1DC21;LEIBNIZIAN	SYSTEM FIVE-FIVE INVERTED PLUS-MINUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
C-14	1DC22;LEIBNIZIAN	SYSTEM FIVE-FIVE INVERTED PLUS OR PLUS-MINUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
C-15	1DC23;LEIBNIZIAN	SYSTEM FIVE-FIVE INVERTED PLUS-MINUS OR PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
C-16	1DC24;LEIBNIZIAN	SYSTEM FIVE PLUS-MINUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
C-17	1DC25;LEIBNIZIAN	SYSTEM FIVE MINUS-PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
C-18	1DC26;LEIBNIZIAN	SYSTEM FIVE PLUS-PLUS-MINUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
C-19	1DC27;LEIBNIZIAN	SYSTEM FIVE PLUS-MINUS-PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
C-20	1DC28;LEIBNIZIAN	SYSTEM FIVE MINUS-PLUS-PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
C-21	1DC29;LEIBNIZIAN	SYSTEM FIVE PLUS-MINUS-MINUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
C-22	1DC2A;LEIBNIZIAN	SYSTEM FIVE MINUS-PLUS-MINUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
C-23	1DC2B;LEIBNIZIAN	SYSTEM FIVE MINUS-MINUS-PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
C-24	1DC2C;LEIBNIZIAN	SYSTEM FIVE PLUS-PLUS-PLUS-MINUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
C-25	1DC2D;LEIBNIZIAN	SYSTEM FIVE PLUS-PLUS-MINUS-PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
C-26	1DC2E;LEIBNIZIAN	SYSTEM FIVE PLUS-MINUS-PLUS-PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
C-27	1DC2F;LEIBNIZIAN	SYSTEM FIVE MINUS-PLUS-PLUS-PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
C-28	1DC30;LEIBNIZIAN	SYSTEM FIVE PLUS-PLUS-MINUS-MINUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
C-29	1DC31;LEIBNIZIAN	SYSTEM FIVE MINUS-PLUS-PLUS-MINUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
C-30	1DC32;LEIBNIZIAN	SYSTEM FIVE MINUS-MINUS-PLUS-PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
C-31	1DC33;LEIBNIZIAN	SYSTEM FIVE MINUS-PLUS-MINUS-MINUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
C-32	1DC34;LEIBNIZIAN	SYSTEM FIVE-FOUR MINUS OR PLUS-MINUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
C-33	1DC35;LEIBNIZIAN	SYSTEM FIVE-FOUR MINUS OR MINUS-PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;

B-13 1DC09 FE00; alternate form; # LEIBNIZIAN SYSTEM TWO PLUS OR PLUS-MINUS AMBIGUOUS SIGN
B-14 1DC0A FE00; alternate form; # LEIBNIZIAN SYSTEM TWO PLUS OR MINUS-PLUS AMBIGUOUS SIGN
B-15 1DC0D FE00; alternate form; # LEIBNIZIAN SYSTEM TWO NEGATED PLUS OR MINUS-PLUS
 AMBIGUOUS SIGN
B-17 1DC12 FE00; alternate form 1; # LEIBNIZIAN SYSTEM TWO NEGATED PLUS OR MINUS-PLUS
 PARTIAL AMBIGUOUS SIGN
B-18 1DC12 FE01; alternate form 2; # LEIBNIZIAN SYSTEM TWO NEGATED PLUS OR MINUS-PLUS
 PARTIAL AMBIGUOUS SIGN

7. Bibliography

LAA – refers to: Leibniz, Gottfried Wilhelm: *Sämtliche Schriften und Briefe*. (‘Leibniz-Akademie-Ausgabe’, many volumes)

LH – refers to: Leibniz’s original manuscripts, GWLB Hanover

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— : *Operum mathematicorum*, Oxford 1657

— : *Treatise of Algebra*. London 1685

**ISO/IEC JTC 1/SC 2/WG 2
PROPOSAL SUMMARY FORM TO ACCOMPANY SUBMISSIONS
FOR ADDITIONS TO THE REPERTOIRE OF ISO/IEC 10646¹**

Please fill all the sections A, B and C below.

Please read Principles and Procedures Document (P & P) from <http://std.dkuug.dk/JTC1/SC2/WG2/docs/principles.html> for guidelines and details before filling this form.

Please ensure you are using the latest Form from <http://std.dkuug.dk/JTC1/SC2/WG2/docs/summaryform.html>.

See also <http://std.dkuug.dk/JTC1/SC2/WG2/docs/roadmaps.html> for latest *Roadmaps*.

A. Administrative

1. Title:	Proposal to encode Leibnizian ambiguous signs		
2. Requester's name:	Uwe Mayer, Siegmund Probst, David Rabouin, Elisabeth Rinner, Andreas Stötzner, Achim Trunk, Charlotte Wahl		
3. Requester type (Member body/Liaison/Individual contribution):	Individual (work group)		
4. Submission date:	2026-05-26		
5. Requester's reference (if applicable):	LUCPL-2618		
6. Choose one of the following:			
This is a complete proposal:			Yes
(or) More information will be provided later:			

B. Technical – General

1. Choose one of the following:			
a. This proposal is for a new script (set of characters):			No
Proposed name of script:			
b. The proposal is for addition of character(s) to an existing block:			No
Name of the existing block:			
2. Number of characters in proposal:			59
3. Proposed category (select one from below - see section 2.2 of P&P document):			
A-Contemporary	B.1-Specialized (small collection)	Yes	B.2-Specialized (large collection)
C-Major extinct	D-Attested extinct		E-Minor extinct
F-Archaic Hieroglyphic or Ideographic			G-Obscure or questionable usage symbols
4. Is a repertoire including character names provided?			Yes
a. If YES, are the names in accordance with the "character naming guidelines" in Annex L of P&P document?			Yes
b. Are the character shapes attached in a legible form suitable for review?			Yes
5. Fonts related:			
a. Who will provide the appropriate computerized font to the Project Editor of 10646 for publishing the standard?	Andreas Stötzner		
b. Identify the party granting a license for use of the font by the editors (include address, e-mail, ftp-site, etc.):	Andreas Stötzner Gestaltung, Klaufügelweg 21, 88400 Biberach/R., Germany, as@signographie.de		
6. References:			
a. Are references (to other character sets, dictionaries, descriptive texts etc.) provided?			Yes
b. Are published examples of use (such as samples from newspapers, magazines, or other sources) of proposed characters attached?			Yes
7. Special encoding issues:			
Does the proposal address other aspects of character data processing (if applicable) such as input, presentation, sorting, searching, indexing, transliteration etc. (if yes please enclose information)?			No

8. Additional Information:

Submitters are invited to provide any additional information about Properties of the proposed Character(s) or Script that will assist in correct understanding of and correct linguistic processing of the proposed character(s) or script. Examples of such properties are: Casing information, Numeric information, Currency information, Display behaviour information such as line breaks, widths etc., Combining behaviour, Spacing behaviour, Directional behaviour, Default Collation behaviour, relevance in Mark Up contexts, Compatibility equivalence and other Unicode normalization related information. See the Unicode standard at <http://www.unicode.org> for such information on other scripts. Also see Unicode Character Database (<http://www.unicode.org/reports/tr44/>) and associated Unicode Technical Reports for information needed for consideration by the Unicode Technical Committee for inclusion in the Unicode Standard.

¹ Form number: N4502-F (Original 1994-10-14; Revised 1995-01, 1995-04, 1996-04, 1996-08, 1999-03, 2001-05, 2001-09, 2003-11, 2005-01, 2005-09, 2005-10, 2007-03, 2008-05, 2009-11, 2011-03, 2012-01)

C. Technical - Justification

1. Has this proposal for addition of character(s) been submitted before?	Yes
If YES explain <i>N5329R3 (L-2608), N5329R2 (L-2529), N5329R (L-2526), N5329 (L-2504)</i>	
2. Has contact been made to members of the user community (for example: National Body, user groups of the script or characters, other experts, etc.)?	Yes
If YES, with whom?	
Leibniz-Archiv, Forschungsstelle der Leibniz-Edition, Niedersächsische Landesbibliothek (GWLb), Hanover, Göttingen Academy of Science and Humanities in Lower Saxony (DE), Philium research group of CNRS (UMR 7219, laboratoire SPHERE) / Université de Paris VII; general: scholars, researchers, authors and editors working in the field of science history and upon editions of historic text corpora (e.g. of G. W. Leibniz, but also many others)	
If YES, available relevant documents: L-2409, L-2410	
3. Information on the user community for the proposed characters (for example: size, demographics, information technology use, or publishing use) is included?	Yes
Reference:	
4. The context of use for the proposed characters (type of use; common or rare)	Common
Reference: mainly specialist usage, scholarly, worldwide	
5. Are the proposed characters in current use by the user community?	Yes
If YES, where? Reference: mainly Europe, Americas; other countries	
6. After giving due considerations to the principles in the P&P document must the proposed characters be entirely in the BMP?	No
If YES, is a rationale provided?	
If YES, reference:	
7. Should the proposed characters be kept together in a contiguous range (rather than being scattered)?	Yes
8. Can any of the proposed characters be considered a presentation form of an existing character or character sequence?	No
If YES, is a rationale for its inclusion provided?	
If YES, reference:	
9. Can any of the proposed characters be encoded using a composed character sequence of either existing characters or other proposed characters?	Yes
If YES, is a rationale for its inclusion provided?	
If YES, reference: <i>5 variation sequences; see p. 14</i>	
10. Can any of the proposed character(s) be considered to be similar (in appearance or function) to, or could be confused with, an existing character?	No
If YES, is a rationale for its inclusion provided?	
If YES, reference:	
11. Does the proposal include use of combining characters and/or use of composite sequences?	No
If YES, is a rationale for such use provided?	
If YES, reference:	
Is a list of composite sequences and their corresponding glyph images (graphic symbols) provided?	
If YES, reference:	
12. Does the proposal contain characters with any special properties such as control function or similar semantics?	No
If YES, describe in detail (include attachment if necessary)	
13. Does the proposal contain any Ideographic compatibility characters?	No
If YES, are the equivalent corresponding unified ideographic characters identified?	
If YES, reference:	