Universal Multiple-Octet Coded Character Set
International Organization for Standardization
Internationale Standardisierungs-Organisation
Organisation Internationale de Normalisation
Διεθνής Οργανισμός Τυποποίησης
Международная организация по стандартизации

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Title: Proposal to add historic scientific characters to the UCS

Source: Uwe Mayer, Siegmund Probst, David Rabouin, Elisabeth Rinner, Andreas Stötzner, Achim Trunk, Charlotte Wahl
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This proposal requests the encoding of 228 historical scientific characters, many of them from the field of mathematics, as testified in works of Gottfried Wilhelm Leibniz (1646–1716), of his contemporaries and in related editions.

1. Background

In the history of mathematics, there is a strong interest in a precise capturing of historical mathematical notations, including an adequate representation of special characters. Thus, the typical scenarios of usage of our proposed characters are:

- Text capturing in digital editions: according to the guidelines of TEI, the "chunks" of mathematical texts and some elements of aggregation such as "(" are represented by their characters. All other elements of nestings belong to the domain of structure elements. As such, they are represented by using markup languages. For an example, see the digital edition of the work of Newton (https://www.newtonproject.ox.ac.uk/).
- Automatic text recognition/transcription: in order to achieve a better result, the characters will be included into a text recognition model.

The background of this proposal is the collaboration of two European institutions: the Leibniz-Archiv: Forschungsstelle der Leibniz-Edition (a department of the Gottfried Wilhelm Leibniz Bibliothek – Niedersächsische Landesbibliothek (GWLB), Hanover (Germany), supervised by the Göttingen Academy of Science and Humanities in Lower Saxony (Germany)) and the Philiumm research group of CNRS (UMR 7219, laboratoire SPHERE) / Université de Paris VII (France), in the Philiumm Project (2021–2026), funded by the European Research Council (N° ADG-101020985), both working on comprehensive editions of Leibniz's scientific legacy (see Philiumm; Leibniz-Archiv). As the focus of editorial work shifts towards digital and online editions, the need of a standard encoding for a larger range of special characters becomes obvious. Most of the characters proposed appear in the works of Leibniz. He was one of the most prolific scholars of Europe in the age of enlightenment. His manuscripts embrace the subjects of mathematics, philosophy, history, law studies, engineering and many others. He maintained a correspondence with more than a thousand scholars in many countries and left a legacy of about 200.000 manuscript pages. Among his well-known achievements are fundamental contributions to infinitesimal calculus and binary mathematics, which make him an eminent author even today, more than 300 years after his passing.

In his writings Leibniz makes extensive use of special ideographic characters which he adopted from other authors or invented himself in order to find suitable means of expression for his concepts. Best known is his introduction of a cursive long s for "summa" which later became generally known as the *integral* sign: ∫.

Besides the traditional production of printed editions currently editorial activities move steadily into the digisphere, towards the internet in particular. Facsimile and diplomatic online transcripions of important historic sources are about to become a new standard in scientific publishing. That development makes it all the more obvious that a given source text is to be created as *text* in the technical sense, as an encoded string of characters which enables copying and searching. The works of authors like Newton, Descartes, Huygens or Leibniz require an advanced repertoire of encoded characters. We see the need to represent such texts reliably in their original form. We see our proposal being in line with other previous or recent encodings of historic characters and specialized notations.

2. General outline of the proposal

This proposal is based to a great extent on recent studies by Uwe Mayer, Siegmund Probst, Elisabeth Rinner, Achim Trunk, Charlotte Wahl (Leibniz-Archiv) and Arilès Remaki (Philiumm), editors of Leibniz's manuscripts, about the special characters occurring in Leibniz's works, in editions of those sources and in works of other authors (mainly from the field of mathematics). Florian Cajori's ground-breaking "A history of mathematical notations" from 1928 is still a valuable reference for the matter elaborated in this proposal.

Regarding the amount and nature of the characters in question, a new block "Scientific characters" or "Historic scientific characters" to the UCS is proposed. The *Leibnizian ambiguity signs* (section c) form the largest subset of this proposal, they may be considered as candidates for a new block of their own. Future additions to this block (and, possibly, to the other sets) are likely to happen, as research goes on and new characters will be discovered in sources which have not been recognized so far. Some of the characters proposed may be seen as candidates for inclusion in existing blocks, e.g. the character pair Ȣ/ȣ as an addition to the 0370 Greek block.

3. Characters overview

The characters proposed are grouped according to their context and nature, as follows:

a) Historical mathematical operators
b) Historical mathematical relations
c) Leibnizian ambiguity signs
d) Geometrical signs
e) Alchemical symbols
f) Miscellaneous scientific signs
g) Superscript characters
h) Letterlike symbols
i) Coss symbols
k) Digit characters

If this proposal gets accepted, the following characters will exist:

a) Historical mathematical operators

LEIBNIZIAN DIVISION SIGN
LEIBNIZIAN PRODUCT SIGN
LEIBNIZIAN DIVISION-PRODUCT SIGN
LEIBNIZIAN DIVISION STAFF SIGN 1
LEIBNIZIAN DIVISION STAFF SIGN 2

b) Historical mathematical relations

LEIBNIZIAN EQUAL SIGN
LEIBNIZIAN DOUBLE EQUAL SIGN
LEIBNIZIAN EQUALITY WITH S SIGN
LEIBNIZIAN GREATER
LEIBNIZIAN LESS
BERNOULLIAN GREATER
BERNOULLIAN LESS
LEIBNIZIAN GREATER WITH P
LEIBNIZIAN LESS WITH P
LEIBNIZIAN GREATER-LESS SIGN
GREATER 2
LESS 2
PARALLEL GREATEREQUAL
PARALLEL LESSEQUAL
FACIT SIGN
CARTESIAN EQUAL SIGN
TSCHIRNHAUS EQUAL SIGN
CONGRUENCE SIGN 1
CONGRUENCE SIGN 2
SIMILARITY SIGN
COINCIDENCE SIGN
LEIBNIZIAN SIMILARITY SIGN 1
LEIBNIZIAN SIMILARITY SIGN 2

c) Leibnizian ambiguity signs

AMBIGUITY SIGN A-01
AMBIGUITY SIGN A-02
AMBIGUITY SIGN A-03
AMBIGUITY SIGN A-04
AMBIGUITY SIGN A-05
AMBIGUITY SIGN A-06
AMBIGUITY SIGN A-07
AMBIGUITY SIGN A-08
AMBIGUITY SIGN B-01
AMBIGUITY SIGN B-02
AMBIGUITY SIGN B-03
AMBIGUITY SIGN B-04
AMBIGUITY SIGN B-05
AMBIGUITY SIGN B-06
AMBIGUITY SIGN B-07

AMBIGUITY SIGN B-08
AMBIGUITY SIGN B-09
AMBIGUITY SIGN B-10
AMBIGUITY SIGN B-11
AMBIGUITY SIGN B-12
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AMBIGUITY SIGN B-14
AMBIGUITY SIGN B-15
AMBIGUITY SIGN B-16
AMBIGUITY SIGN B-17
AMBIGUITY SIGN B-18
AMBIGUITY SIGN C-01
AMBIGUITY SIGN C-02
AMBIGUITY SIGN C-03
AMBIGUITY SIGN C-04
AMBIGUITY SIGN C-05
AMBIGUITY SIGN C-06
AMBIGUITY SIGN C-07
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AMBIGUITY SIGN C-12
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AMBIGUITY SIGN C-25
AMBIGUITY SIGN C-26
AMBIGUITY SIGN C-27
AMBIGUITY SIGN C-28
AMBIGUITY SIGN C-29
AMBIGUITY SIGN C-30
AMBIGUITY SIGN C-31
LEFT VIRGULA PARANTHESIS
RIGHT VIRGULA PARANTHESIS
PLUSMINUS SIGN
MINUSPLUS SIGN

d) Geometrical signs

DOUBLE CIRCLE WITH DOT
CIRCLE WITH DOUBLE VERTICAL LINE
CIRCLE WITH DOUBLE VERTICAL AND HORIZONTAL LINE
DOUBLE CIRCLE WITH DOUBLE HORIZONTAL LINE
CIRCLE WITH HALF MOON OBLIQUE
HALF RIGHTHAND CIRCLE WITH DIAMETER
SMALL SECTOR WITH CHORD
SMALL SECTOR
SMALL SECTOR WITH DOUBLE ARC
SMALL SECTOR TRIANGLE
SMALL SEGMENT
RIGHT TRIANGLE POINTING RIGHT
KITE SIGN
ANGLE 1
ANGLE 2
ANGLE 3
ANGLE 4
ANGLE VERTICAL
CUBUS 1
CUBUS 2
HORIZONTAL DOUBLE SQUARE
VERTICAL DOUBLE SQUARE
THREE-PART BIG SQUARE 1
THREE-PART BIG SQUARE 2
FOUR-PART BIG SQUARE
HYPERBOLE

e) Alchemical symbols

ALCHEMICAL SYMBOL FOR ALUMEN-PISCES
ALCHEMICAL SYMBOL FOR OIL BOILED
ALCHEMICAL SYMBOL FOR MOON-JUPITER
ALCHEMICAL SYMBOL FOR TARTAR-SALT
ALCHEMICAL SYMBOL ENCLOSED SUN
ALCHEMICAL SYMBOL ENCLOSED MOON
ALCHEMICAL SYMBOL FOR REALGAR 3
ALCHEMICAL SYMBOL FOR HORA 2
ALCHEMICAL SYMBOL FOR RETORT 2

f) Miscellaneous scientific signs

CASTING-OUT-NINES
LUNATE ENCIRCLED DIGIT ONE
PROPORTION 1
PROPORTION 2
RIGHTHAND RELATION SIGN
LEFTHAND RELATION SIGN
CLOVERLEAF SIGN
INFINITY SIGN WITH DOTS
INVOLVED SIGN
LEIBNIZIAN ENCIRCLED V SIGN

⧇ LEIBNIZIAN BOXED ENCIRCLED V SIGN
- - BROKEN EMDASH
⸺ CROSSED EMDASH
• BOLD PERIOD
ʌ√ RADIX SIGN 1
ʌʌ√ RADIX SIGN 2
ʌʌʌ√ RADIX SIGN 3
◌̆ COMBINING BOMBELLI POWER MARK
◌̸ COMBINING DOUBLE-WIDE SLASH
◌̮ COMBINING HALF CIRCLE BELOW
◌⃝ COMBINING ENCLOSING SPIRAL MARK
◌⃝ COMBINING DOUBLE-WIDE ENCLOSING SPIRAL MARK
◌̰̃ COMBINING FACTOR MARK
◌̅ COMBINING OVERLINE WITH TERMINALS
◌̅ COMBINING DOUBLE-WIDE OVERLINE WITH TERMINALS
◌͡ COMBINING HORIZONTAL PARANTHESIS

g) Superscript characters

g⃞ SUPERSCRIPT ENCLOSED SMALL G SIGN
n⃞ SUPERSCRIPT ENCLOSED SMALL N SIGN
t⃞ SUPERSCRIPT ENCLOSED SMALL T SIGN
x⃞ SUPERSCRIPT ENCLOSED SMALL X SIGN
z⃞ SUPERSCRIPT ENCLOSED SMALL Z SIGN
ⓩ SUPERSCRIPT ENCIRCLED SMALL Z SIGN
〰 SUPERSCRIPT WAVE
〰̅ SUPERSCRIPT WAVE WITH TOP LINE

h) Letterlike symbols

αx BERNOULLIAN ALPHA-X SIGN
Đ LATIN CAPITAL D WITH TOP BAR AND CROSSBAR
⅃ LATIN CAPITAL REVERSED L
ʅ LATIN LOWERCASE REVERSED L
ꝑ LOWERCASE P WITH DOUBLE CROSSBAR
ɣ LOWERCASE KURRENT X SIGN
XX LATIN CAPITAL DOUBLE X
xx LATIN LOWERCASE DOUBLE X
σσ SIGMA-SIGMA SIGN
Ȣ GREEK CAPITAL OMICRON-UPSILON
ȣ GREEK LOWERCASE OMICRON-UPSILON

i) Coss symbols

ȼ LOWERCASE C WITH SMALL SLASH
ç LOWERCASE C WITH DESCENDER
ꜿ LOWERCASE C WITH RIGHT LOOP
ꝺ LOWERCASE D ROTUNDA WITH CROSSING LOOP
℞ SMALL CAPITAL R WITH SLASH
ꝛ LOWERCASE R ROTUNDA WITH LOOP
ß DOUBLE S ABBREVIATION SIGN
ſ LOWERCASE LONG S WITH TOP LOOP
ʒ LOWERCASE KURRENT Z SIGN

k) Digit characters

0̸ SLASHED DIGIT ZERO
1̸ SLASHED DIGIT ONE
2̸ SLASHED DIGIT TWO
3̸ SLASHED DIGIT THREE
4̸ SLASHED DIGIT FOUR
5̸ SLASHED DIGIT FIVE
6̸ SLASHED DIGIT SIX
7̸ SLASHED DIGIT SEVEN
8̸ SLASHED DIGIT EIGHT
9̸ SLASHED DIGIT NINE
0⃫ DOUBLE SLASHED DIGIT ZERO
1⃫ DOUBLE SLASHED DIGIT ONE
2⃫ DOUBLE SLASHED DIGIT TWO
3⃫ DOUBLE SLASHED DIGIT THREE
4⃫ DOUBLE SLASHED DIGIT FOUR
5⃫ DOUBLE SLASHED DIGIT FIVE
6⃫ DOUBLE SLASHED DIGIT SIX
7⃫ DOUBLE SLASHED DIGIT SEVEN
8⃫ DOUBLE SLASHED DIGIT EIGHT
9⃫ DOUBLE SLASHED DIGIT NINE
0⃫̸ TRIPLE SLASHED DIGIT ZERO
1⃫̸ TRIPLE SLASHED DIGIT ONE
2⃫̸ TRIPLE SLASHED DIGIT TWO
3⃫̸ TRIPLE SLASHED DIGIT THREE
4⃫̸ TRIPLE SLASHED DIGIT FOUR
5⃫̸ TRIPLE SLASHED DIGIT FIVE
6⃫̸ TRIPLE SLASHED DIGIT SIX
7⃫̸ TRIPLE SLASHED DIGIT SEVEN
8⃫̸ TRIPLE SLASHED DIGIT EIGHT
9⃫̸ TRIPLE SLASHED DIGIT NINE
0⃥ BACKSLASHED DIGIT ZERO
1⃥ BACKSLASHED DIGIT ONE
2⃥ BACKSLASHED DIGIT TWO
3⃥ BACKSLASHED DIGIT THREE
4⃥ BACKSLASHED DIGIT FOUR
5⃥ BACKSLASHED DIGIT FIVE
6⃥ BACKSLASHED DIGIT SIX
7⃥ BACKSLASHED DIGIT SEVEN
8⃥ BACKSLASHED DIGIT EIGHT
9⃥ BACKSLASHED DIGIT NINE
0̸⃥ CROSSED DIGIT ZERO
1̸⃥ CROSSED DIGIT ONE
2̸⃥ CROSSED DIGIT TWO
3̸⃥ CROSSED DIGIT THREE
4̸⃥ CROSSED DIGIT FOUR
5̸⃥ CROSSED DIGIT FIVE
6̸⃥ CROSSED DIGIT SIX
7̸⃥ CROSSED DIGIT SEVEN
8̸⃥ CROSSED DIGIT EIGHT
9̸⃥ CROSSED DIGIT NINE

4. Figures and explanations

a) Historical mathematical operators
b) Historical mathematical relations
c) Leibnizian ambiguity signs
d) Geometrical signs
e) Alchemical symbols
f) Miscellaneous scientific signs
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i) Coss symbols
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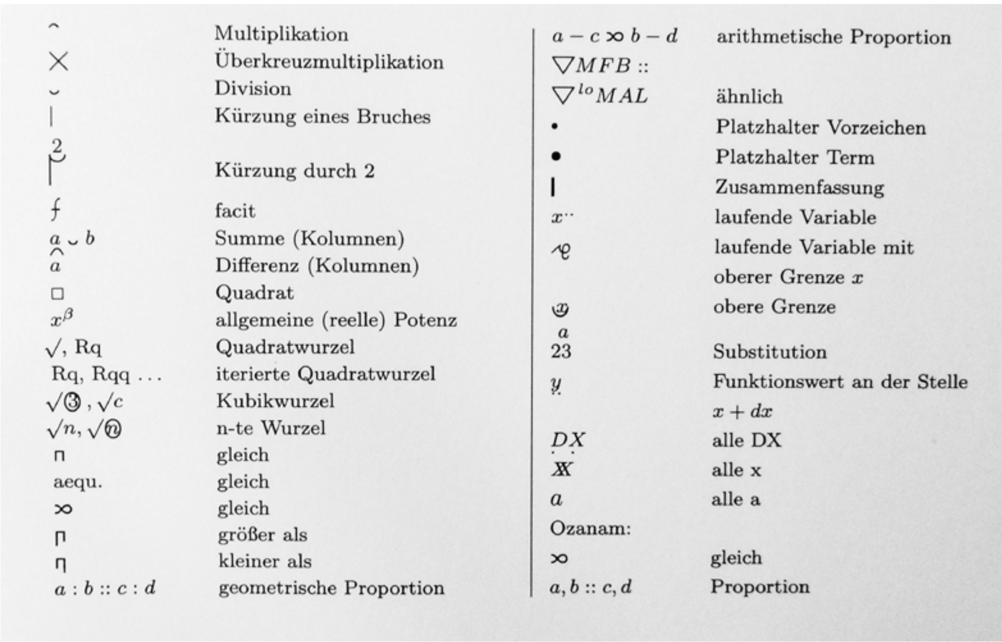
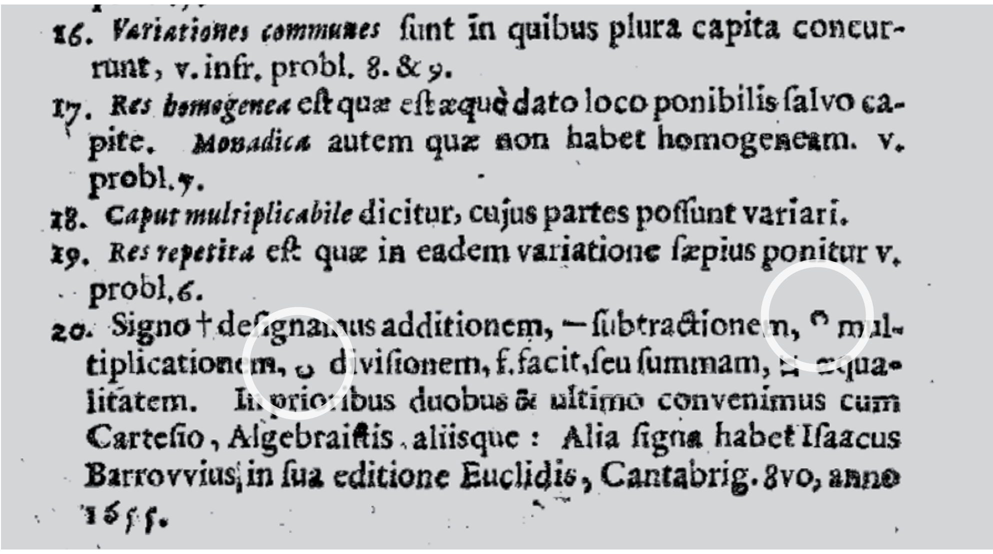


⌢	Multiplikation	$a - c \infty b - d$	arithmetische Proportion
×	Überkreuzmultiplikation	$\bigtriangledown MFB$ ::	
⌣	Division	$\bigtriangledown^{lo} MAL$	ähnlich
|	Kürzung eines Bruches	•	Platzhalter Vorzeichen
2 over |	Kürzung durch 2	●	Platzhalter Term
ƒ	facit	|	Zusammenfassung
$a \smile b$	Summe (Kolumnen)	$x^{\cdot\cdot}$	laufende Variable
$\overset{\frown}{a}$	Differenz (Kolumnen)	[illegible]	laufende Variable mit oberer Grenze x
□	Quadrat	[illegible]	obere Grenze
x^{β}	allgemeine (reelle) Potenz	$\overset{a}{23}$	Substitution
√, Rq	Quadratwurzel	$\underset{\cdot}{y}$	Funktionswert an der Stelle $x + dx$
Rq, Rqq ...	iterierte Quadratwurzel	$\underset{\cdot\,\cdot}{DX}$	alle DX
√③, $\sqrt{c}$	Kubikwurzel	[illegible]	alle x
$\sqrt{n}$, √ⓝ	n-te Wurzel	a	alle a
⊓	gleich	Ozanam:	
aequ.	gleich	∞	gleich
∞	gleich	$a, b :: c, d$	Proportion
⊓	größer als		
⊓	kleiner als		
$a : b :: c : d$	geometrische Proportion		

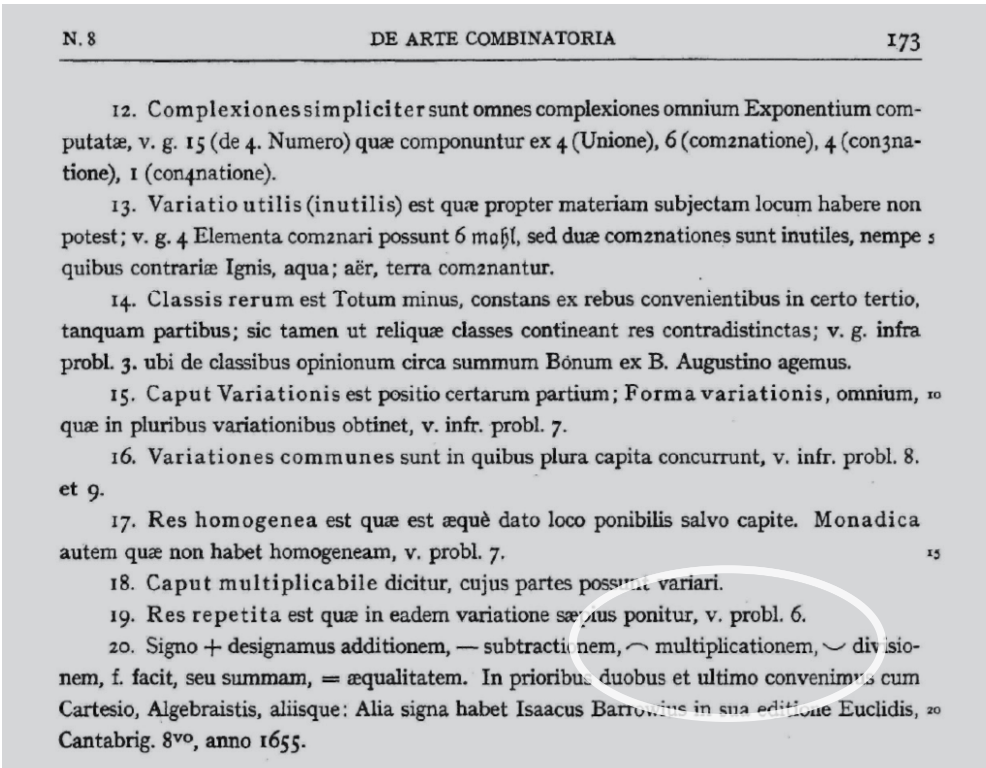
Leibniz-Akademie-Ausgabe (LAA), Series VII / mathematical manuscripts, volume 4, p. 873.
A typical example of a legend at the end of a volume of the Leibniz Academy Edition.

4.a) Historical mathematical operators



16. *Variationes communes* ſunt in quibus plura capita concurrunt, v. infr. probl. 8. & 9.
17. *Res homogenea* eſt quæ eſt æquè dato loco ponibilis ſalvo capite. *Monadica* autem quæ non habet homogeneam. v. probl. 7.
18. *Caput multiplicabile* dicitur, cujus partes poſſunt variari.
19. *Res repetita* eſt quæ in eadem variatione ſæpius ponitur v. probl. 6.
20. Signo + deſignamus additionem, − ſubtractionem, ⌒ multiplicationem, ◡ diviſionem, f. facit, ſeu ſummam, = æqualitatem. In prioribus duobus & ultimo convenimus cum Carteſio, Algebraiſtis, aliiſque: Alia ſigna habet Iſaacus Barrovvius in ſua editione Euclidis, Cantabrig. 8vo, anno 1655.

◡ LEIBNIZIAN DIVISION SIGN, ⌒ LEIBNIZIAN PRODUCT SIGN
Leibniz used these division and multiplication signs in print from the year 1666 onwards and continued to make use of them in his manuscripts in later years.
Leibniz, Dissertatio de arte combinatoria, 1666, p. 5



N. 8 DE ARTE COMBINATORIA 173

12. Complexiones simpliciter sunt omnes complexiones omnium Exponentium computatæ, v. g. 15 (de 4. Numero) quæ componuntur ex 4 (Unione), 6 (com2natione), 4 (con3natione), 1 (con4natione).

13. Variatio utilis (inutilis) est quæ propter materiam subjectam locum habere non potest; v. g. 4 Elementa com2nari possunt 6 mahl, sed duæ com2nationes sunt inutiles, nempe quibus contrariæ Ignis, aqua; aër, terra com2nantur.

14. Classis rerum est Totum minus, constans ex rebus convenientibus in certo tertio, tanquam partibus; sic tamen ut reliquæ classes contineant res contradistinctas; v. g. infra probl. 3. ubi de classibus opinionum circa summum Bonum ex B. Augustino agemus.

15. Caput Variationis est positio certarum partium; Forma variationis, omnium, quæ in pluribus variationibus obtinet, v. infr. probl. 7.

16. Variationes communes sunt in quibus plura capita concurrunt, v. infr. probl. 8. et 9.

17. Res homogenea est quæ est æquè dato loco ponibilis salvo capite. Monadica autem quæ non habet homogeneam, v. probl. 7.

18. Caput multiplicabile dicitur, cujus partes possunt variari.

19. Res repetita est quæ in eadem variatione sæpius ponitur, v. probl. 6.

20. Signo + designamus additionem, — subtractionem, ⌒ multiplicationem, ◡ divisionem, f. facit, seu summam, = æqualitatem. In prioribus duobus et ultimo convenimus cum Cartesio, Algebraistis, aliisque: Alia signa habet Isaacus Barrovius in sua editione Euclidis, Cantabrig. 8vo, anno 1655.

◡ LEIBNIZIAN DIVISION SIGN, ⌒ LEIBNIZIAN PRODUCT SIGN
LAA VI-1 p.173

44 I. GEOMETRISCHE STUDIEN 1672–1676 N. 5_3

cuius latus unum est differentia linearum duarum primae secundaeque, quod est proportionale triangulo linearum. Cum ergo sit hypotenusa trianguli linearum, linea 2da seu AA + DD,rq. et hypotenusa trianguli residui per altitudinem secti AA + DD,rq. − D. erit altitudo ad altidudinem et basis ad basin ut hypotenusa ad hypotenusam, fiet ergo: AA + DD,rq. dat AA + DD,rq. − D. quid dat altitudo D. dabit AA + DD,rq. − D,, ⌒ D,,, ◡ AA + DD,rq. Et quid dat basis A. dabit AA + DD,rq. − D,,⌒ A,,, ◡ AA + DD,rq. Detrahatur haec basis a basi A. fiet

A,,,, − AA + DD,rq. − D,, ⌒ A,,, ◡ AA + DD,rq.

huius Q. addatur quadrato altitudinis fiet Q. cuius rq. est basis quaesita

A,,,, − AA + DD,rq. − D,, ⌒ A,,, ◡ AA + DD,rq.,,,,,Q. + AA + DD,rq. − D,, ⌒ D,,, ◡ AA + DD,rq.,,,,Q.,,,,,,Rq.

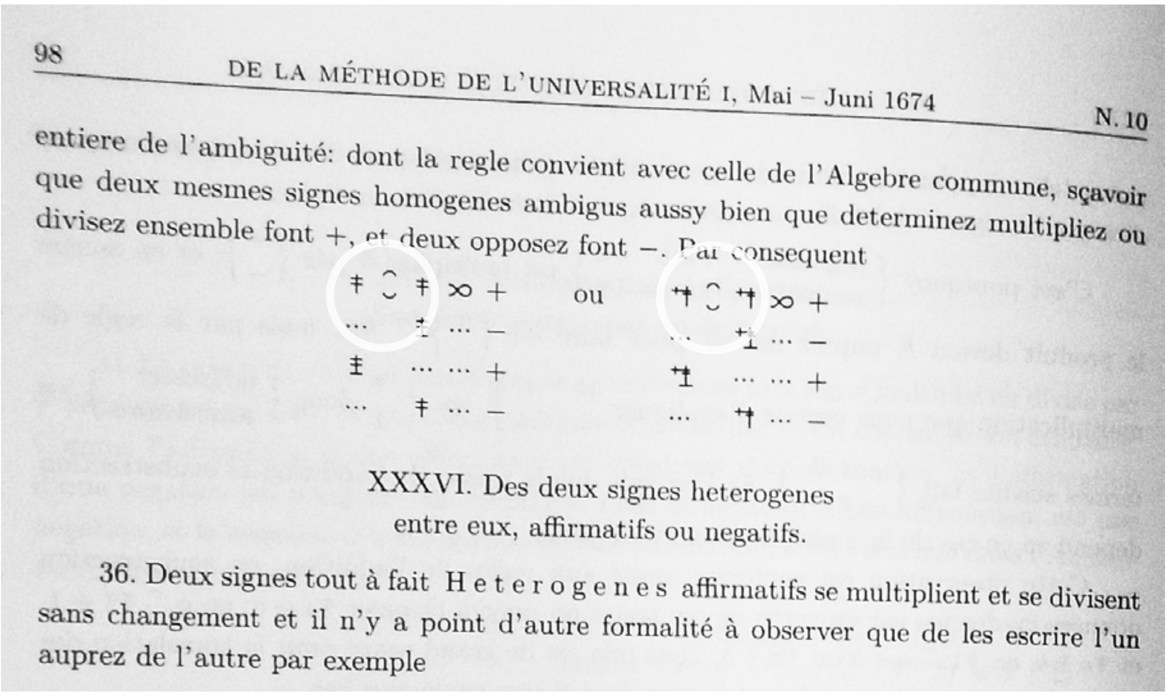
Basis isoscelis dimidii quadratum detrahatur a quadrato lineae primae habebitur altitudo isoscelis

DD,,,,,,,,, − A,,,, − AA + DD,rq. − D,, ⌒ A ,,, ◡ AA + DD,rq.,,,,,Q. + AA + DD,rq. − D,, ⌒ D,,, ◡ AA + DD,rq.,,,,Q.,,,,,,Rq.,,,,,,, ◡ 2,,,,,,,,Q.

Nunc bases quoque et altitudines caeterorum duorum isoscelium investig...

◡ LEIBNIZIAN DIVISION SIGN, ⌒ LEIBNIZIAN PRODUCT SIGN
LAA VII-1 p. 44; VII-3 p. 566 (below)
These two characters should neither be unified with 25E0 and 25E1 (Geometric shapes) nor with 2312 ARC (Miscellaneous technical), because the semantics (and also the expected typographic rendering) are considerably different from these mathematical operators.

idem est ac si spatio $AMCDA$ adderetur segmentum $ACDA$ unde fiet triangulum AMC vel ABC seu semirectangulum sub abscissa et applicata. Igitur $PM \sqcap BC - \frac{AH}{2}$ ducta in $DE \sqcap \beta$, seu βPM, aequatur differentiae inter $\frac{AB \frown BC}{2}$, et $\frac{AB - DE, \frown BC - EC}{2}$ sive $\beta \frown PM \sqcap \frac{\boxed{AB \frown BC - AB \frown BC} - DE \frown BC, -AB \frown EC + DE \frown EC}{2}$. Iam $PM \sqcap BC - \frac{AH}{2}$. et $DE \sqcap \beta$. Ergo $2\beta BC - \beta AH \sqcap -\beta BC - AB \frown EC + \beta EC$, cumque $\beta \frown EC$ negligi possit, fiet: $-3\beta BC + \beta AH \sqcap AB \frown EC$. Est autem $\frac{AH}{FB - AB} \sqcap \frac{BC}{AB}$. sive $AH \sqcap \frac{BC, \frown FB - AB}{AB}$. et $FB \sqcap \frac{BC^2}{BG}$. Ergo $AH \sqcap \frac{BC}{AB}, \frown \frac{BC^2}{BG} - AB$. Idemque $AH \sqcap \frac{AB \frown EC + 3\beta BC}{\beta}$. fiet ergo aequatio inter $\frac{BC^3, -AB^2 \frown BG}{AB \frown BG}$ et $\frac{AB \frown EC + 3\beta BC}{\beta}$, sive inter: $BC^3\beta \; - \; AB^2, BG, \beta \sqcap AB^2, EC, BG \; + \; 3\beta BC, AB, BG$. Pro BG substituatur $\frac{a^2}{BC}$. fiet: $BC^3\beta \; - \; AB^2, \frac{a^2}{BC}, \beta \sqcap AB^2, EC, \frac{a^2}{BC} \; + \; 3\beta BC, AB, \frac{a^2}{BC}$ sive multiplicatis omnibus per BC fiet: $BC^4\beta \; - \; AB^2, a^2\beta \sqcap AB^2, EC, a^2 \; + \; 3\beta BC, AB, a^2$.



entiere de l'ambiguité: dont la regle convient avec celle de l'Algebre commune, sçavoir que deux mesmes signes homogenes ambigus aussy bien que determinez multipliez ou divisez ensemble font +, et deux opposez font −. Par consequent

$\pm \; ◠◡ \; \pm \; \infty \; +$ ou $^{+}\!\pm \; ◠◡ \; ^{+}\!\pm \; \infty \; +$

$\cdots \; \mp \cdots -$ $\cdots \; ^{+}\!\mp \cdots -$

$\mp \cdots \cdots +$ $^{+}\!\mp \cdots \cdots +$

$\cdots \; \pm \cdots -$ $\cdots \; ^{+}\!\pm \cdots -$

XXXVI. Des deux signes heterogenes entre eux, affirmatifs ou negatifs.

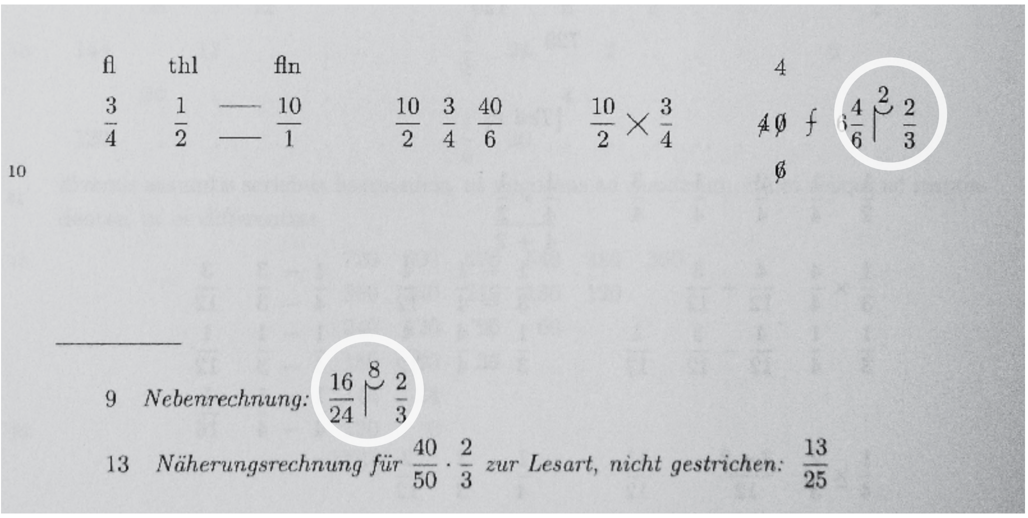
36. Deux signes tout à fait Heterogenes affirmatifs se multiplient et se divisent sans changement et il n'y a point d'autre formalité à observer que de les escrire l'un auprez de l'autre par exemple

◠◡ LEIBNIZIAN DIVISION-PRODUCT SIGN

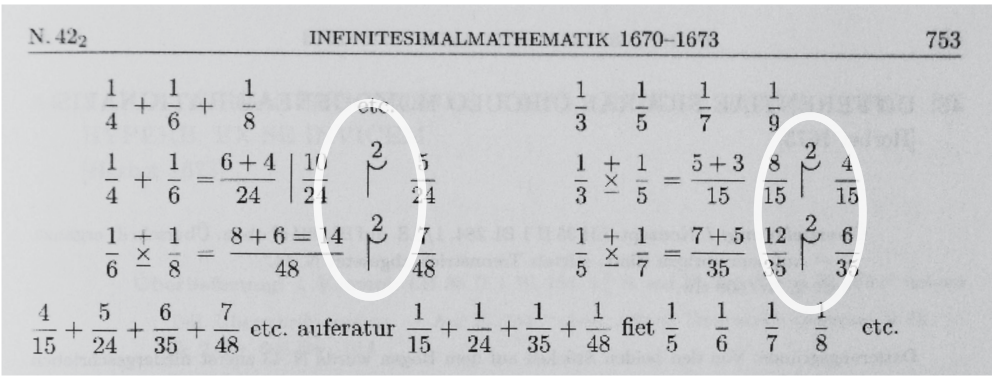
An ambiguity operator sign that combines the Leibnizian division and product signs to denote a product in one and a division in the other case.

Using ambiguity signs (c.f. section c) can result in the need of a product sign in one and a division sign in the second case. To write this down, Leibniz combines his product sign with his division sign.

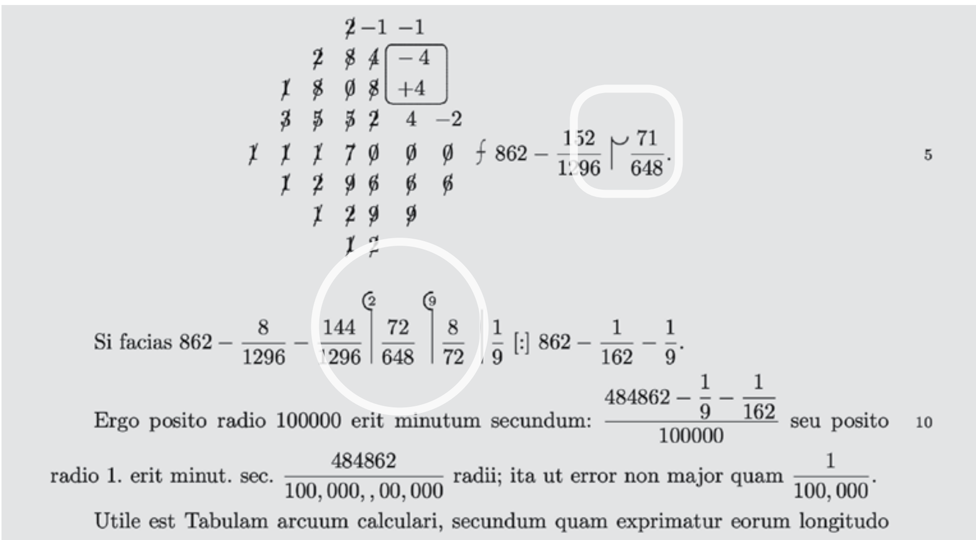
LAA VII-7 p. 98



ᛚ LEIBNIZIAN DIVISION STAFF SIGN 1
LAA VII-3 p. 138



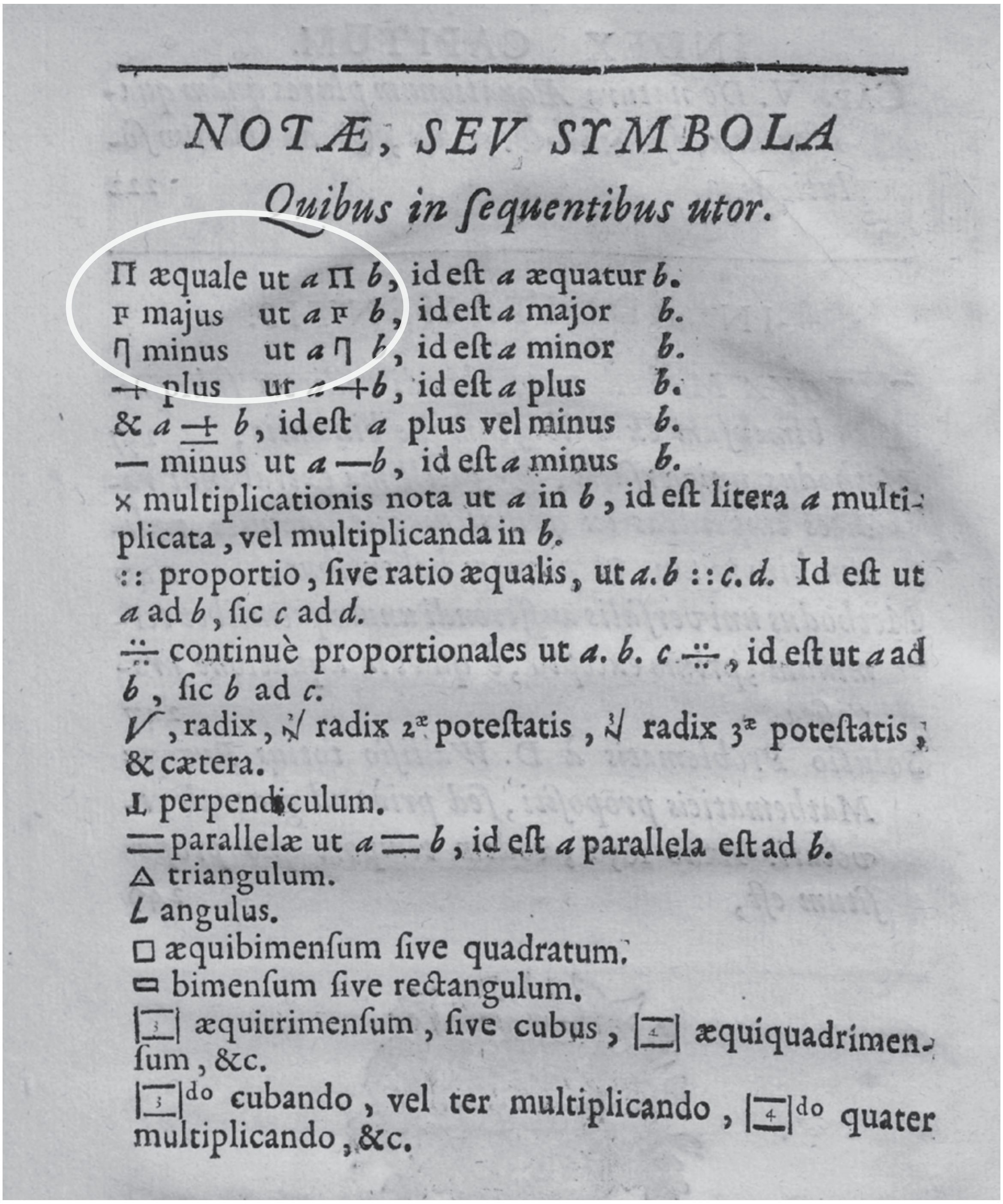
ᛚ LEIBNIZIAN DIVISION STAFF SIGN 1
LAA VII-4 p. 753



ᛚ LEIBNIZIAN DIVISION STAFF SIGN 1 and ⸢ LEIBNIZIAN DIVISION STAFF SIGN 2
LAA VII-6 p. 379

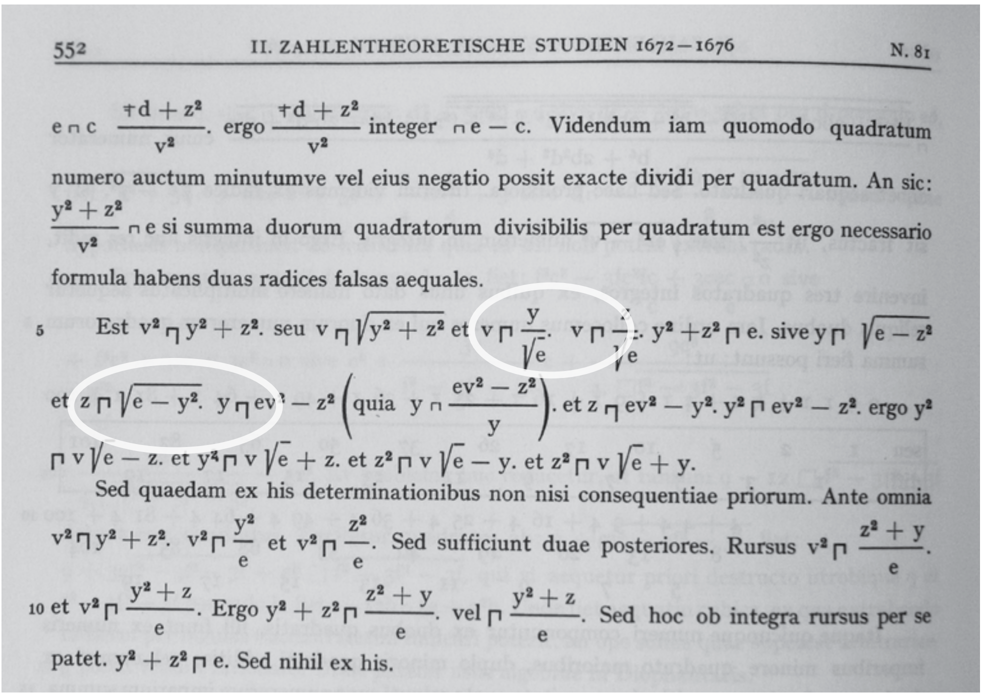
4.b) Historical mathematical relations

Leibniz made use of a fine differentiation of notions of equality and inequality in his mathematical writings. The character ⊓ LEIBNIZIAN EQUAL SIGN signifies in many of his mathematical writings equality in the common meaning as it denotes the equality of two things with regard to some property. Leibniz adopted the symbol (as well as the related symbols for “greater than” and “less than”) probably in 1674, after reading François Dulaurens: Specimina Mathematica Duobus Libris Comprehensa, Paris, 1667 (http://digitale-sammlungen.gwlb.de/resolve?PPN=1066520976).


NOTÆ, SEV SYMBOLA
Quibus in ſequentibus utor.

⊓ æquale ut *a* ⊓ *b*, id eſt *a* æquatur *b*.
╒ majus ut *a* ╒ *b*, id eſt *a* major *b*.
╕ minus ut *a* ╕ *b*, id eſt *a* minor *b*.
+ plus ut *a* + *b*, id eſt *a* plus *b*.
& *a* ± *b*, id eſt *a* plus vel minus *b*.
— minus ut *a* — *b*, id eſt *a* minus *b*.
× multiplicationis nota ut *a* in *b*, id eſt litera *a* multiplicata, vel multiplicanda in *b*.
:: proportio, ſive ratio æqualis, ut *a*. *b* :: *c*. *d*. Id eſt ut *a* ad *b*, ſic *c* ad *d*.
∻ continuè proportionales ut *a*. *b*. *c* ∻, id eſt ut *a* ad *b*, ſic *b* ad *c*.
√, radix, √2 radix 2æ poteſtatis, √3 radix 3æ poteſtatis, & cætera.
⊥ perpendiculum.
= parallelæ ut *a* = *b*, id eſt *a* parallela eſt ad *b*.
△ triangulum.
∠ angulus.
□ æquibimenſum ſive quadratum.
▭ bimenſum ſive rectangulum.
|3| æquitrimenſum, ſive cubus, |4| æquiquadrimenſum, &c.
|3|do cubando, vel ter multiplicando, |4|do quater multiplicando, &c.

╒ LEIBNIZIAN GREATER, ╕ LEIBNIZIAN LESS
Dulaurens, Specimina Mathematica, 1667

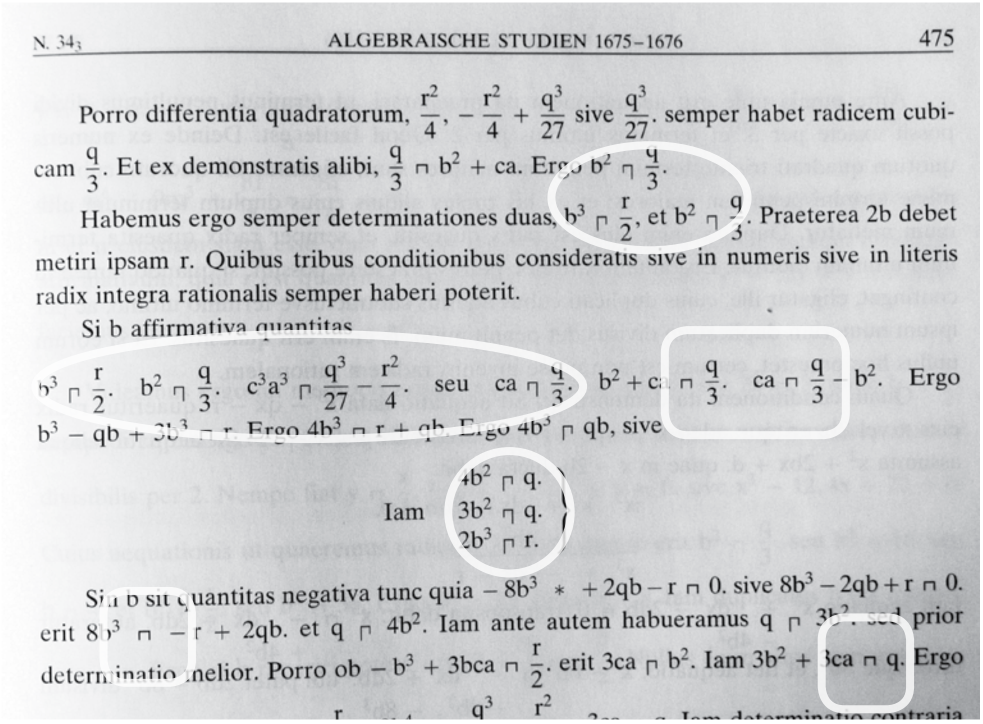

552 II. ZAHLENTHEORETISCHE STUDIEN 1672–1676 N. 81

$e \sqcap c\ \frac{\mp d + z^2}{v^2}$. ergo $\frac{\mp d + z^2}{v^2}$ integer $\sqcap e - c$. Videndum iam quomodo quadratum numero auctum minutumve vel eius negatio possit exacte dividi per quadratum. An sic: $\frac{y^2 + z^2}{v^2} \sqcap e$ si summa duorum quadratorum divisibilis per quadratum est ergo necessario formula habens duas radices falsas aequales.

Est $v^2 \sqcap y^2 + z^2$. seu $v \sqcap \sqrt{y^2 + z^2}$ et $v \sqcap \frac{y}{\sqrt{e}}$. $v \sqcap \frac{z}{\sqrt{e}}$. $y^2 + z^2 \sqcap e$. sive $y \sqcap \sqrt{e - z^2}$ et $z \sqcap \sqrt{e - y^2}$. $y \sqcap ev^2 - z^2$ (quia $y \sqcap \frac{ev^2 - z^2}{y}$). et $z \sqcap ev^2 - y^2$. $y^2 \sqcap ev^2 - z^2$. ergo $y^2 \sqcap v\sqrt{e} - z$. et $y^2 \sqcap v\sqrt{e} + z$. et $z^2 \sqcap v\sqrt{e} - y$. et $z^2 \sqcap v\sqrt{e} + y$.

Sed quaedam ex his determinationibus non nisi consequentiae priorum. Ante omnia $v^2 \sqcap y^2 + z^2$. $v^2 \sqcap \frac{y^2}{e}$ et $v^2 \sqcap \frac{z^2}{e}$. Sed sufficiunt duae posteriores. Rursus $v^2 \sqcap \frac{z^2 + y}{e}$. et $v^2 \sqcap \frac{y^2 + z}{e}$. Ergo $y^2 + z^2 \sqcap \frac{z^2 + y}{e}$. vel $\sqcap \frac{y^2 + z}{e}$. Sed hoc ob integra rursus per se patet. $y^2 + z^2 \sqcap e$. Sed nihil ex his.

⊓ LEIBNIZIAN GREATER, ⊓ LEIBNIZIAN LESS
LAA VII-1 p. 552


N. 34_3 ALGEBRAISCHE STUDIEN 1675–1676 475

Porro differentia quadratorum, $\frac{r^2}{4}, -\frac{r^2}{4} + \frac{q^3}{27}$ sive $\frac{q^3}{27}$. semper habet radicem cubicam $\frac{q}{3}$. Et ex demonstratis alibi, $\frac{q}{3} \sqcap b^2 + ca$. Ergo $b^2 \sqcap \frac{q}{3}$.

Habemus ergo semper determinationes duas, $b^3 \sqcap \frac{r}{2}$, et $b^2 \sqcap \frac{q}{3}$. Praeterea 2b debet metiri ipsam r. Quibus tribus conditionibus consideratis sive in numeris sive in literis radix integra rationalis semper haberi poterit.

Si b affirmativa quantitas

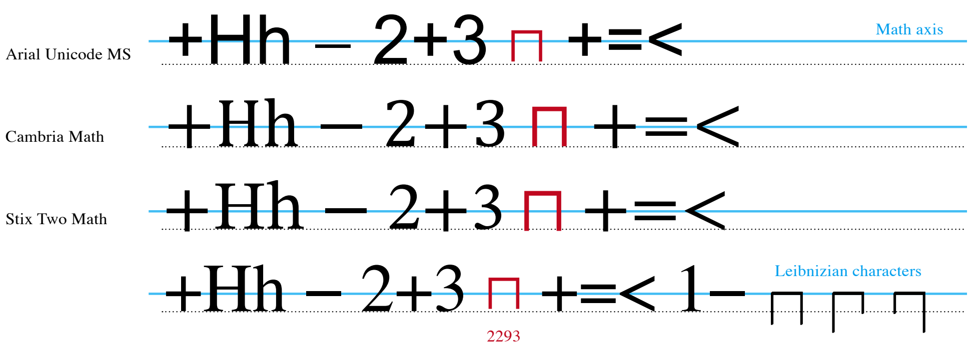
$b^3 \sqcap \frac{r}{2}$. $b^2 \sqcap \frac{q}{3}$. $c^3a^3 \sqcap \frac{q^3}{27} - \frac{r^2}{4}$. seu $ca \sqcap \frac{q}{3}$. $b^2 + ca \sqcap \frac{q}{3}$. $ca \sqcap \frac{q}{3} - b^2$. Ergo $b^3 - qb + 3b^3 \sqcap r$. Ergo $4b^3 \sqcap r + qb$. Ergo $4b^3 \sqcap qb$, sive

Iam $4b^2 \sqcap q$. $3b^2 \sqcap q$. $2b^3 \sqcap r$.

Si b sit quantitas negativa tunc quia $-8b^3 * +2qb - r \sqcap 0$. sive $8b^3 - 2qb + r \sqcap 0$. erit $8b^3 \sqcap -r + 2qb$. et $q \sqcap 4b^2$. Iam ante autem habueramus $q \sqcap 3b^2$. sed prior determinatio melior. Porro ob $-b^3 + 3bca \sqcap \frac{r}{2}$. erit $3ca \sqcap b^2$. Iam $3b^2 + 3ca \sqcap q$. Ergo

⊓ LEIBNIZIAN EQUAL SIGN, ⊓ LEIBNIZIAN GREATER, ⊓ LEIBNIZIAN LESS
LAA VII-2 p. 475

Leibniz made use of subtle distinctions with notions of equality and inequality, in his mathematical writings. He adopted the symbol ⊓ (as well as the related symbols for “greater than” and “less than”) probably in 1674, after reading François Dulaurens: Specimina Mathematica Duobus Libris Comprehensa, Paris, 1667.
Whereas the printer of Dulaurens’ book used a capital letter Greek pi type as a symbol for equality and made the signs for greater and less ad hoc and uneven, in Leibniz’s manuscripts we encounter a well-considered coordination of these signs: The equals sign represents, as it were, a balance beam with two equal weights symbolized by the vertical strokes. For greater and less, respectively, vertical strokes of unequal length are used. The signs are aligned vertically according to the minus sign, with it’s horizontal bar matching at the same height. This establishes a significant difference to the otherwise quite similar character SQUARECAP (2293).
Translated to font technique, Leibniz’s original alignment of his equal/greater/less signs with minus requires a position of the glyph’s horizotal parts with the *math axis*. This alignment would, on the other hand, be inappropriate for 2293 and related characters.



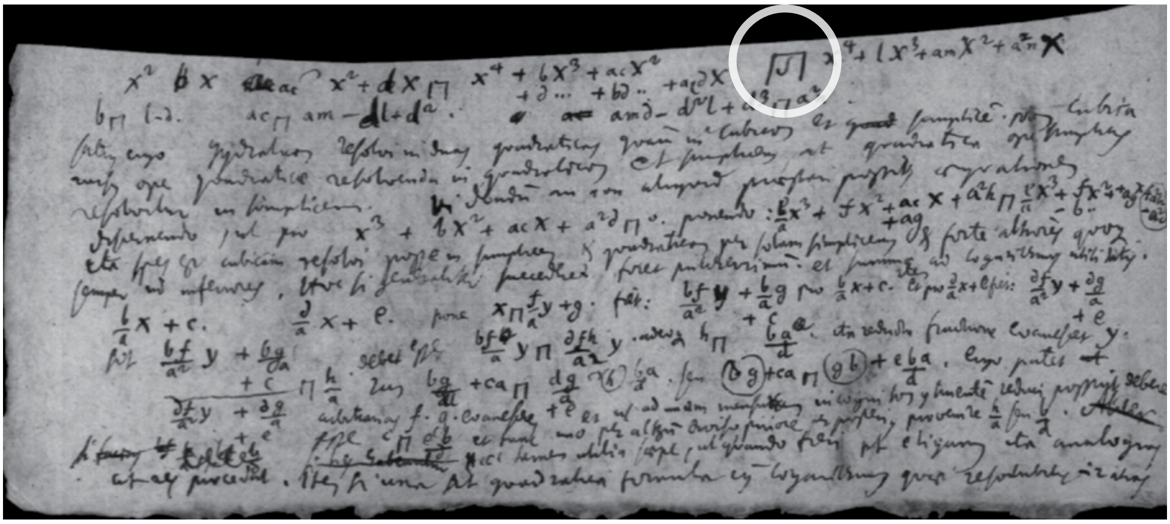
The character 2293 is positioned typically on the baseline in most fonts, whereas the Leibnizian characters (on the right) require a vertical adjustment of their top part with the math axis.

Due to their semantical connections, the 2293 ⊓ SQUARE CAP, 2229 ∩ INTERSECTION, 222A ∪ UNION and 2294 ⊔ SQUARE CUP characters need a strong consistency in their visual representation. On the other hand, the same is needed for ⊓ LEIBNIZIAN DOUBLE EQUAL SIGN, ⊓ LEIBNIZIAN EQUALITY WITH S SIGN, ⊓ LEIBNIZIAN GREATER, ⊓ LEIBNIZIAN LESS, ⊓ LEIBNIZIAN GREATER WITH P, ⊓ LEIBNIZIAN LESS WITH P, ⊓ LEIBNIZIAN GREATER-LESS SIGN. Whereas all these Leibnizian characters have their horizontal line matching the vertical position of 2212 − MINUS SIGN (the math axis), the existing characters of modern set theory are situated on the baseline, reaching a height usually between x-height and capital height.

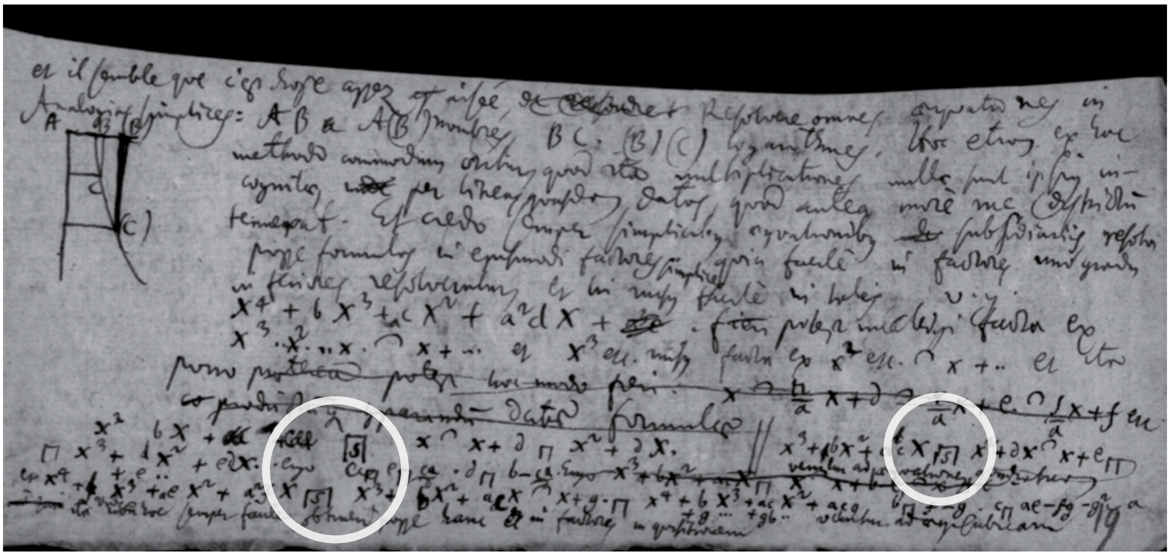
ab ac ad ae af	tions (B). The solution: let the number be multiplied by
bc bd be bf	one less than the number; half of the product will be what
cd ce cf	is required. That is, (A ∩ (A – 1)) ∪ 2 = B. For example, let
de df	the Number be 6 ∩ 5, f. 30 ∪ 2, makes 15.’ The Reason for
ef	the Solution: draw Table ℷ, in which the possible
	com2nations of 6 things *abcdef* are enumerated.

This example of replacing the Leibnizian product and division signs by 2229 ∩ INTERSECTION and 222A ∪ UNION leads to misunderstanding and confusion in reading for mathematicians and historians of mathematics. As literature on the history of Leibniz’s mathematics and on the history of more recent mathematics is published in the same journals and collective volumes and historic and modern notation has to be used in interpreting the source texts, there is the need to distinguish both character groups within a math font. The same applies to the Leibnizian equality/inequality sign group.

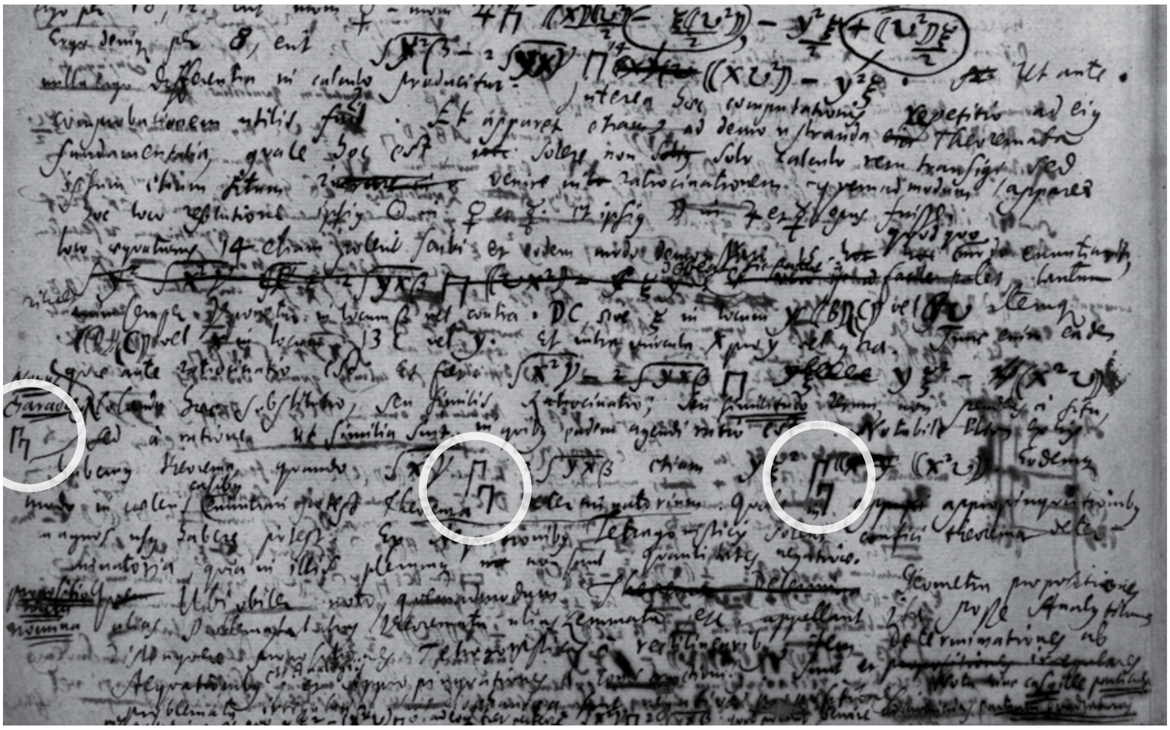
Leibniz derived the configurations of several other signs from ⊓ LEIBNIZIAN EQUAL SIGN:
The sign ⊓s LEIBNIZIAN EQUALITY WITH S SIGN denotes a kind of equality by definition that originates from equating two expressions with each other as in the phrase “let *a* be equal to *b*”. Unlike the definition sign in modern mathematics, there is no specific direction in Leibniz’s sign. The “s” in the sign is an abbreviation of the Latin word “sit”.
Combining both signs (┌ and ┐) into ┌┐ LEIBNIZIAN GREATER-LESS SIGN leads to an ambiguous inequality sign that denotes “greater than” in the first case, and “less than” in the second.



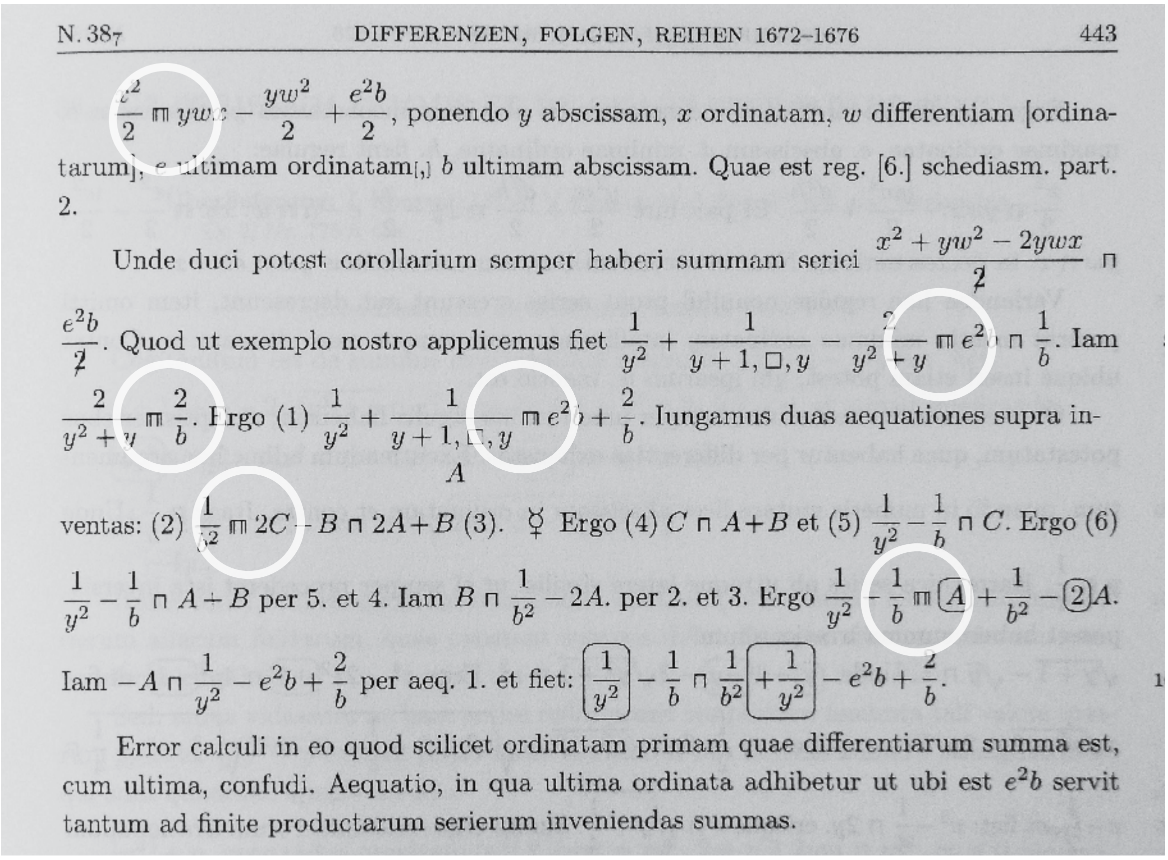
⊓s LEIBNIZIAN EQUALITY WITH S SIGN
LH 35 V 14, fol. 18r. *The edition of this manuscript is currently in progress.*



⊓s LEIBNIZIAN EQUALITY WITH S SIGN
LH 35 V 14, fol. 19r. *The edition of this manuscript is currently in progress.*



$\sqcap\!\!\Gamma$ LEIBNIZIAN GREATER-LESS SIGN
LH 35 XIII 3, fol. 150v. *The edition of this manuscript is currently in progress*.



N. 38₇ DIFFERENZEN, FOLGEN, REIHEN 1672–1676 443

$\frac{e^2}{2} \sqcap\!\!\sqcap yw..x - \frac{yw^2}{2} + \frac{e^2b}{2}$, ponendo y abscissam, x ordinatam, w differentiam [ordinatarum], e ultimam ordinatam[,] b ultimam abscissam. Quae est reg. [6.] schediasm. part. 2.

Unde duci potest corollarium semper haberi summam seriei $\frac{x^2 + yw^2 - 2ywx}{2} \sqcap \frac{e^2b}{2}$. Quod ut exemplo nostro applicemus fiet $\frac{1}{y^2} + \frac{1}{y+1,\square,y} - \frac{2}{y^2+y} \sqcap\!\!\sqcap e^2b \sqcap \frac{1}{b}$. Iam $\frac{2}{y^2+y} \sqcap\!\!\sqcap \frac{2}{b}$. Ergo (1) $\frac{1}{y^2} + \underbrace{\frac{1}{y+1,\square,y}}_{A} \sqcap\!\!\sqcap e^2b + \frac{2}{b}$. Iungamus duas aequationes supra inventas: (2) $\frac{1}{b^2} \sqcap\!\!\sqcap 2C - B \sqcap 2A + B$ (3). ☿ Ergo (4) $C \sqcap A + B$ et (5) $\frac{1}{y^2} - \frac{1}{b} \sqcap C$. Ergo (6) $\frac{1}{y^2} - \frac{1}{b} \sqcap A + B$ per 5. et 4. Iam $B \sqcap \frac{1}{b^2} - 2A$. per 2. et 3. Ergo $\frac{1}{y^2} - \frac{1}{b} \sqcap\!\!\sqcap \boxed{A} + \frac{1}{b^2} - \textcircled{2}A$. Iam $-A \sqcap \frac{1}{y^2} - e^2b + \frac{2}{b}$ per aeq. 1. et fiet: $\boxed{\frac{1}{y^2}} - \frac{1}{b} \sqcap \frac{1}{b^2}\boxed{+\frac{1}{y^2}} - e^2b + \frac{2}{b}$.

Error calculi in eo quod scilicet ordinatam primam quae differentiarum summa est, cum ultima, confudi. Aequatio, in qua ultima ordinata adhibetur ut ubi est e^2b servit tantum ad finite productarum serierum inveniendas summas.

$\sqcap\!\!\sqcap$ LEIBNIZIAN DOUBLE EQUAL SIGN
LAA VII-3 p. 443

Notae Algebraicae usitatiores

Additio:	$a+b$	
Substractio	$a-b$	
Multiplicatio	$a \frown b$	vel ab vel $2\cdot 3$ id est 6, vel $2\cdot 3\cdot 5$ id est 30
Divisio	$a:b$	vel $\frac{a}{b}$. Eodem modo exprimitur r a t i o a ad b
Ductio in se seu potentia	$\boxed{3}a$	id est a^3 vel cubus ab a
Extractio	$\sqrt[3]{a}$	radix cubica de a, vel $\boxed{\frac{1}{3}}a$
Aequalitas,	$a=b$	
Majoritas	$a \sqcap b$	a majus quam b
Minoritas	$a \sqcap b$	a minus quam b

⊓ LEIBNIZIAN GREATER, ⊓ LEIBNIZIAN LESS
LAA III-7 p. 597

$$2+\frac{1}{99}$$

$$v \sqcap_p \frac{zc}{100^5}. \qquad v \sqcap_p \frac{zc}{100^5}+1.$$

$$\frac{v}{c} \sqcap \frac{z}{100^5}+e. \qquad \frac{v}{c} \sqcap_p \frac{z}{100^5}. \text{ Ergo } \frac{v100^5}{c100^5} \sqcap_p \frac{zc}{c100^5}.$$

$$\frac{v}{c} \sqcap_p \frac{z}{100^5}+1. \qquad \frac{v100^5}{c100^5} [\sqcap_p] \frac{zc}{c100^5}+1.$$

[Tschirnhaus mit Ergänzungen von Leibniz]

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}+\frac{1}{e}$$

⊓p LEIBNIZIAN GREATER WITH P, ⊓p LEIBNIZIAN LESS WITH P, ⊓ LEIBN. EQUAL SIGN
These signs denote "a little bit greater" and "a little bit less", the letter "p" abbreviating the Latin word "paulo" (little).
LAA VII-3 p. 732

(7) Ungleichungen:

Zusätzlich zu den üblichen Symbolen ⊓ für „größer" und ⊓ für „kleiner" (N. 66) führt Leibniz noch Zeichen für „ein wenig größer" (⊓p) bzw. „ein wenig kleiner" (⊓p) ein (N. 54).

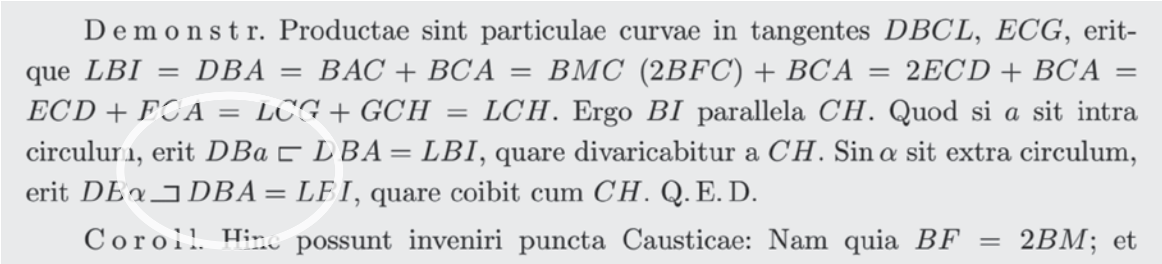
⊓p LEIBNIZIAN GREATER WITH P, ⊓p LEIBNIZIAN LESS WITH P
LAA VII-3 p. XXXI

$$v \sqcap_p \frac{zc}{100^5}. \qquad v \sqcap_p \frac{zc}{100^5}+1.$$

$$\frac{v}{c} \sqcap \frac{z}{100^5}+e. \qquad \frac{v}{c} \sqcap_p \frac{z}{100^5}. \text{ Ergo } \frac{v100^5}{c100^5} \sqcap_p \frac{zc}{c100^5}.$$

$$\frac{v}{c} \sqcap_p \frac{z}{100^5}+1. \qquad \frac{v100^5}{c100^5} [\sqcap_p] \frac{zc}{c100^5}+1.$$

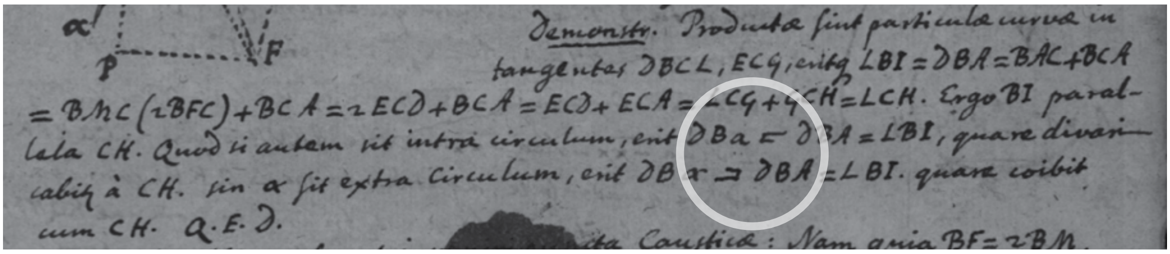
⊓p LEIBNIZIAN GREATER WITH P, ⊓p LEIBNIZIAN LESS WITH P
LAA VII-3 p. 732



Demonstr. Productae sint particulae curvae in tangentes $DBCL$, ECG, eritque $LBI = DBA = BAC + BCA = BMC$ $(2BFC) + BCA = 2ECD + BCA = ECD + ECA = LCG + GCH = LCH$. Ergo BI parallela CH. Quod si a sit intra circulum, erit DBa ⊏ $DBA = LBI$, quare divaricabitur a CH. Sin α sit extra circulum, erit $DB\alpha$ ⊐ $DBA = LBI$, quare coibit cum CH. Q. E. D.

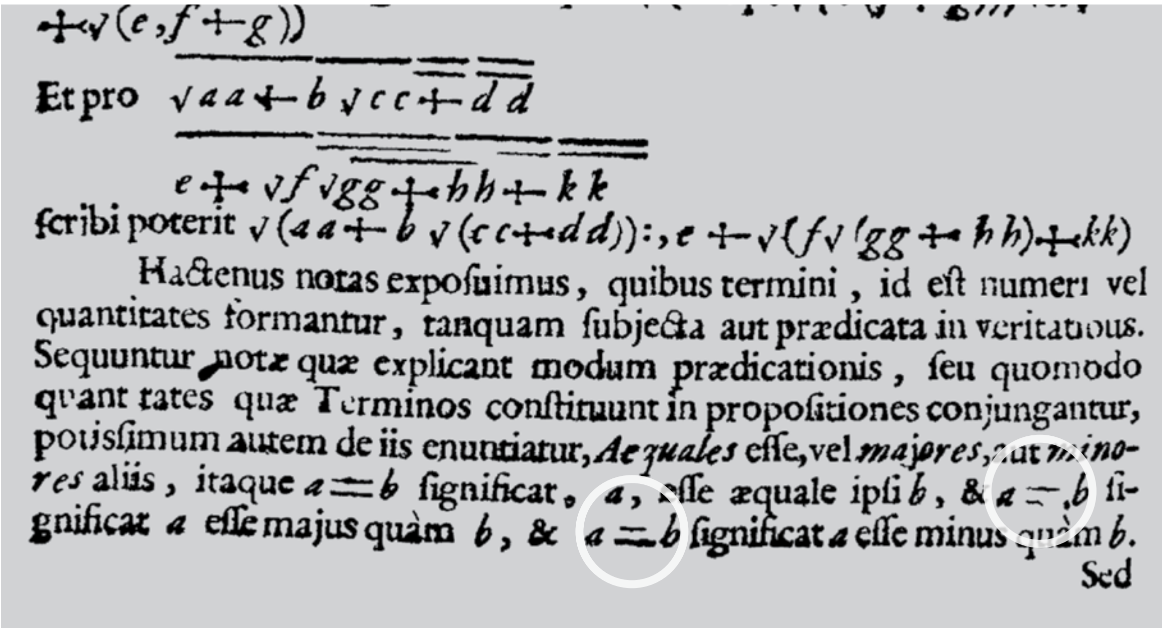
Coroll. Hinc possunt inveniri puncta Causticae: Nam quia $BF = 2BM$; et

⊏ BERNOULLIAN GREATER, ⊐ BERNOULLIAN LESS
LAA III-6 p. 688 and corresponding manuscript part (below)



Demonstr. Productae sint particulae curvae in tangentes DBCL, ECG, eritque LBI = DBA = BAC + BCA = BMC (2BFC) + BCA = 2ECD + BCA = ECD + ECA = LCG + GCH = LCH. Ergo BI parallela CH. Quod si autem sit intra circulum, erit DBa ⊏ DBA = LBI, quare divaricabitur à CH. Sin α sit extra Circulum, erit DBα ⊐ DBA = LBI. quare coibit cum CH. Q. E. D.
... Caustica: Nam quia BF = 2BM.

Distinct from the above signs are these two greater / less signs, which lack the vertical part.
A distinction of the two character pairs is neccessary for editorial reasons.



+√(e,f+g))

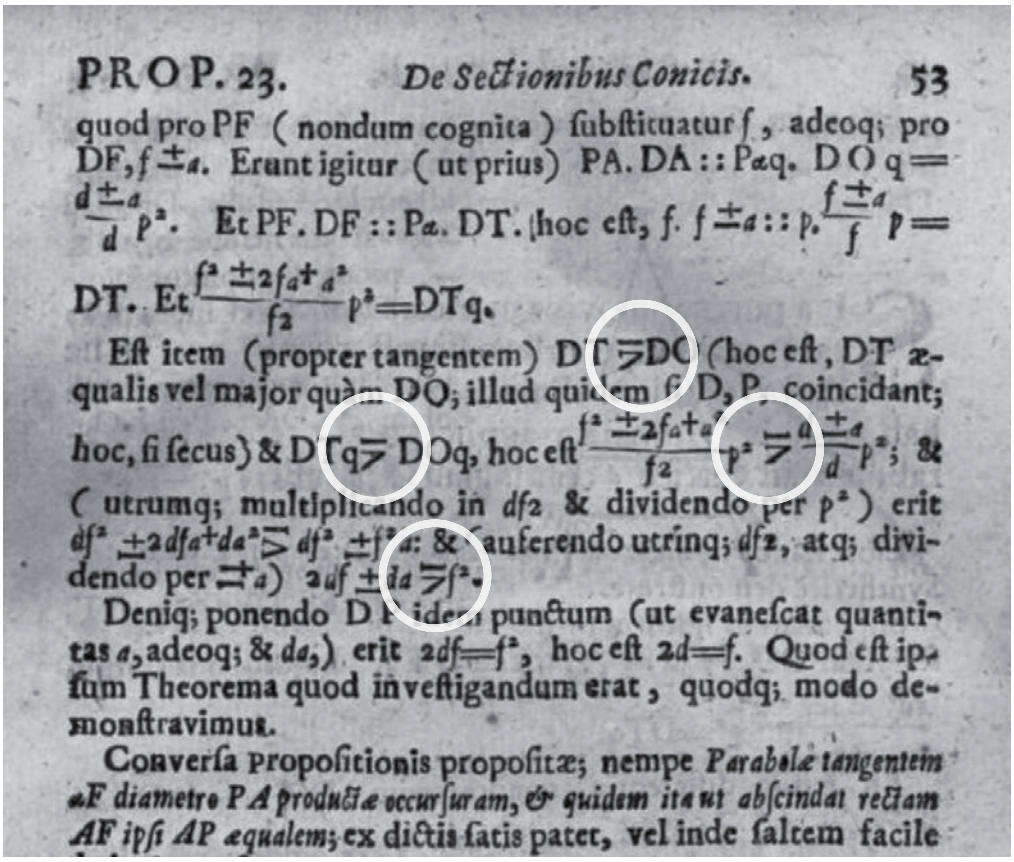
Et pro $\overline{\sqrt{aa+b\sqrt{cc+dd}}}$

$e+\sqrt{f\sqrt{gg+hh}+kk}$

ſcribi poterit √(aa+b√(cc+dd)):,e+√(f√(gg+hh)+kk)

Hactenus notas expoſuimus, quibus termini, id eſt numeri vel quantitates formantur, tanquam ſubjecta aut prædicata in veritatibus. Sequuntur notæ quæ explicant modum prædicationis, ſeu quomodo quantitates quæ Terminos conſtituunt in propoſitiones conjungantur, potiſſimum autem de iis enuntiatur, *Aequales* eſſe, vel *majores*, aut *minores* aliis, itaque *a* = *b* ſignificat, *a*, eſſe æquale ipſi *b*, & *a* ≓ *b* ſignificat *a* eſſe majus quàm *b*, & *a* ≒ *b* ſignificat *a* eſſe minus quàm *b*. Sed

≓ GREATER 2, ≒ LESS 2
Monitum de Characteribus Algebraica, Miscellanea Berolinensia, 1710, p. 158



PROP. 23. *De Sectionibus Conicis.* 53

quod pro PF (nondum cognita) substituatur f, adeoq; pro DF, $f \pm a$. Erunt igitur (ut prius) PA.DA :: Pαq. DOq = $\frac{d \pm a}{d} p^2$. Et PF.DF :: Pα.DT. (hoc est, $f.f \pm a :: p.\frac{f \pm a}{f} p =$ DT. Et $\frac{f^2 \pm 2fa + a^2}{f2} p^2 =$ DTq.

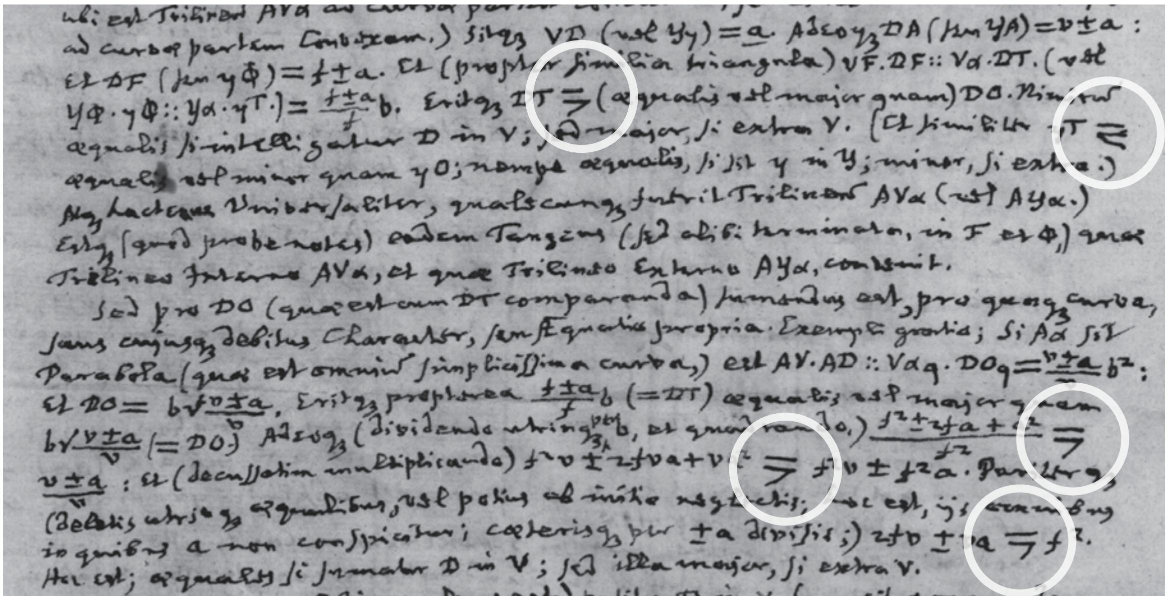
Est item (propter tangentem) DT ⋝ DO (hoc est, DT æqualis vel major quàm DO; illud quidem si D, P coincidant; hoc, si secus) & DTq ⋝ DOq, hoc est $\frac{f^2 \pm 2fa + a^2}{f2} p^2$ ⋝ $\frac{d \pm a}{d} p^2$; & (utrumq; multiplicando in $df2$ & dividendo per p^2) erit $df^2 \pm 2dfa + da^2$ ⋝ $df^2 \pm f^2 a$: & auferendo utrinq; $df2$, atq; dividendo per $\pm a$) $2df \pm da$ ⋝ f^2.

Deniq; ponendo D P idem punctum (ut evanescat quantitas a, adeoq; & da,) erit $2df = f^2$, hoc est $2d = f$. Quod est ipsum Theorema quod investigandum erat, quodq; modo demonstravimus.

Conversa propositionis propositæ; nempe *Parabolæ tangentem αF diametro PA productæ occursuram, & quidem ita ut abscindat rectam AF ipsi AP æqualem*; ex dictis satis patet, vel inde saltem facile

⋝ PARALLEL GREATEREQUAL,
Wallis, De sectionibus conicis nova methodo expositis tractatus, 1655; p. 53

In these historic symbols for "lessequal" and "greaterequal" the "=" strokes are on top of the glyphs, whereas in the existing characters 29A4 and 29A5 they appear on the bottom of the glyphs. We reagard this a sufficient difference to disunify the two character pairs.

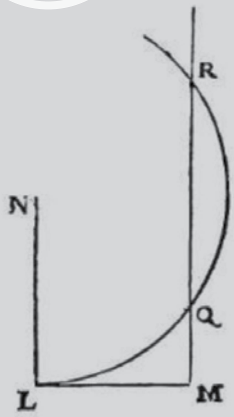


⋝ PARALLEL GREATEREQUAL, ⋜ PARALLEL LESSEQUAL
Manuscript of J. Wallis, LBr 974, 28v.

angle, iuſques a O, en ſorte qu'N O ſoit eſgale a N L, la toute OM eſt z la ligne cherchée. Et elle s'exprime en cete ſorte

z ꝏ $\frac{1}{2}a + \sqrt{\frac{1}{4}aa + bb}$.

Que ſi iay yy ꝏ $-\,ay + bb$, & qu'y ſoit la quantité qu'il faut trouuer, ie fais le meſme triangle rectangle N L M, & de ſa baze M N i'oſte N P eſgale a N L, & le reſte P M eſt y la racine cherchée. De façon que iay y ꝏ $-\,\frac{1}{2}a + \sqrt{\frac{1}{4}aa + bb}$. Et tout de meſme ſi i'auois x^4 ꝏ $-\,ax^2 + b^2$. P M ſeroit x^2. & i'aurois x ꝏ $\sqrt{-\,\frac{1}{2}a + \sqrt{\frac{1}{4}aa + bb}}$: & ainſi des autres.



Enfin ſi i'ay

z^2 ꝏ $az - bb$:

ie fais N L eſgale à $\frac{1}{2}a$, & L M eſgale à b cõme deuãt, puis, au lieu de ioindre les poins M N, ie tire M Q R parallele a L N. & du centre N par L ayant deſcrit vn cercle qui la couppe aux poins Q & R, la ligne cherchée z eſt M Q, oubiẽ M R, car en ce cas elle s'exprime en deux façons, a ſçauoir z ꝏ $\frac{1}{2}a + \sqrt{\frac{1}{4}aa - bb}$, & z ꝏ $\frac{1}{2}a - \sqrt{\frac{1}{4}aa - bb}$.

Et ſi le cercle, qui ayant ſon centre au point N, paſſe par le point L, ne couppe ny ne touche la ligne droite M Q R, il n'y a aucune racine en l'Equation, de façon qu'on peut aſſurer que la conſtruction du probleſme propoſé eſt impoſſible.

ꝏ CARTESIAN EQUAL SIGN
Descartes, La Géométrie, 1637, p. 303
The type composer seems to have utilized a turned *æ* letter as a makeshift for that special symbol here, from which he carved off the horizontal bar of the *e* in some instances.

2°.

— in — facit +

Esto +*d* — *e* ducendum in +*c* — *b*, quod erit per praecedentem +*dc* — *db* — *ec* · *eb*

Sumpto +*v* ꝏ +*d* — *e*, et +*x* ꝏ +*c* — *b* erit etiam

+*v* + *e* ꝏ +*d*, et +*x* + *b* ꝏ +*c* et ad invicem ductis

+*vx* ꝏ +*dc* — *db* — *ec* · *eb* et

+*vx* + *vb* + *ex* + *eb* ꝏ +*dc*

Et locis +*v* et +*x* in hac ultima aequatione, ubi simul non conjunguntur, substitutis ipsarum valoribus, erit

+*vx* + *bd* — *be* + *ec* — *eb* + *eb* ꝏ +*dc* quae reducitur ad

ꝏ CARTESIAN EQUAL SIGN
LAA III-2 p. 698. – Equal sign introduced and mainly used by René Descartes.

43. JOHANN JAKOB FERGUSON FÜR LEIBNIZ

[Hannover, Frühjahr 1680]. [42. 44.]

Überlieferung:

K Abfertigung: *LH* XXXV 12,2 Bl. 32—33. 1 Bog. 2°. 1 S. (Bl. 33 v°). Bemerkung von Leibniz' Hand. Auf Bl. 32 r° Aufzeichnung von Leibniz zur gleichen Thematik; auf Bl. 33 r° und Bl. 32 v° Aufzeichnung von Leibniz zum Alhazenschen Problem. — (Unsere Druckvorlage)

Ponatur latus quadrati ꝏ *ax* + *b* eritque

quadratum *aaxx* + 2*abx* + *bb*

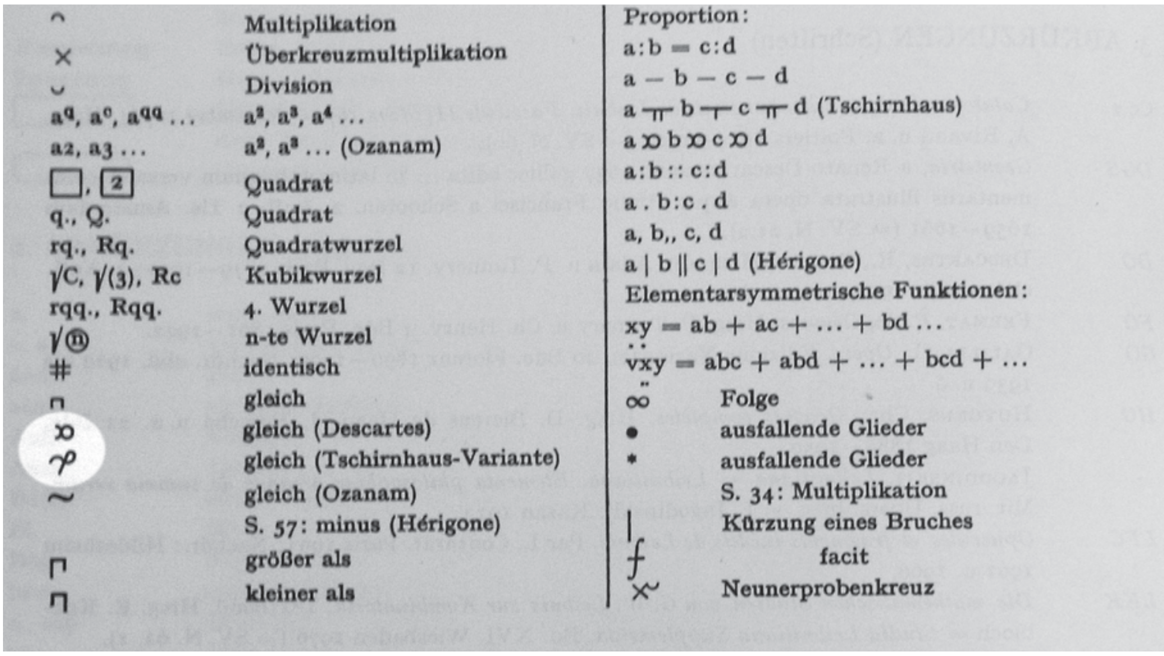
addatur *c*

et Cubus *aaxx* + 2*abx* + *bb* + *c*, aequale cubo cujus latus *dx* + *f* ergo ꝏ d^3x^3 + 3*ddfxx* + 3*dffx* + f^3, sit jam *bb* + *c* ꝏ f^3 habebitur d^3x^3 + 3*ddfxx* + 3*dffx* ꝏ *aaxx* + 2*abx* sive d^3xx + 3*ddfx* + 3*dff* ꝏ *aax* + 2*ab* sit iterum 3*dff* ꝏ 2*ab*, et erit d^3xx + 3*ddfx* ꝏ *aax* sive d^3x + 3*ddf* ꝏ *aa* vel *x* ꝏ $\frac{aa - 3ddf}{d^3}$ unde *dx* + *f* ꝏ $\frac{aa - 2ddf}{dd}$ sive ꝏ $\frac{aa}{dd}$ — 2*f* latus Cubi.

ꝏ CARTESIAN EQUAL SIGN. LAA III-3 p. 102.

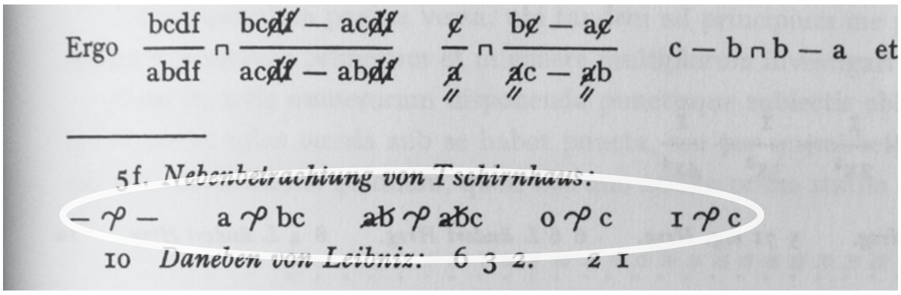
			neum $\delta\gamma\pi\rho$	tur spatium $\delta\gamma\pi\rho$.
ibid.		l. 24.	$\frac{trdy}{z} = rdx$	v. pag. 284. l. 14. ubi *dy* ꝏ $\frac{zdx}{t}$.
pag.	286.	l. 9.	- - - - -	Nam $\frac{zdz}{\sqrt{rr - zz}}$ ꝏ *FE*, omnia autem *FE* ꝏ *CA* seu $\sqrt{rr - zz}$, *z* indefinite accipitur pro quavis *DG*.

ꝏ CARTESIAN EQUAL SIGN
LAA III-7 p. 137



⌢	Multiplikation	Proportion:	
×	Überkreuzmultiplikation	a:b = c:d	
⌣	Division	a − b − c − d	
a^q, a^c, a^{qq} ...	a^2, a^3, a^4 ...	a ⊤ b ⊤ c ⊤ d (Tschirnhaus)	
a2, a3 ...	a^2, a^3 ... (Ozanam)	a ꝏ b ꝏ c ꝏ d	
□, [2]	Quadrat	a:b :: c:d	
q., Q.	Quadrat	a . b:c . d	
rq., Rq.	Quadratwurzel	a, b,, c, d	
√C, √(3), Rc	Kubikwurzel	a | b || c | d (Hérigone)	
rqq., Rqq.	4. Wurzel	Elementarsymmetrische Funktionen:	
√(n)	n-te Wurzel	xy = ab + ac + ... + bd ...	
#	identisch	vxy = abc + abd + ... + bcd + ...	
⊓	gleich	∞	Folge
ꝏ	gleich (Descartes)	•	ausfallende Glieder
℘	gleich (Tschirnhaus-Variante)	*	ausfallende Glieder
~	gleich (Ozanam)		S. 34: Multiplikation
	S. 57: minus (Hérigone)	|	Kürzung eines Bruches
⊏	größer als	∫	facit
⊐	kleiner als	✗	Neunerprobenkreuz

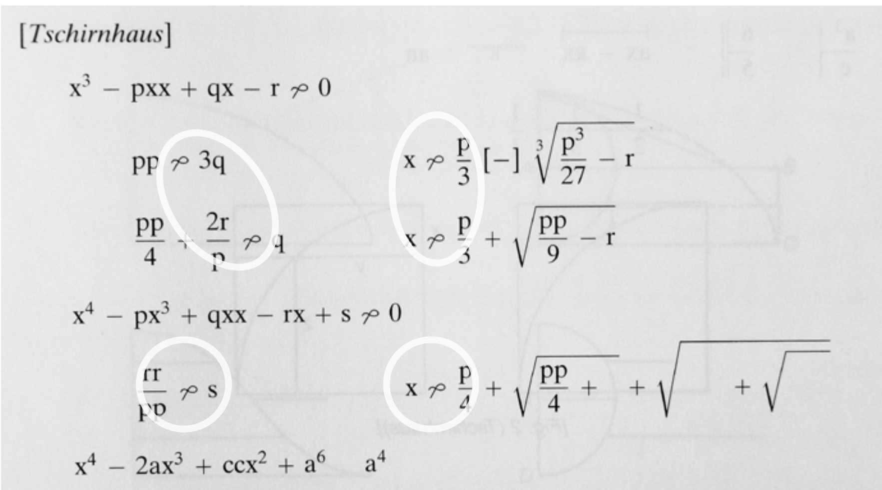
ꝏ CARTESIAN EQUAL SIGN, ℘ TSCHIRNHAUS EQUAL SIGN
This example shows the distinction of the two similar historic equal signs in the Leibniz edition.



Ergo $\frac{bcdf}{abdf}$ ⊓ $\frac{bcdf - acdf}{acdf - abdf}$ ⊓ $\frac{c}{a}$ ⊓ $\frac{bc - ac}{ac - ab}$ c − b ⊓ b − a et

5f. *Nebenbetrachtung von Tschirnhaus:*
− ℘ − a ℘ bc ab ℘ abc 0 ℘ c 1 ℘ c
10 *Daneben von Leibniz:* 6 3 2. 2 1

℘ TSCHIRNHAUS EQUAL SIGN
LAA III-1 p. 595



[*Tschirnhaus*]

$x^3 - pxx + qx - r$ ℘ 0

pp ℘ 3q   x ℘ $\frac{p}{3}$ [−] $\sqrt[3]{\frac{p^3}{27} - r}$

$\frac{pp}{4} + \frac{2r}{p}$ ℘ q   x ℘ $\frac{p}{3} + \sqrt{\frac{pp}{9} - r}$

$x^4 - px^3 + qxx - rx + s$ ℘ 0

$\frac{rr}{pp}$ ℘ s   x ℘ $\frac{p}{4} + \sqrt{\frac{pp}{4} + \quad + \sqrt{\quad + \sqrt{\quad}}}$

$x^4 - 2ax^3 + ccx^2 + a^6 \quad a^4$

℘ TSCHIRNHAUS EQUAL SIGN
LAA VII-2 p. 715

N. 26 ARITHMETISCHE KREISQUADRATUR 1673–1676 301

kan sien daer, AB is $\frac{1}{8}$ van AC dat het differ. ontrent is $\frac{1}{2}$ sec: soude dan diff: van de geheele AB. ontrent 3 secunden.

Maer soo men de $\angle ACB$, 2 mahl, in 2 gelijcke deelen deelt, dan is AB, een weijnig kleijnder als $\frac{1}{5}$ deel van AC (wen AB is $\propto AC$) en de $\angle$en differ. als men kan sien in de wercking bouen, daer AB is $\frac{1}{5}$ deel van AC, dat de differentie is ontrent 12 sec.

Daerom wen de sijde AB is $\propto AC$ ofte een wenig kleijnder, het is genoeg om de $\angle ACB$, te deelen in 2 mahl, in 2 gelijcke deel, de $\angle$ sal ontrent $\frac{4}{5}$ deel, van 1 minut differen (als men met de 2 eerste termen, als $\frac{b}{1} - \frac{b^3}{3} \propto$ de arcus ADE werckt) van de Tab. sinus; ende hoe naeder het kombt tot $\frac{1}{3}$ deel van AC, hoeweeniger het verschiet.

Soo AB is $\frac{1}{3}$ deel van AC ofte een wenig groter soo heeft men van nooden de $\angle ACB$

∝ TSCHIRNHAUS EQUAL SIGN
LAA VII-6 p. 301

66 EHRENFRIED WALTHER VON TSCHIRNHAUS AN LEIBNIZ, 17. IV. 1677 N. 30

sive cubica $x^3 - pxx + qx - r \propto o$ etc.; si jam saltem unicus terminus debeat auferri supponatur $x \propto a + y$ et transmutatur aequatio in qua unicus terminus debet auferri; ope $x \propto a + y$ in aliam; ubi y incognita radix, in qua ponatur ille terminus auferendus $\propto o$ atque sic inveniemus qua ratione a sit assumenda ad terminum illum auferendum. Sit eg. in hac aequatione $xx - px + q \propto o$ auferendus secundus terminus fiat $x \propto y + a$ jam vero

$$\begin{array}{llll} xx \propto & yy + 2ay + aa & \propto o & \text{adeoque ponendo } 2ay - by \propto o \text{ erit } 2a - p \propto o \\ -px \propto & \quad - py - pa & & \\ + q \propto & \qquad\qquad q & & \end{array}$$

et $a \propto \frac{p}{2}$, hinc patet debere fieri $x \propto y + \frac{p}{2}$ ad secundum terminum in aequatione quadratica

∝ TSCHIRNHAUS EQUAL SIGN
LAA III-2 p. 66; III-2 p. 285 (below)

incognitae potestates ordine per divisionem inserendo ac assumendo semper quotientes aequaliter compositas, quarum omnium possibilium modorum determinatus semper numerus facile exhibetur; hanc vero Methodum in praesentia abunde declaravi et specimina exhibui; sed non ita pridem ad majorem perfectionem deduxi. 2[da] est supponendo formulas omnes possibiles radicalium $x \propto \sqrt{a} + \sqrt{b}$, $x \propto \sqrt[3]{a} + b$, $x \propto \sqrt{a + \sqrt{b + \sqrt{c}}}$ quae facile omnes quot esse possunt numero determinantur et tunc liberandae sunt ab signis radicalibus atque comparatio instituenda. Specimen Tibi exhibebo ad formulas Cardanicas obtinendas sit $x \propto \sqrt[3]{a} + \sqrt[3]{b}$ supponatur jam $\sqrt[3]{a} \propto c$ et $\sqrt[3]{b} \propto d$ et habebimus has tres aequationes $x \propto c + d$, $a \propto c^3$ et $b \propto d^3$ quibus reductis inveniemus aequationem absque signo radicali (ut Tibi jam notum erit juxta Methodum D. de Beaune radicalia signa auferendi, quaeque

N. 46 NOTAE AD TRIANGULA NUMERORUM ET AD ALGEBRAM, Erste Hälfte Mai 1676 287

[*Vierter Teil*]

$$a + b \frown cc + 2cd + dd$$
$$a \frown cc \qquad b \frown 2cd$$

$$a^2 + 2ab + b^2 \frown c^3 + 3cd^3 + 3c^3d + d^3$$

$$a^2 \frown c^3 \qquad 2ab \frown 3c^2d \qquad b^2 \frown 3cd^2 + d^3$$

$$a \frown \sqrt{c^3} \qquad b \frown \frac{3c^2d}{2a} \qquad \frac{9c^4dd}{4c^3} \frown 3c^3d + d^3$$

$$\frac{9cdd}{4}$$

$$9cdd \frown 12c^3d + d^3$$
$$9cd \frown 12c^3 + dd$$
$$dd \frown 9cd - 12c^3$$
$$d \frown 3c + \sqrt{9cc - 12c^3}$$
$$d \frown 3c + c\sqrt{9 - 12c}$$

$\frown$ TSCHIRNHAUS EQUAL SIGN
LAA VII-8 p. 287; III-2 p. 380 (below)

380 EHRENFRIED WALTHER VON TSCHIRNHAUS AN LEIBNIZ, 10. IV. 1678 N. 154

ratione determinentur. Atque sic haec porro sese ita in infinitum habere; sed prolixioribus non opus, cum operanti juxta ea quae diximus haec sese statim manifestabunt. Attamen ut omni ex parte satisfaciam, Demonstratio possibilitatis poterat universalius et facilius sic absolvi; aequationes seu quaestiones ex aequaliter compositis primis et simplicissimis quantitatibus $x + y \frown a$ et $xy \frown b$ reducuntur ad quadraticam $yy - ay + b \frown o$; $x + y + z \frown a$, $xy + xz + yz \frown b$, $xyz \frown c$ ad Cubicam $y^3 - ayy + by - c \frown o$; $x + y + z + t \frown a$, $xy + xz + xt + yz + yt + zt \frown b$, $xyz + xyt + xzt + yzt \frown c$, $xyzt \frown d$ ad quadrato-quadraticam $y^4 - ay^3 + byy - cy + d \frown o$ atque sic porro ubi jam notum et facillime demonstratur.

Jam vero 2do aequationes

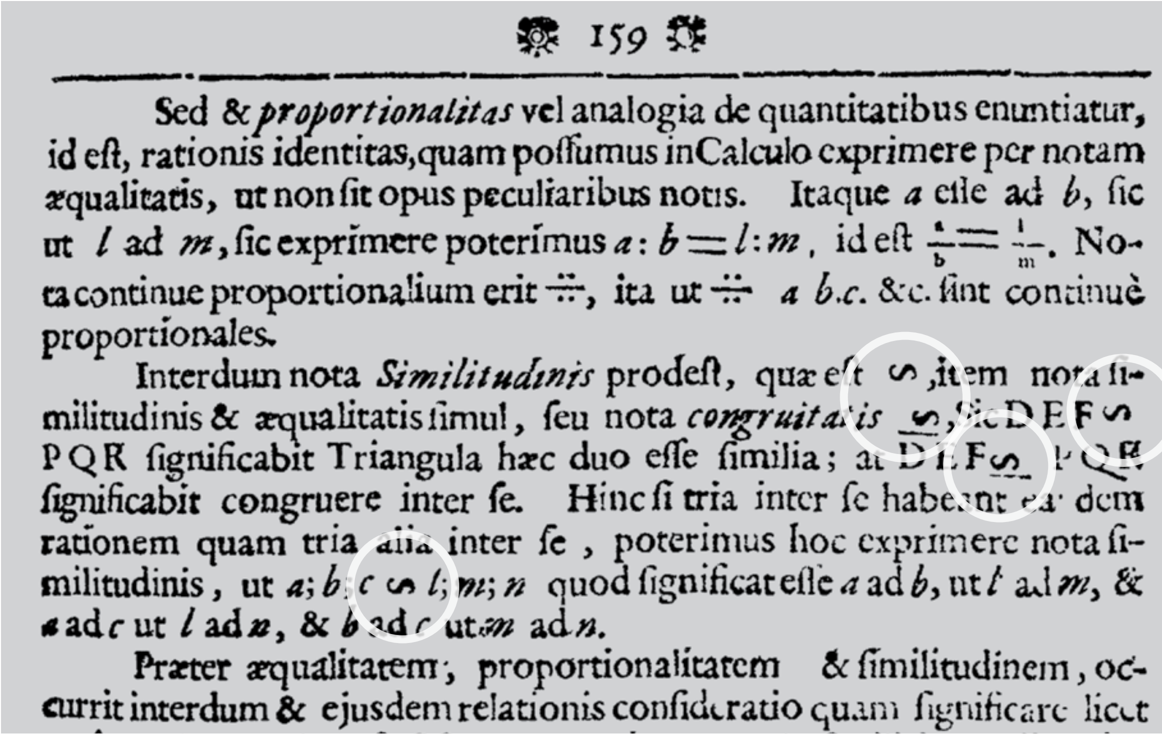
$xx + yy \frown a$, $xy \frown b$ possunt reduci ad $xx + yy \frown a$ et $xxyy \frown bb$ etc.
$x^3 + y^3 \frown a$ $\qquad$ $x^3 + y^3 \frown a$ $\quad$ $x^3y^3 \frown b^3$
$x^4 + y^4 \frown a$ $\qquad$ $x^4 + y^4 \frown a$ $\quad$ $x^4y^4 \frown b^4$

item per superiora Theoremata aequationes

$xx + yy + zz \frown a$, $\quad xy + xz + yz \frown b$, $\quad xyz \frown c$
$x^3 + y^3 + z^3 \frown a$
$x^4 + y^4 + z^4 \frown a$

reducuntur ad aequationes

$xx + yy + zz \frown a$,	$xxyy + yyzz + xxzz \frown$	cognitae	$xxyyzz \frown cc$
$x^3 + y^3 + z^3 \frown a$	$x^3y^3 + y^3z^3 + x^3z^3$	quantitati	$x^3y^3z^3 \frown c^3$
$x^4 + y^4 + z^4 \frown a$	$x^4y^4 + y^4z^4 + x^4z^4$		$x^4y^4z^4 \frown c^4$



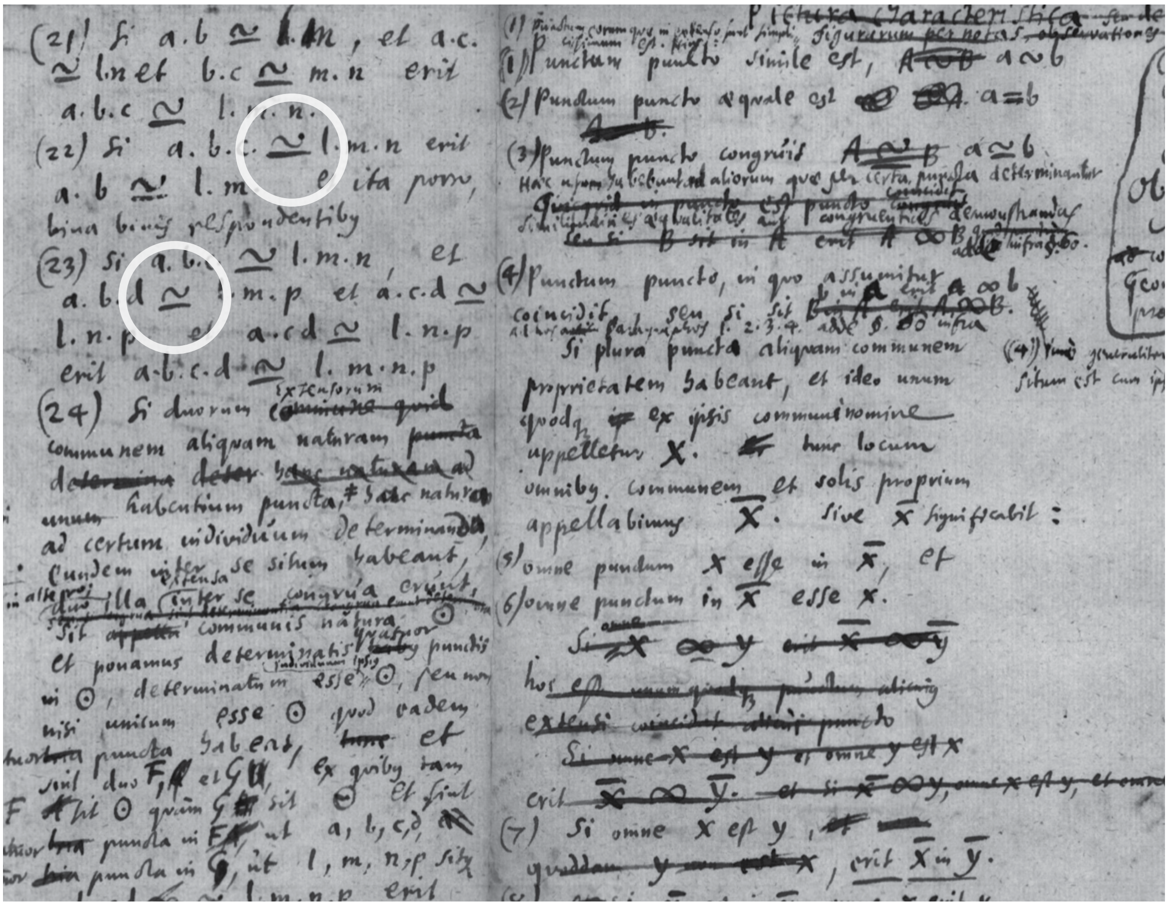
159

Sed & *proportionalitas* vel analogia de quantitatibus enuntiatur, id est, rationis identitas, quam possumus in Calculo exprimere per notam æqualitatis, ut non sit opus peculiaribus notis. Itaque *a* esse ad *b*, sic ut *l* ad *m*, sic exprimere poterimus $a : b = l : m$, id est $\frac{a}{b} = \frac{l}{m}$. Nota continue proportionalium erit ∺, ita ut ∺ *a b.c.* &c. sint continuè proportionales.

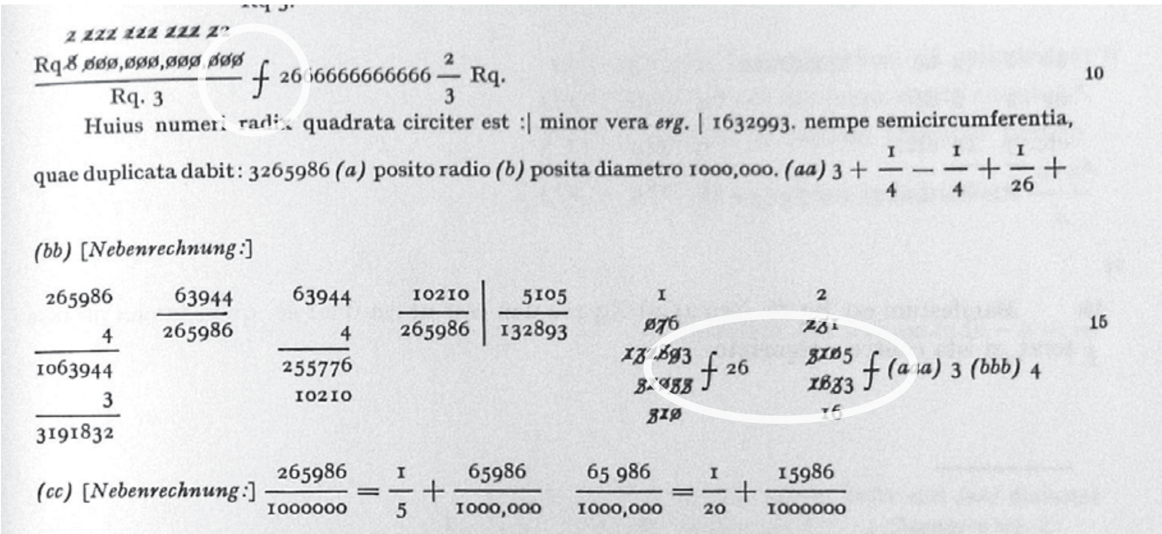
Interdum nota *Similitudinis* prodest, quæ est ∽, item nota similitudinis & æqualitatis simul, seu nota *congruitatis* ≌, sic DEF ∽ PQR significabit Triangula hæc duo esse similia; at DEF ≌ PQR significabit congruere inter se. Hinc si tria inter se habeant eandem rationem quam tria alia inter se, poterimus hoc exprimere nota similitudinis, ut *a*; *b*; *c* ∽ *l*; *m*; *n* quod significat esse *a* ad *b*, ut *l* ad *m*, & *a* ad *c* ut *l* ad *n*, & *b* ad *c* ut *m* ad *n*.

Præter æqualitatem, proportionalitatem & similitudinem, occurrit interdum & ejusdem relationis consideratio quam significare licet

∽ SIMILARITY SIGN, ≌ CONGRUENCE SIGN 2
Monitum de Characteribus Algebraica, Miscellanea Berolinensia, 1710, p. 159

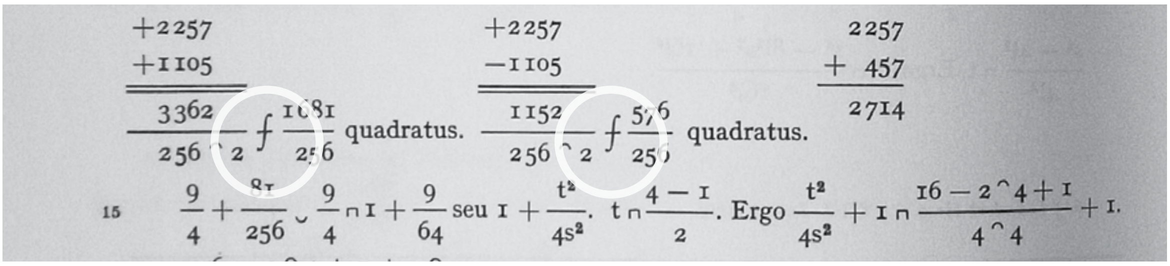


≅ CONGRUENCE SIGN 1
LH 35 I 14 fol. 1r

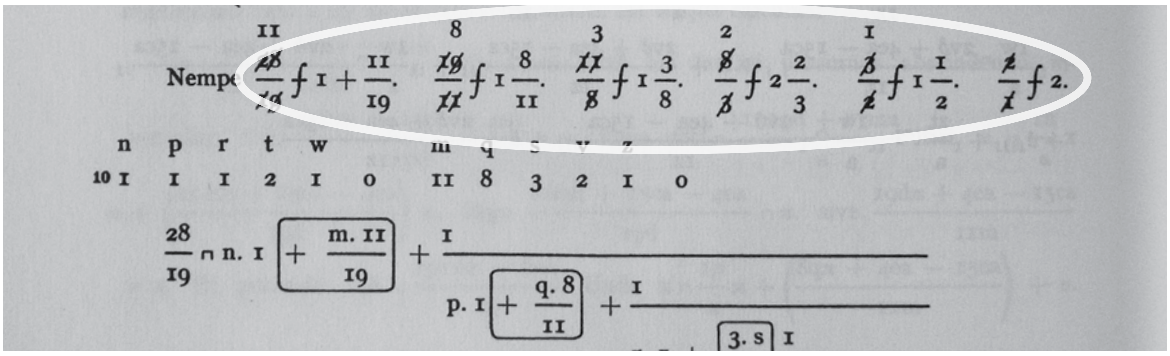


ꞙ FACIT SIGN – LAA VII-1 p. 65

Leibniz uses various script-style forms of the lowercase *f* for *facit* in his writings. It is an established practice in the LAA edition for many decades to represent this expression by a specially shaped, “upright cursive” f with a reversed stress pattern, in order to distinguish it from the ordinary lowercase f. There is a similar looking character, LATIN SMALL LETTER F WITH HOOK (0192) which is defined as a currency character for *Florin* but which also gets used as an alphabetic character in the Ewe language. Since this unification is rather problematic already, we advocate that 0192 not getting further loaded with other meanings. Regardless of a certain optical likeness the reason for including this character is mainly its distinctive purpose and function as an element of mathematical notation. The meaning is also different from that of the modern “function symbol” as which 0192 is annotated, additionally.



ꞙ FACIT SIGN
LAA VII-1 p. 352



ꞙ FACIT SIGN
LAA VII-1 p. 508

N. 38_{18} DIFFERENZEN, FOLGEN, REIHEN 1672–1676 551

$$\text{sive (30) } c \sqcap \frac{\begin{array}{l} +9,\ 25fa^2 \\ ∓\quad 81\ldots \end{array}}{\begin{array}{l} ∓3 \frown 125\beta^2 \\ \quad 27\,.. \\ +3\frown 45\,.. \\ \quad 75\,.. \end{array}} \sqcap \frac{\begin{array}{l} +3\frown 25fa^2 \\ ∓3\frown\ 9\ldots \end{array}}{\begin{array}{l} ∓125\beta^2 \\ ..\ 27\,.. \\ +\ 45\,.. \\ ..\ 75\,.. \end{array}} \sqcap \frac{\begin{array}{l} +3\frown 25fa^2 \\ ∓\,..\quad 9\ldots \end{array}}{\begin{array}{l} ∓[152]\beta^2 \\ +\ 120\,.. \end{array}}.$$

$$\text{Ac denique erit (31) } b \sqcap \frac{\begin{array}{ll} -4,125a^3f & -6,3,25a^3f \\ \ldots\ 27\ldots & ‡\,..\,..\ 9\ldots \\ ‡\,..\ 45\ldots & \\ ..\,..\ 75\ldots & \end{array}}{\begin{array}{l} ∓\ 9,125\beta^3 \\ ..\,..\ 27\,.. \\ +\,..\ 45\,.. \\ ..\,..\ 75\,.. \end{array}},\ \text{seu } b \sqcap \frac{\begin{array}{l} ‡\ 642fa^3 \\ -\ 802\ldots \end{array}}{\begin{array}{l} ∓1368\beta^3 \\ +1080\,.. \end{array}}.$$

550,15–551,5 *Nebenrechnungen:*

zu Z. 15: 4 / 2̸2̸5 ⨍ 25 / 9̸ — 15 ⌢ 25 / 9 ⌢ 9

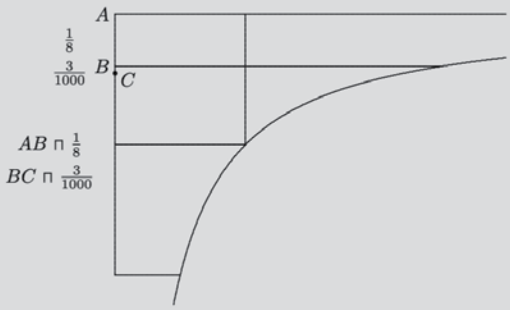
zu Z. 1–5: +9, 25 ∓99 ∓3 ⌢ 125 9 ⌢ 15 / ±18 ∓3 ⌢ 27 9 ⌢ 25 / ∓81 3 ⌢ ∓152 3 ⌢ 45 / 3 ⌢ 75

⨍ FACIT SIGN
LAA VII-3 p. 553 (top),
VII-6 p. 449 (right)

These samples demonstrate the intentional use of a specific character for “facit” in order to distinguish it from the the ordinary italic *f*.

N. 43 ARITHMETISCHE KREISQUADRATUR 1673–1676 449

Quaeritur log. a 10. Inveniamus a 250 id est a 25 in 10. Habebimus et a 10 ex dato a 2. Est enim 5^3 in 2. Inveniemus a 250. si habeamus a $\frac{1}{250}$. Est autem notus log. ab $\frac{1}{256}$. Quaeratur differentia inter $\frac{1}{250}$ et $\frac{1}{256}$. Ea est $\frac{256-250}{250,256} \Big| \frac{6}{64000} \Big| \frac{3}{32000}$ eritque $\frac{1}{250} \sqcap \frac{1}{256} + \frac{3}{32000}$ vel $\sqcap \frac{1}{8} + \frac{3}{1000} \sqcap \frac{1024}{8000} \sqcap \frac{16}{125}$. Nam si hoc dividas per 32. habebis $\frac{1}{250}$ nam fit $\frac{1024}{8000}$ in $\frac{1}{32}$ dat $\frac{1024}{256000}$. Ergo quaerenda quantitas $\frac{d}{f} - \frac{d^2}{2f^2} + \frac{d^3}{3f^3}$ etc. ita ut d sit $\frac{3}{1000}$. et f. $\frac{1}{8}$.



[*Fig. 2*]

1–5 *Nebenbetrachtung:* $\frac{1}{250} - \frac{1}{256} \sqcap \frac{6}{64000} \Big| \frac{3}{32000}$. Ergo $\frac{1}{250} \sqcap \frac{1}{256} + \frac{3}{32000}$ cujus quaeritur logarithmus.

256 / 250 / 12800 / 512 / 64000 — 2̸5̸6̸0̸00 ⨍ 250 (with cancelled figures 0̸, 1̸, 2̸2̸, 1̸0̸2̸4̸, 1̸0̸2̸2, 10)

(10) Weitere neue Notationen

Wohl im April 1676 verwendet Leibniz mit ∽ ein neues Symbol für die Ähnlichkeit von Dreiecken. Ob er es auch andernorts einsetzt, ist bislang nicht bekannt. Das Beispiel:

$\underbrace{ABD}$ ∽ $\underbrace{TMN}$ (N. 66)

Im gleichen Stück entwickelt er schrittweise eine neue Notation für die eindeutige Zuordnung bestimmter geometrischer Größen zueinander. Er geht von einer Kurve aus,

∽ LEIBNIZIAN SIMILARITY SIGN 1
LAA VII-7 p. LIII

N. 66 EXPRESSIO LOGARITHMICA AEQUATIOQUE IDENTICA, April (?) 1676 595

$BC^2 \sqcap 1, AB.$ $AB \sqcap 1.$ erit $BC \sqcap 1.$ $DC \sqcap 2.$ $AD \sqcap \sqrt{2}.$

$\underbrace{ABD}$ ∽ $\underbrace{TMN}$ seu $\dfrac{TM \sqcap 2AM}{MN \sqcap \sqrt{AM}} \sqcap \dfrac{AB}{BD}$ et $\sqrt{AM} \sqcap \dfrac{AB}{2BD}$ et $AM \sqcap \dfrac{AB^2}{4BD \sqcap AB}$.

Ergo $AM \sqcap \dfrac{AB}{4}$ et $\sqrt{AM^2 \boxed{+NM^2} AM} \sqcap AN \sqcap \sqrt{\dfrac{AB^2}{16} + \dfrac{AB}{4}}$.

$\overline{AB} \sqcap \overline{x}.$ $\overline{DB} \sqcap \overline{y}.$ $\overline{TM} \sqcap \overline{z}.$

$AD \sqcap \sqrt{x^2+y^2} \sqcap \omega.$ $\dfrac{z\,[\sqcap]\,TM}{(y) \sqcap MN}$ seu $\dfrac{d\overline{x}}{d\overline{y}} \sqcap A$ [*bricht ab*]

∽ LEIBNIZIAN SIMILARITY SIGN 1
LAA VII-7 p. 595

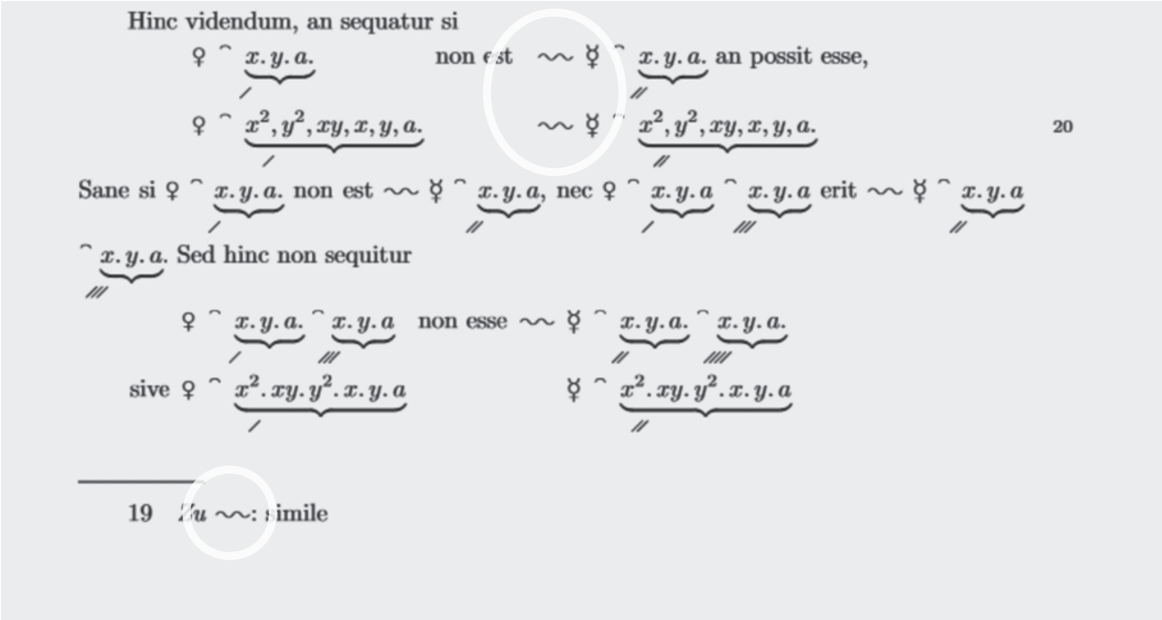
altero nulla in re differt, itaque quod alteri possibile est, etiam ipsi possibile est.

> Locus rei est in quo ipsa sita est, res autem in alia esse intelligitur hoc loco, si omne extremum ejus extremo parti alterius congruit. Est autem omne extremum puncti, lineae superficiei, ipsum punctum linea superficies.

Puncta Extensi determinati habent inter se situm determinatum. Ergo duo puncta determinato extenso connexa habent inter se situm determinatum.

Dari possunt duo puncta eum habentia situm inter se, quem habent duo alia inter se, ut $A.B$ ♉ $C.D$. Alioqui poterit demonstrari ipsa coincidere: sed hoc admisso quaero utrum demonstretur hinc A ♉ C et B ♉ D an A ♉ D et B ♉ C. Nulla enim reddi potest ratio cur unum potius quam alterum. Ergo vel non sequitur inde coincidentia, vel sequitur omnia quatuor sibi coincidere. Verum ex una congruentia quatuor rerum congruentiae concludi non possunt. Assertio haec nihil aliud significat, quam extensum aliquod posse moveri seu extensum ex loco cujus termini A et B posse transferri in locum cujus termini C et D. idque ex eo etiam ostendi potest quod spatium illimitatum est indifferens respectu extensi propositi. Eodem modo probatur mille dari posse puncta, eum habentia situm inter se,

♉ COINCIDENCE SIGN
PHILIUMM. p. 83



Hinc videndum, an sequatur si

$$♀ \frown \underbrace{x.y.a.}_{/} \quad \text{non est} \quad \sim\!\sim ☿ \frown \underbrace{x.y.a.}_{//} \text{ an possit esse,}$$

$$♀ \frown \underbrace{x^2, y^2, xy, x, y, a.}_{/} \quad \sim\!\sim ☿ \frown \underbrace{x^2, y^2, xy, x, y, a.}_{//}$$

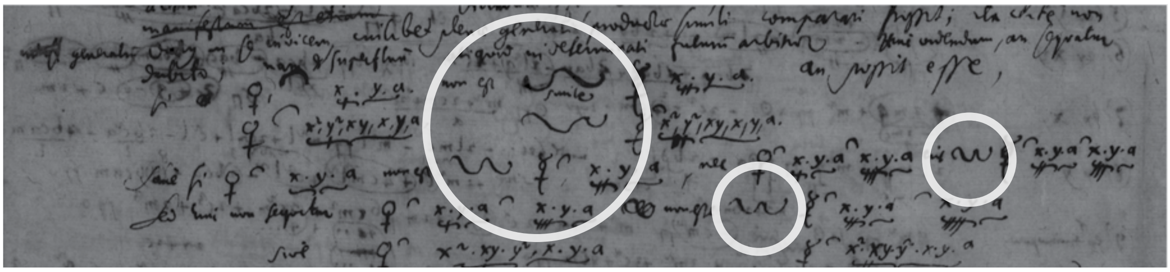
Sane si $♀ \frown \underbrace{x.y.a.}_{/}$ non est $\sim\!\sim ☿ \frown \underbrace{x.y.a}_{//}$, nec $♀ \frown \underbrace{x.y.a}_{/} \frown \underbrace{x.y.a}_{///}$ erit $\sim\!\sim ☿ \frown \underbrace{x.y.a}_{//} \frown \underbrace{x.y.a.}_{////}$ Sed hinc non sequitur

$$♀ \frown \underbrace{x.y.a.}_{/} \frown \underbrace{x.y.a}_{///} \quad \text{non esse} \sim\!\sim ☿ \frown \underbrace{x.y.a.}_{//} \frown \underbrace{x.y.a.}_{////}$$

$$\text{sive } ♀ \frown \underbrace{x^2.xy.y^2.x.y.a}_{/} \qquad ☿ \frown \underbrace{x^2.xy.y^2.x.y.a}_{//}$$

19 *Zu* ~~: simile

~~ LEIBNIZIAN SIMILARITY SIGN 2
LAA VII-3 p. 75

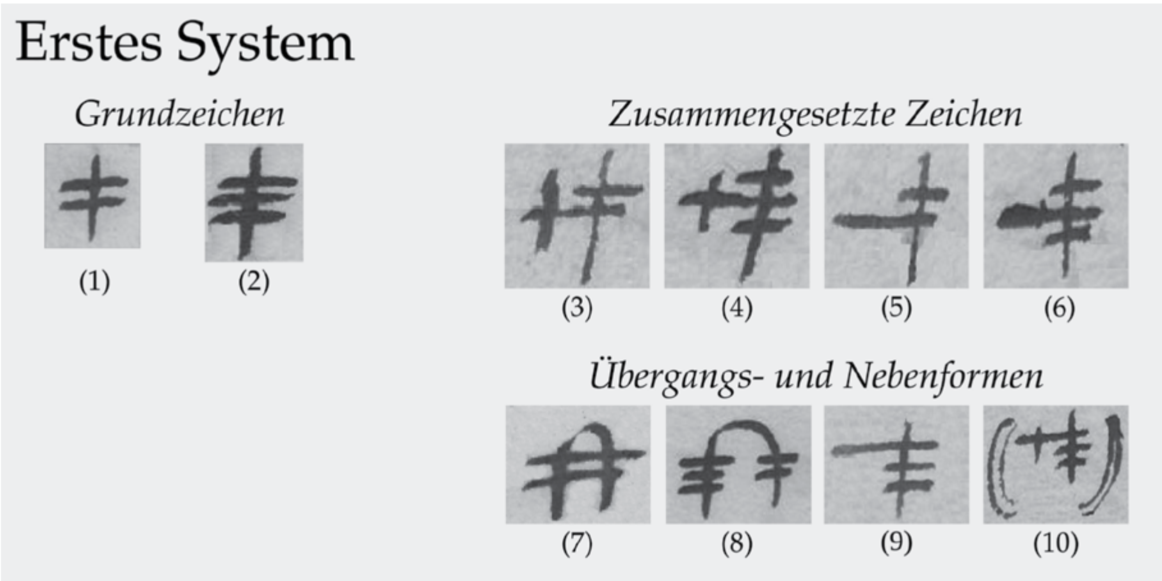


~~ LEIBNIZIAN SIMILARITY SIGN 2
LH 35 V 1 fol. 4v°

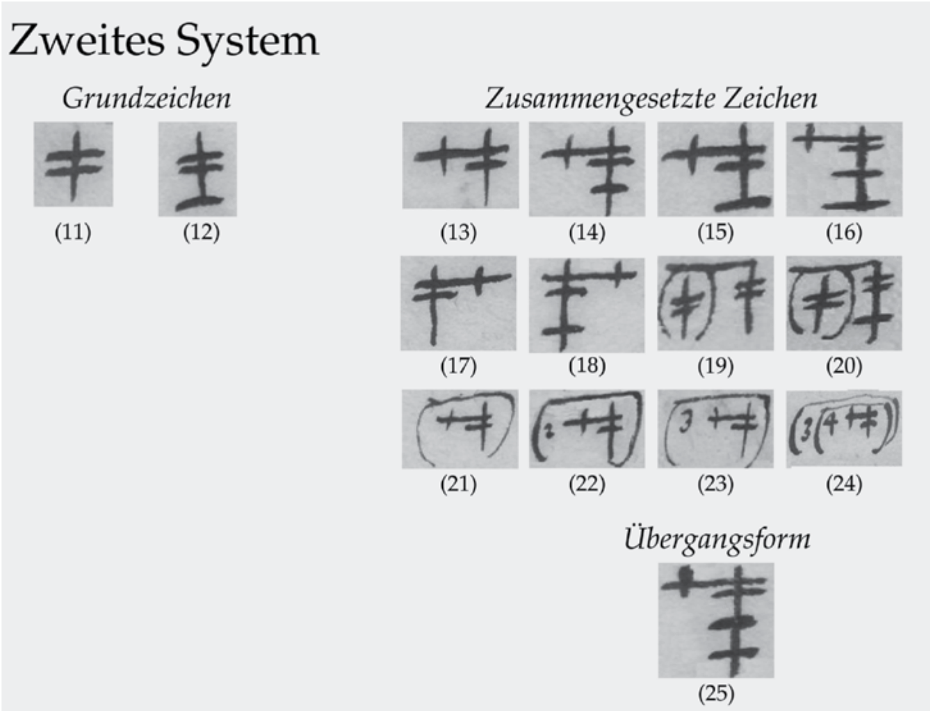
4.c) Leibnizian ambiguity signs

The term “ambiguity signs” (lt. *signa ambigua* or fr. *caracteres ambigus*) has been introduced by Leibniz in the 1670ies. He developed and used several series of these multiple-meaning characters in the framework of his mathematical studies and correspondences. They served for a combined consideration and handling of multiple equations which were distinguished by different prescriptions.
The ambiguity characters are related to the well-known ± and ∓ characters (00B1, 2213), both by their graphical structure and historically. For editorial work the ambiguity signs are important for e.g. ascribing dates to manuscript sources which lack an original *datum*. The signs also inform about Leibniz’s way of systematic thinking about how to notate certain logical concepts.
We propose an encoding scheme of complete sets of ambiguity signs because incomplete sets would be of no much use for editorial purposes. Achim Trunk (GWLB Hanover) describes six different systems, invented by Leibniz. System 3 deploys the same characters as system 3, mostly. The fourth system employes Greek letters and the sixth system uses ordinary numbers, so basically three systems remain (1., 2. and 5.) which consist of special graphic symbols.
The technical numbering of the characters in this proposal (A-xx, B-xx, C-xx) relates to what A. Trunk describes as (sub-)systems 1, 2 and 5.

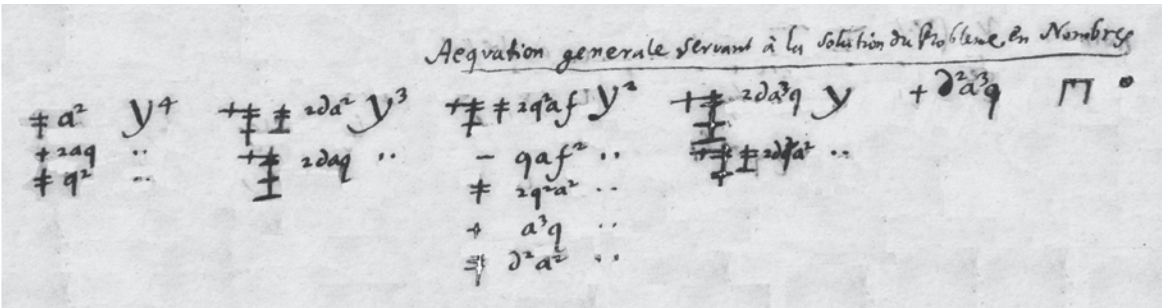
We show overviews compiled by A. Trunk first.



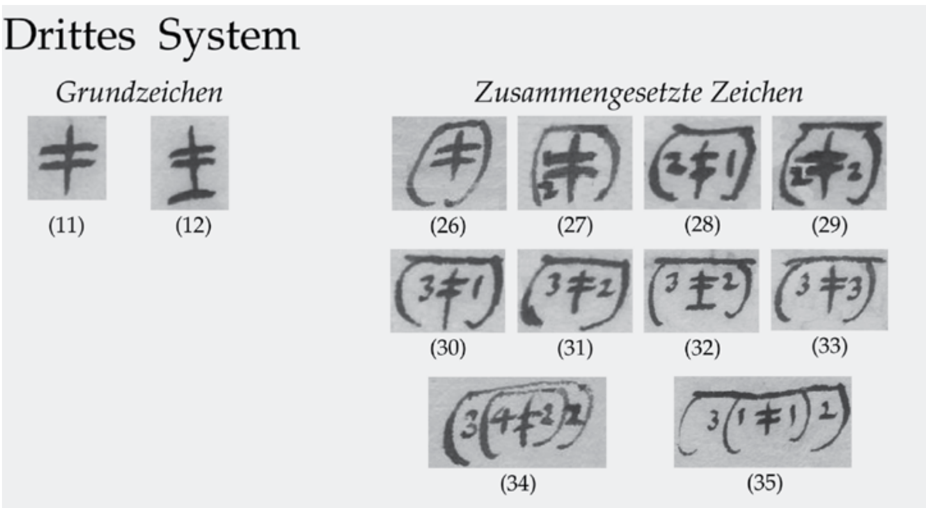
Leibniz’s ambiguity signs, 1st system (A. Trunk)



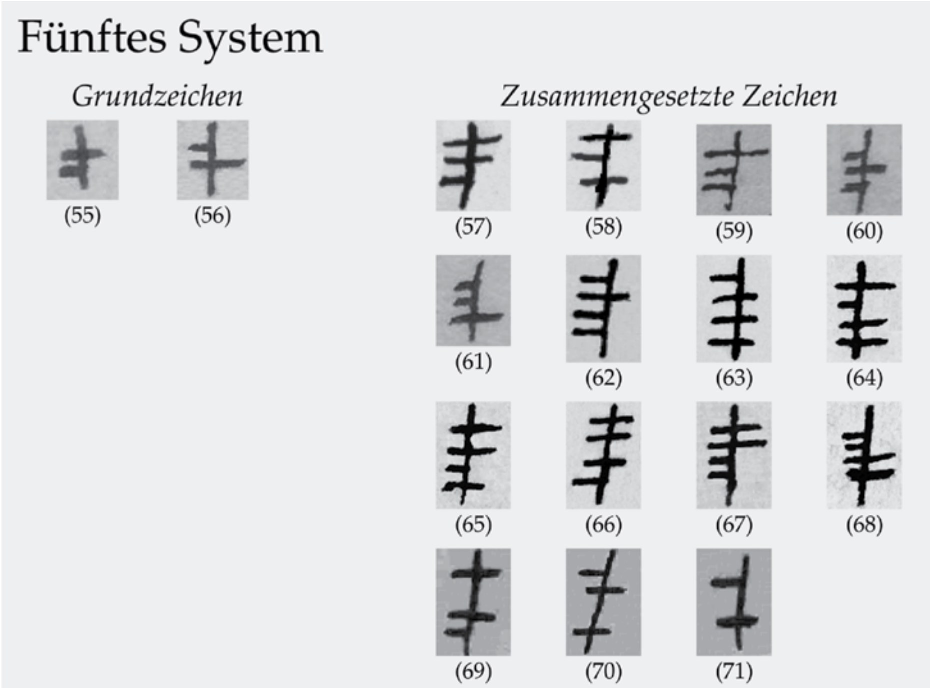
Leibniz’s ambiguity signs, 2nd system (A. Trunk)



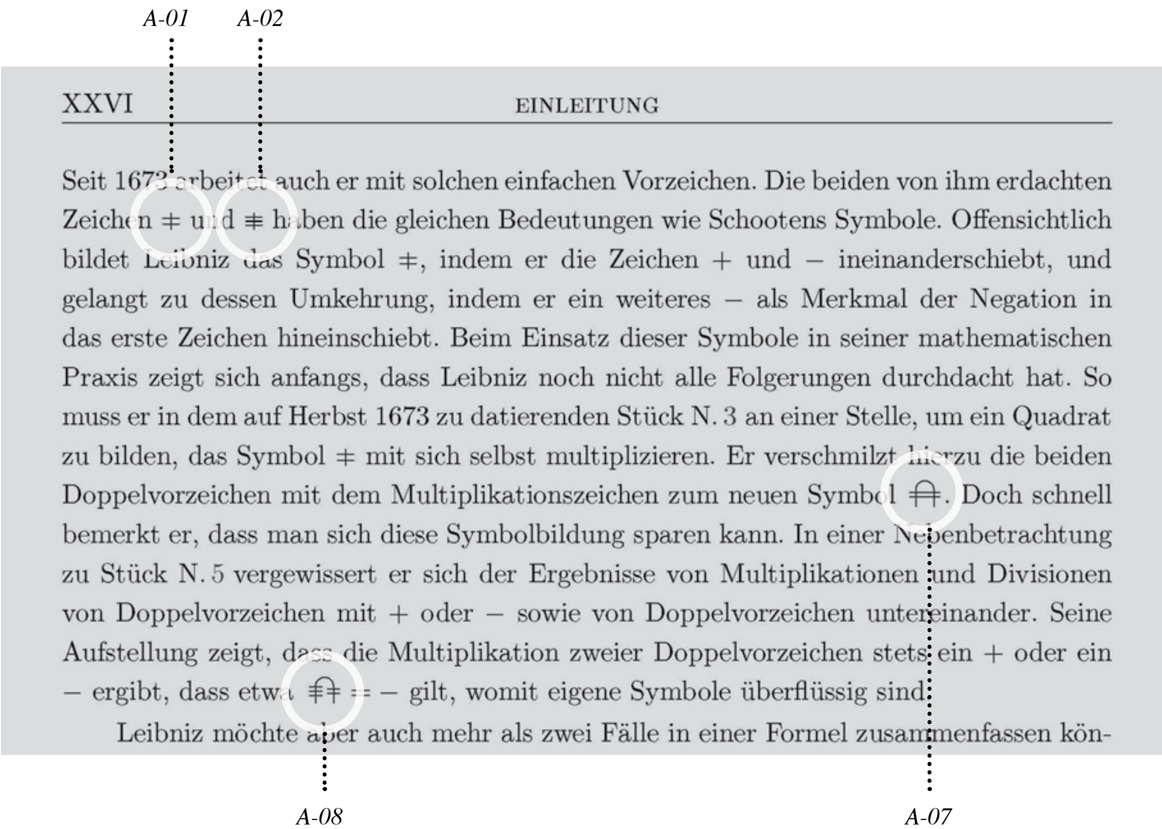
A notation of an algebraic problem by Leibniz, using symbols of the 2nd system. (after A. Trunk)
LH 35, 13, 3, fol. 168v



Leibniz’s ambiguity signs, 3rd system (A. Trunk)



Leibniz's ambiguity signs, 5th system (A. Trunk)

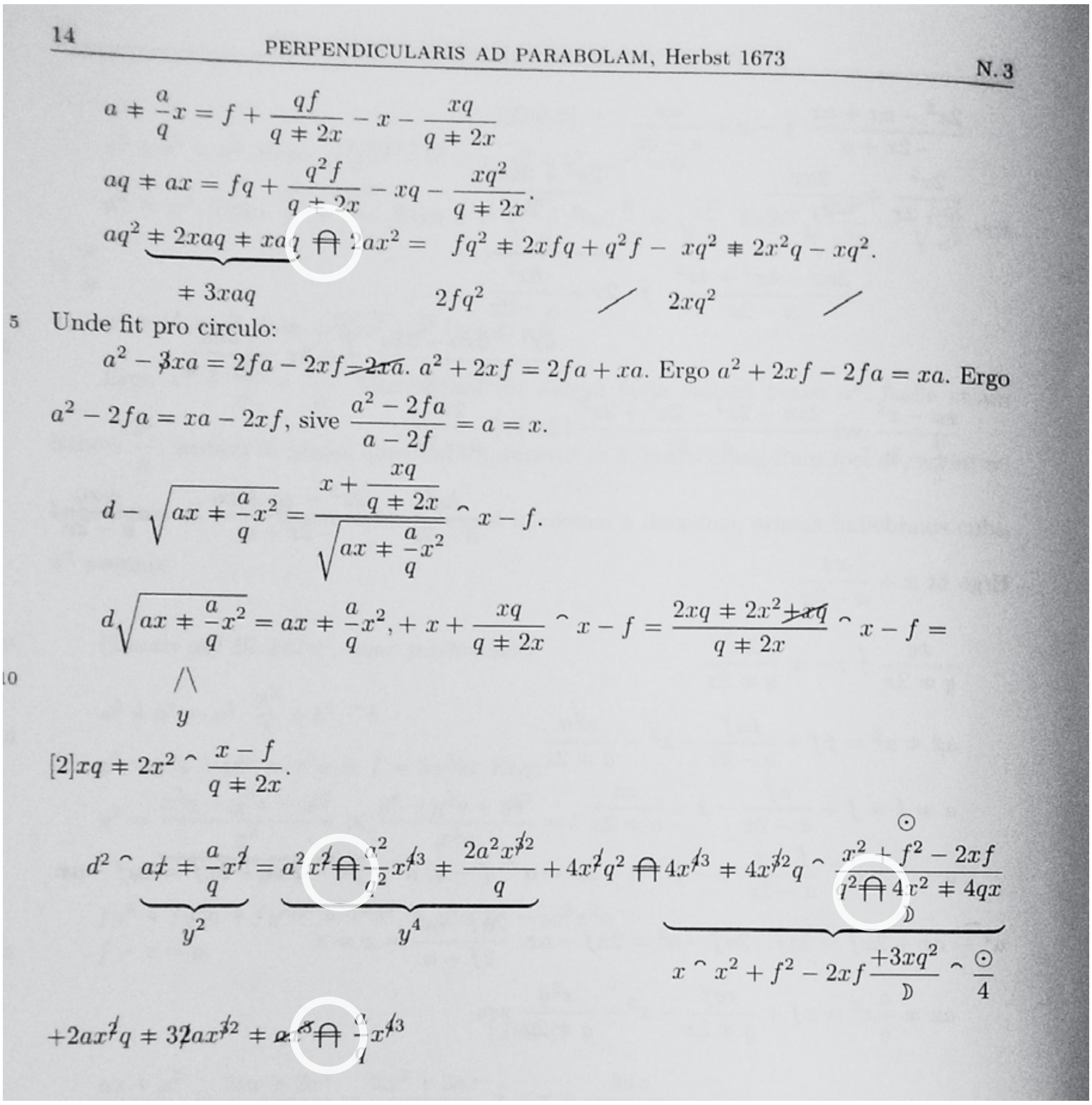


XXVI EINLEITUNG

Seit 1673 arbeitet auch er mit solchen einfachen Vorzeichen. Die beiden von ihm erdachten Zeichen ± und ∓ haben die gleichen Bedeutungen wie Schootens Symbole. Offensichtlich bildet Leibniz das Symbol ±, indem er die Zeichen + und − ineinanderschiebt, und gelangt zu dessen Umkehrung, indem er ein weiteres − als Merkmal der Negation in das erste Zeichen hineinschiebt. Beim Einsatz dieser Symbole in seiner mathematischen Praxis zeigt sich anfangs, dass Leibniz noch nicht alle Folgerungen durchdacht hat. So muss er in dem auf Herbst 1673 zu datierenden Stück N. 3 an einer Stelle, um ein Quadrat zu bilden, das Symbol ± mit sich selbst multiplizieren. Er verschmilzt hierzu die beiden Doppelvorzeichen mit dem Multiplikationszeichen zum neuen Symbol ⩲. Doch schnell bemerkt er, dass man sich diese Symbolbildung sparen kann. In einer Nebenbetrachtung zu Stück N. 5 vergewissert er sich der Ergebnisse von Multiplikationen und Divisionen von Doppelvorzeichen mit + oder − sowie von Doppelvorzeichen untereinander. Seine Aufstellung zeigt, dass die Multiplikation zweier Doppelvorzeichen stets ein + oder ein − ergibt, dass etwa ⩲ = − gilt, womit eigene Symbole überflüssig sind.

Leibniz möchte aber auch mehr als zwei Fälle in einer Formel zusammenfassen kön-

Example of ambiguity signs, 1st system. LAA VII-7 p. XXVI



$a \mp \frac{a}{q}x = f + \frac{qf}{q \mp 2x} - x - \frac{xq}{q \mp 2x}$

$aq \mp ax = fq + \frac{q^2 f}{q \mp 2x} - xq - \frac{xq^2}{q \mp 2x}.$

$\underbrace{aq^2 \mp 2xaq \mp xaq}_{\mp 3xaq} \;⋒\; 2ax^2 = \underbrace{fq^2 \mp 2xfq + q^2 f}_{2fq^2} - \underbrace{xq^2 \mp 2x^2 q - xq^2}_{2xq^2}.$

Unde fit pro circulo:

$a^2 - 3xa = 2fa - 2xf - 2xa$. $a^2 + 2xf = 2fa + xa$. Ergo $a^2 + 2xf - 2fa = xa$. Ergo $a^2 - 2fa = xa - 2xf$, sive $\frac{a^2 - 2fa}{a - 2f} = a = x$.

$d - \sqrt{ax \mp \frac{a}{q}x^2} = \frac{x + \frac{xq}{q \mp 2x}}{\sqrt{ax \mp \frac{a}{q}x^2}} \frown x - f.$

$d\underbrace{\sqrt{ax \mp \frac{a}{q}x^2}}_{y} = ax \mp \frac{a}{q}x^2, + x + \frac{xq}{q \mp 2x} \frown x - f = \frac{2xq \mp 2x^2 \pm xq}{q \mp 2x} \frown x - f =$

$[2]xq \mp 2x^2 \frown \frac{x - f}{q \mp 2x}.$

$d^2 \frown \underbrace{ax \mp \frac{a}{q}x^2}_{y^2} = \underbrace{a^2x^2 ⋒ \frac{a^2}{q^2}x^4 \mp \frac{2a^2x^3}{q}}_{y^4} + 4x^2q^2 ⋒ 4x^4 \mp 4x^3 q \frown \underbrace{\frac{x^2 + f^2 - 2xf}{q^2 ⋒ 4x^2 \mp 4qx}}_{x \frown x^2 + f^2 - 2xf \frac{+3xq^2}{☽} \frown \frac{\odot}{4}}$

$+2ax^2q \mp 3 2ax^3 \mp a^{3} ⋒ \frac{a}{q}x^4$

⋒ AMBIGUITY SIGN A-07;
Example of ambiguity signs, 1st system. LAA VII-7 p. XXVI

$$\left.\begin{array}{lllll} +4g & +8ag & +4ag^2 & -2c^2g^2 \\ & -2c^2 & -4ae^2 & -2c^2e^2 \\ & +6g^2 & -4gc^2 & +\ \ g^4 \\ & +2e^2 & +\ 4g^3 & +2g^2e^2 \\ & & +4ge^2 & +\ \ e^4 \end{array}\right\} = 0$$

Examinato ergo Canone, per exempla circuli, et parabolae, pergem⟨us⟩ ⟨cum⟩ Calculo generali. Habuimus paulo ante valorem ipsius g. indagemus eum adhuc semel ope terminorum tertiorum, collatorum, seu ope multiplicantium secundae dimensionis incognitos. Fiet

$$\frac{\mp 2\frac{a}{q}g^2\,(\mp\ \urcorner\!\mp)\,8ag\ \dfrac{+16a^{[2]}}{\mp 2\frac{a}{q}+6},,\frown a^2 \quad +6}{h^2} = \frac{\mp 2\frac{a}{q}e^2 \mp 2\frac{a}{q}c^2 - 2e^2 + 2c^2 - 4a^2, \frown \frac{a^2}{h^2},, \quad ((\dagger\ \urcorner\!\mp))\ \mp 2q^2f, -qf^2 \mp 2q^2a + a^2q \mp d^2a}{\mp a + 2q \mp \frac{q^2}{a}}$$

---

20 *Kontrollansatz zur quadratischen Ergänzung:* $\sqrt{\mp 2\frac{a}{q}+6}\ g \frown \dfrac{(\mp\ \urcorner\!\mp)\,4a}{\sqrt{\mp 2\frac{a}{q}+6}}$

15 f. $\} = 0$ *(1)* Ponendo jam $x^2 = z^2\frac{h}{a}$ *(2)* Examinato *L*

---

20 $\ldots = \ldots$: Die Koeffizienten, die Leibniz vergleicht, bezieht er wie oben aus den Gleichungen in N. 5 S. 35 sowie auf S. 48 Z. 3–12. Erneut vergisst er den Faktor $\frac{a^2}{q^2} \mp 2\frac{a}{q} + 1$. Zudem nimmt er die

⌐∓ AMBIGUITY SIGN B-04; a character belonging to the 2nd system. LAA VII-7 p. 52

N. 7 AEQUATIO EX INTERSECTIONE ORIENS, Ende Dezember 1673 – Juni 1674 53

seu extracta utrobique Radice

$$\frac{g\sqrt{\mp 2\frac{a}{q}+6}\ \ (\mp\ \urcorner\!\mp)\ \dfrac{4a}{\sqrt{\mp 2\frac{a}{q}+6}}}{\frac{h}{a}} = \sqrt{\begin{array}{c}\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\end{array}} \qquad \text{sive}$$

$$g = \sqrt{\frac{\mp 2\frac{a}{q}e^2 \mp 2\frac{a}{q}c^2 - 2e^2 + 2c^2 - 4a^2,, \ \aleph\ \dfrac{((\dagger\ \urcorner\!\mp))\ \mp 2q^2f - qf^2 \mp 2q^2a + a^2q \mp d^2a}{\mp a + 2q \mp \frac{q^2}{a}\quad \gamma} \ \beth \frown \frac{h^2}{a^2}\ \frown a^2}{[\mp]\,2\frac{a}{q}+6 \quad \delta}}$$

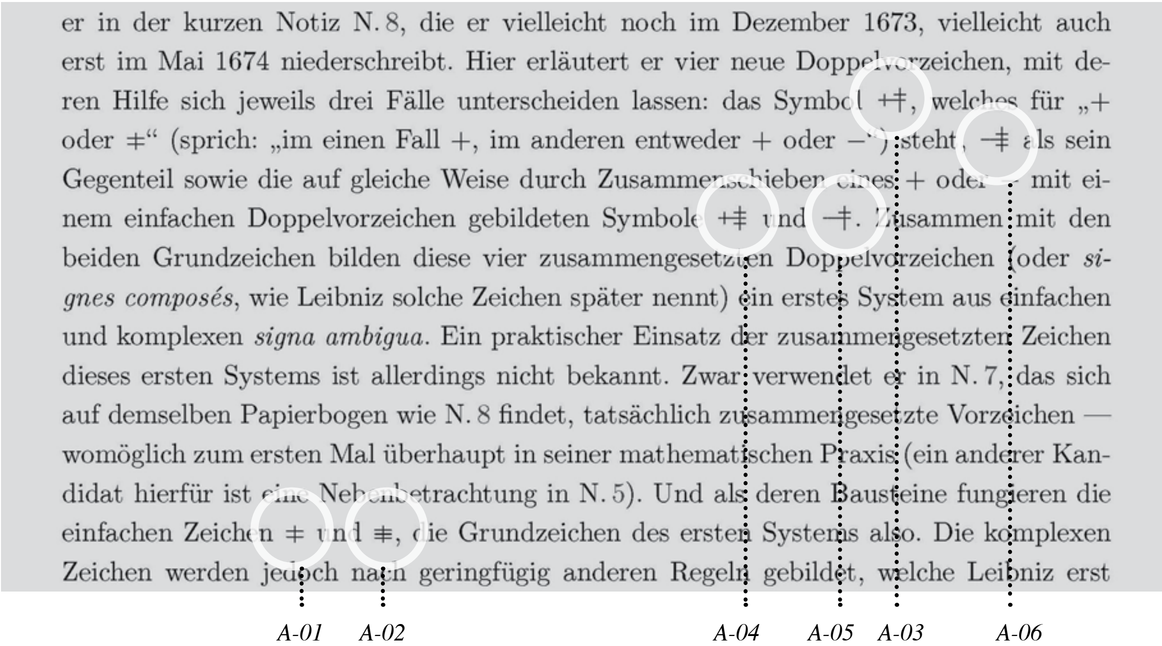
$$(\pm\ {}^{+}\!\mp) - \frac{4 \frown h}{\mp 2\frac{a}{q}+6} \quad \delta$$

Unde evanescit incognita g. valore ejus jam aliter supra dato. Ubi erat:

$$g = \frac{\dfrac{((^{+}\!\mp)) \mp a\,((^{+}\!\pm))\,q,}{\mp a + 2q \mp \frac{q^2}{a}\quad \gamma} \frown 2d \frown \frac{h^2}{a^2}(\theta) \frown h^2 \mp 4\frac{a^2}{q} - 4a \quad ל}{\mp (\mp\ {}^{+}\!\mp)\,4\frac{a}{q}\,(\mp\ \urcorner\!\mp)\,4 \quad ט}$$

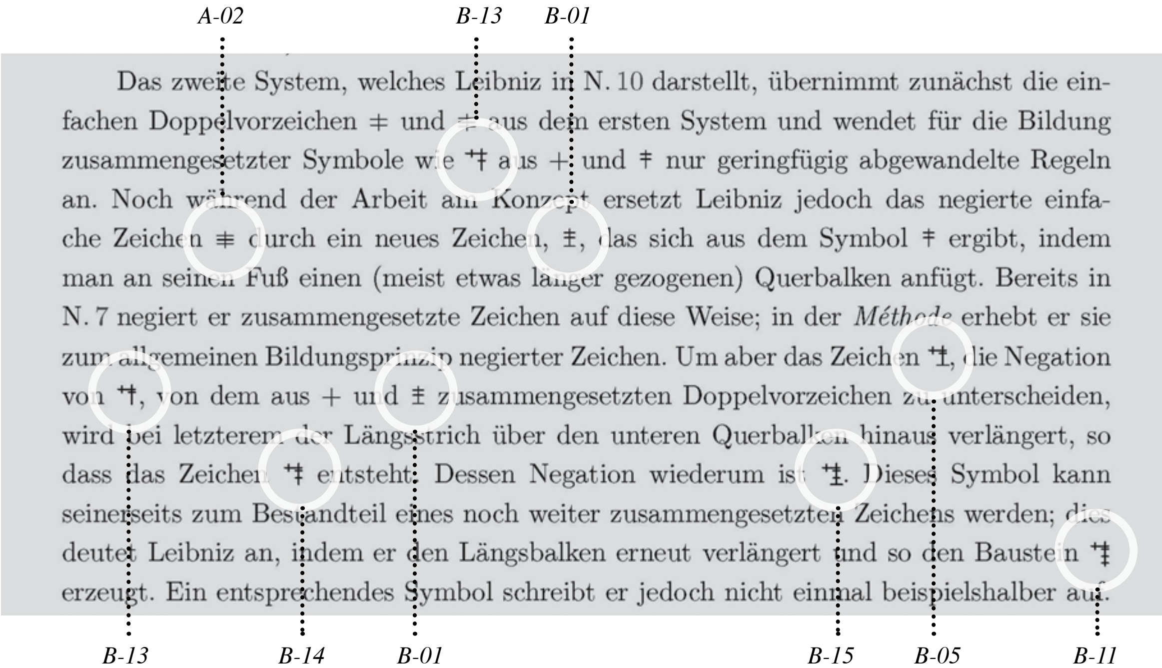
Atque ita novam habemus aequationem inter hos duos valores, cujus aequationis ope

⌐∓ AMBIGUITY SIGN B-04; LAA VII-7 p. 53



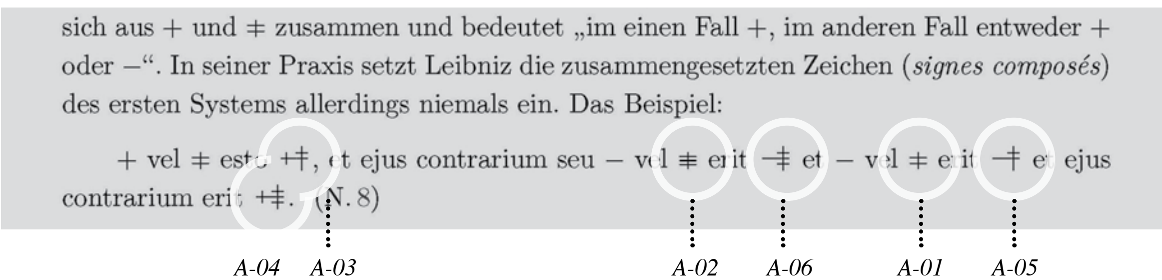
er in der kurzen Notiz N. 8, die er vielleicht noch im Dezember 1673, vielleicht auch erst im Mai 1674 niederschreibt. Hier erläutert er vier neue Doppelvorzeichen, mit deren Hilfe sich jeweils drei Fälle unterscheiden lassen: das Symbol +ǂ, welches für „+ oder ∓“ (sprich: „im einen Fall +, im anderen entweder + oder −“) steht, −ǂ als sein Gegenteil sowie die auf gleiche Weise durch Zusammenschieben eines + oder − mit einem einfachen Doppelvorzeichen gebildeten Symbole +ǂ und −ǂ. Zusammen mit den beiden Grundzeichen bilden diese vier zusammengesetzten Doppelvorzeichen (oder *signes composés*, wie Leibniz solche Zeichen später nennt) ein erstes System aus einfachen und komplexen *signa ambigua*. Ein praktischer Einsatz der zusammengesetzten Zeichen dieses ersten Systems ist allerdings nicht bekannt. Zwar verwendet er in N. 7, das sich auf demselben Papierbogen wie N. 8 findet, tatsächlich zusammengesetzte Vorzeichen — womöglich zum ersten Mal überhaupt in seiner mathematischen Praxis (ein anderer Kandidat hierfür ist eine Nebenbetrachtung in N. 5). Und als deren Bausteine fungieren die einfachen Zeichen ∓ und ⩱, die Grundzeichen des ersten Systems also. Die komplexen Zeichen werden jedoch nach geringfügig anderen Regeln gebildet, welche Leibniz erst

Example of ambiguity signs, 1st system. LAA VII-7 p. XXVI



Das zweite System, welches Leibniz in N. 10 darstellt, übernimmt zunächst die einfachen Doppelvorzeichen ∓ und ⩱ aus dem ersten System und wendet für die Bildung zusammengesetzter Symbole wie +ǂ aus + und ǂ nur geringfügig abgewandelte Regeln an. Noch während der Arbeit am Konzept ersetzt Leibniz jedoch das negierte einfache Zeichen ⩱ durch ein neues Zeichen, ǂ, das sich aus dem Symbol ǂ ergibt, indem man an seinen Fuß einen (meist etwas länger gezogenen) Querbalken anfügt. Bereits in N. 7 negiert er zusammengesetzte Zeichen auf diese Weise; in der *Méthode* erhebt er sie zum allgemeinen Bildungsprinzip negierter Zeichen. Um aber das Zeichen +ǂ, die Negation von +ǂ, von dem aus + und ǂ zusammengesetzten Doppelvorzeichen zu unterscheiden, wird bei letzterem der Längsstrich über den unteren Querbalken hinaus verlängert, so dass das Zeichen +ǂ entsteht. Dessen Negation wiederum ist +ǂ. Dieses Symbol kann seinerseits zum Bestandteil eines noch weiter zusammengesetzten Zeichens werden; dies deutet Leibniz an, indem er den Längsbalken erneut verlängert und so den Baustein +ǂ erzeugt. Ein entsprechendes Symbol schreibt er jedoch nicht einmal beispielshalber auf.

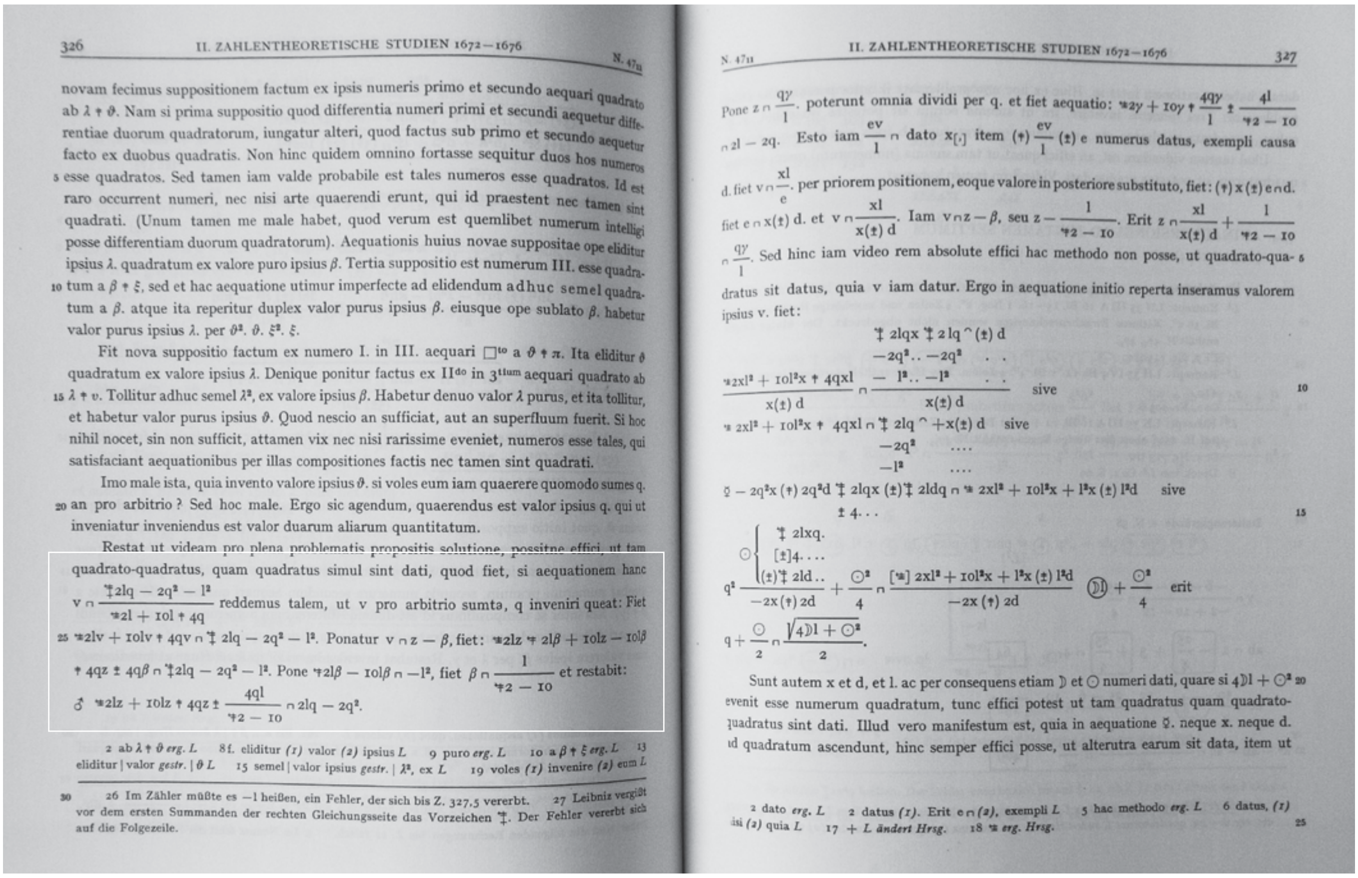
Example of ambiguity signs, 1st and 2nd system. LAA VII-7 p. XXVIII



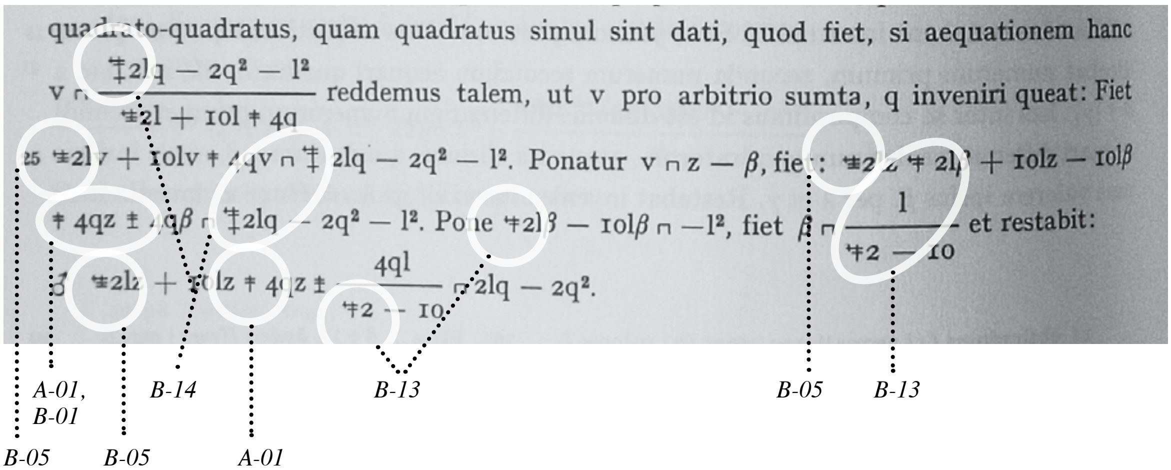
sich aus + und ∓ zusammen und bedeutet „im einen Fall +, im anderen Fall entweder + oder −“. In seiner Praxis setzt Leibniz die zusammengesetzten Zeichen (*signes composés*) des ersten Systems allerdings niemals ein. Das Beispiel:

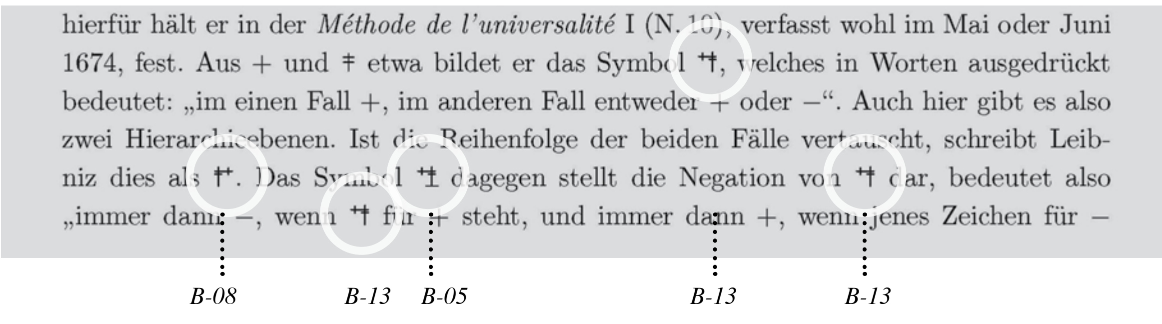
+ vel ∓ est +ǂ, et ejus contrarium seu − vel ⩱ erit −ǂ et − vel ∓ erit −ǂ et ejus contrarium erit +ǂ. (N. 8)

Example of ambiguity signs, 1st system. LAA VII-7 p. XLI

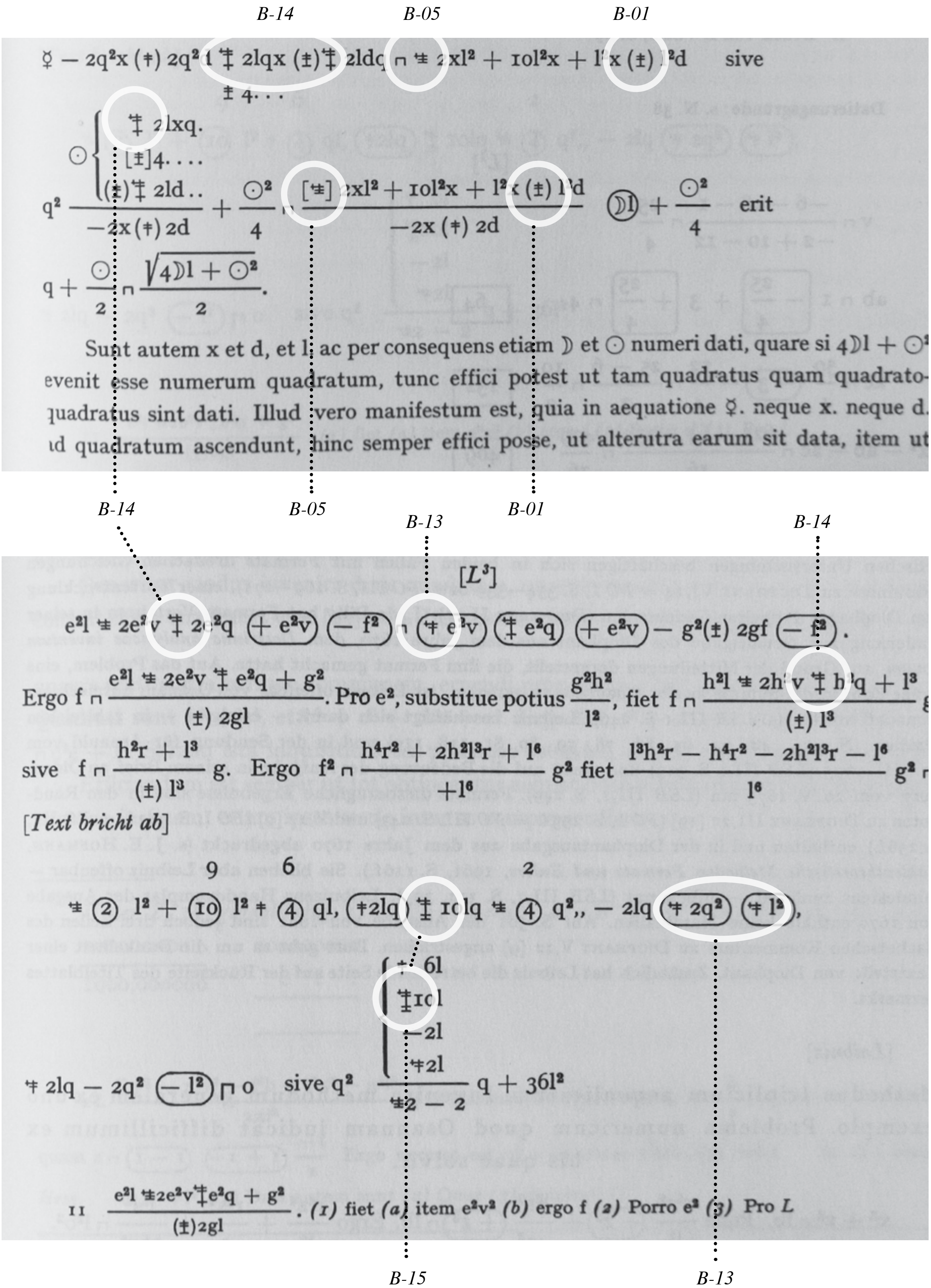


Example of ambiguity signs, 2nd system. LAA VII-1 p. 326–329. See following figures for details.

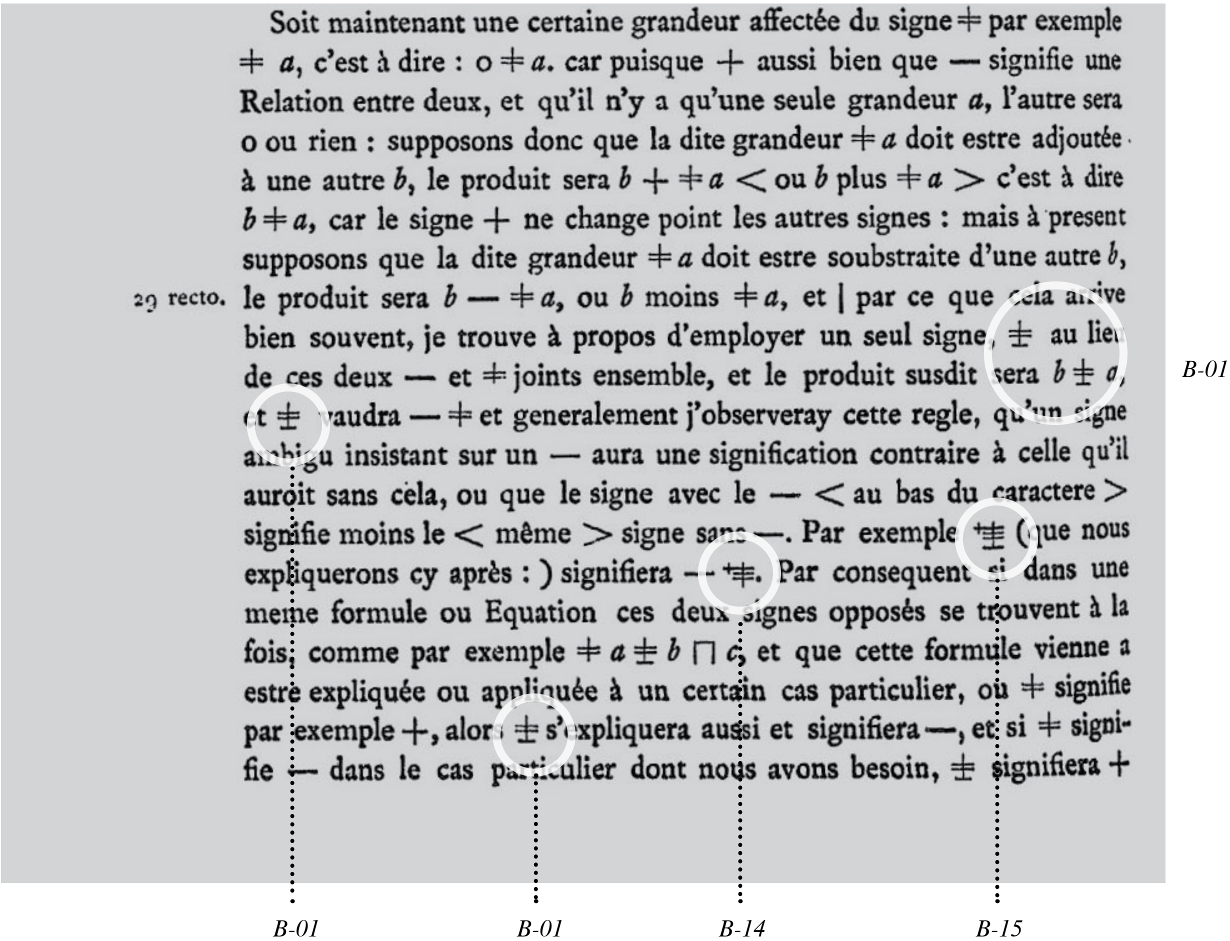




Example of ambiguity signs, 2nd system. LAA VII-7 p. 14

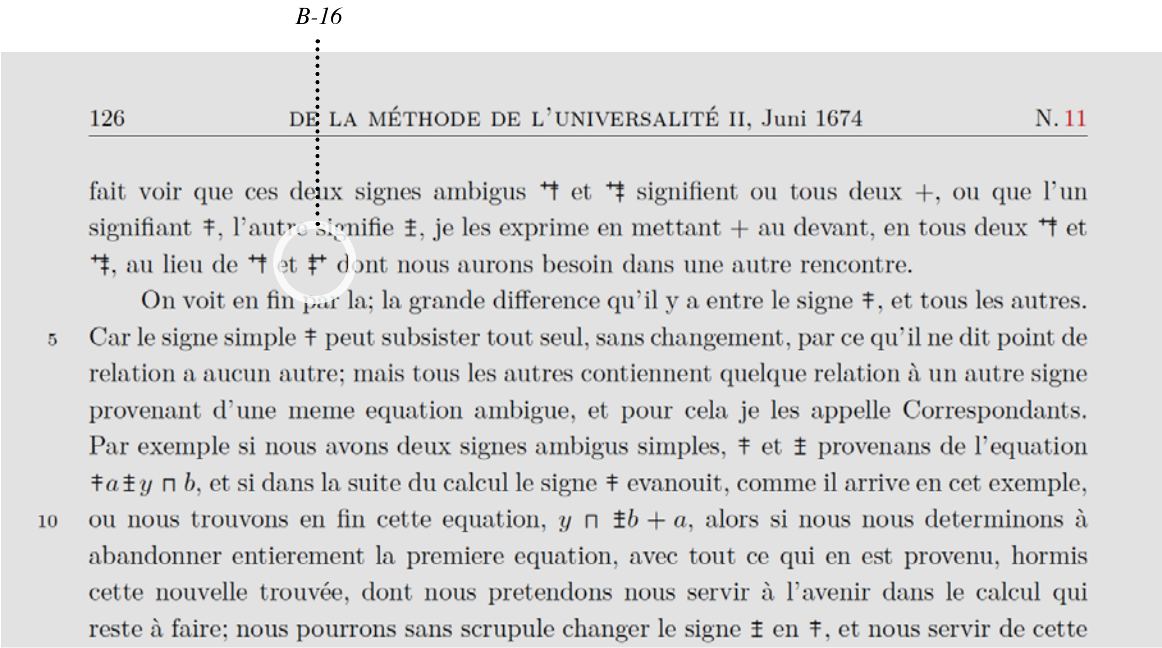


Example of ambiguity signs, 2nd system. LAA VII-1 p. 327 (top), 329



Soit maintenant une certaine grandeur affectée du signe ∓ par exemple ∓ *a*, c'est à dire : o ∓ *a*. car puisque + aussi bien que — signifie une Relation entre deux, et qu'il n'y a qu'une seule grandeur *a*, l'autre sera o ou rien : supposons donc que la dite grandeur ∓ *a* doit estre adjoutée à une autre *b*, le produit sera *b* + ∓ *a* < ou *b* plus ∓ *a* > c'est à dire *b* ∓ *a*, car le signe + ne change point les autres signes : mais à present supposons que la dite grandeur ∓ *a* doit estre soubstraite d'une autre *b*,
29 recto. le produit sera *b* — ∓ *a*, ou *b* moins ∓ *a*, et | par ce que cela arrive bien souvent, je trouve à propos d'employer un seul signe, ± au lieu de ces deux — et ∓ joints ensemble, et le produit susdit sera *b* ± *a*, et ± vaudra — ∓ et generalement j'observeray cette regle, qu'un signe ambigu insistant sur un — aura une signification contraire à celle qu'il auroit sans cela, ou que le signe avec le — < au bas du caractere > signifie moins le < même > signe sans —. Par exemple +± (que nous expliquerons cy après :) signifiera — +∓. Par consequent si dans une meme formule ou Equation ces deux signes opposés se trouvent à la fois, comme par exemple ∓ *a* ± *b* ⊓ *c*, et que cette formule vienne a estre expliquée ou appliquée à un certain cas particulier, ou ∓ signifie par exemple +, alors ± s'expliquera aussi et signifiera —, et si ∓ signifie — dans le cas particulier dont nous avons besoin, ± signifiera +

Example of ambiguity signs, 2nd system.
Couturat 1903 (1961) p. 126



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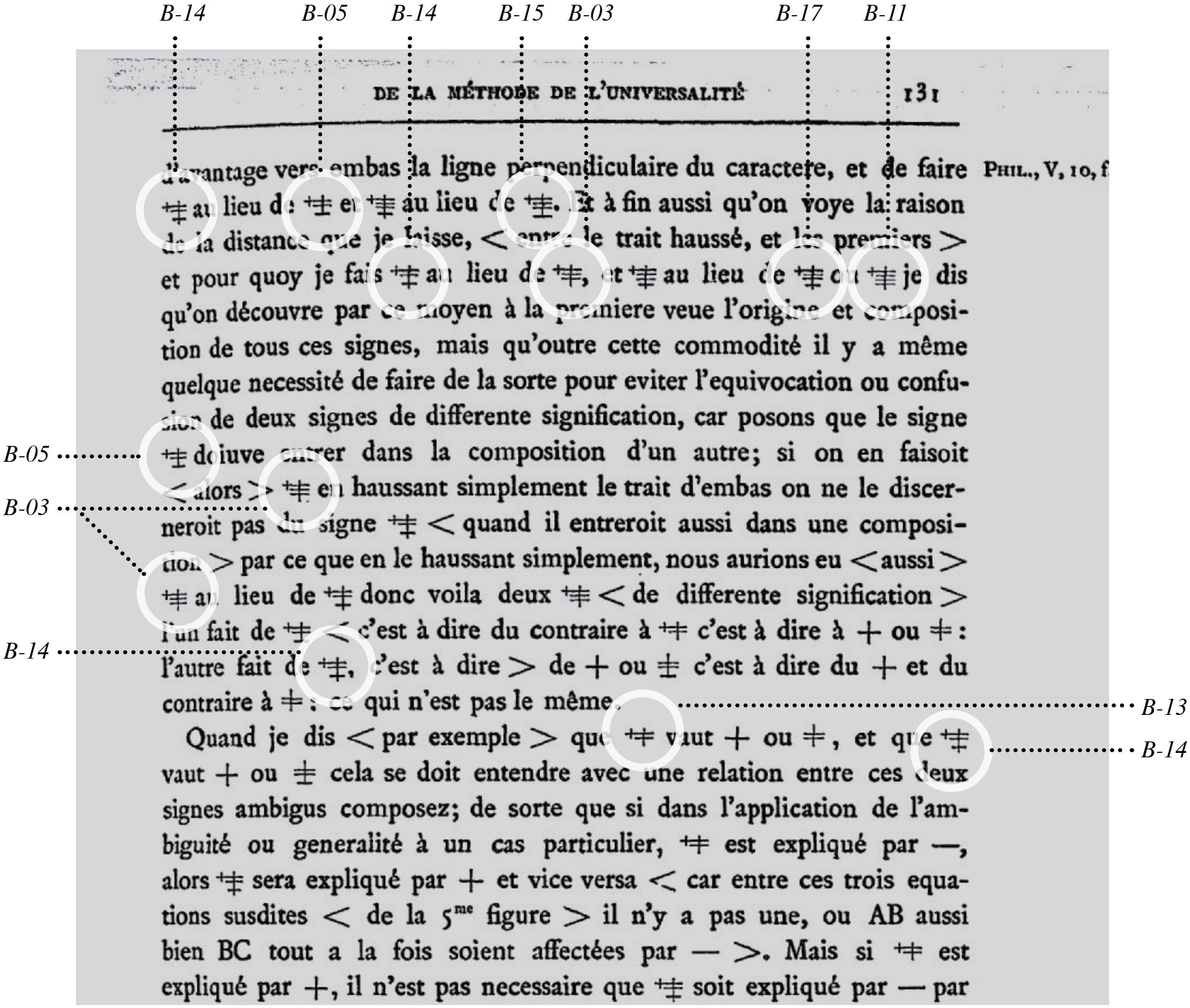
fait voir que ces deux signes ambigus +∓ et +± signifient ou tous deux +, ou que l'un signifiant ∓, l'autre signifie ±, je les exprime en mettant + au devant, en tous deux +∓ et +±, au lieu de +∓ et ±+ dont nous aurons besoin dans une autre rencontre.

On voit en fin par la; la grande difference qu'il y a entre le signe ∓, et tous les autres. Car le signe simple ∓ peut subsister tout seul, sans changement, par ce qu'il ne dit point de relation a aucun autre; mais tous les autres contiennent quelque relation à un autre signe provenant d'une meme equation ambigue, et pour cela je les appelle Correspondants. Par exemple si nous avons deux signes ambigus simples, ∓ et ± provenans de l'equation $∓a ± y ⊓ b$, et si dans la suite du calcul le signe ∓ evanouit, comme il arrive en cet exemple, ou nous trouvons en fin cette equation, $y ⊓ ±b + a$, alors si nous nous determinons à abandonner entierement la premiere equation, avec tout ce qui en est provenu, hormis cette nouvelle trouvée, dont nous pretendons nous servir à l'avenir dans le calcul qui reste à faire; nous pourrons sans scrupule changer le signe ± en ∓, et nous servir de cette

Example of ambiguity sign B-16, 2nd system. LAA VII-7 p. 126



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davantage vers embas la ligne perpendiculaire du caractere, et de faire ⧺ au lieu de ⧺ et ⧺ au lieu de ⧺. Et à fin aussi qu'on voye la raison de la distance que je laisse, < entre le trait haussé, et les premiers > et pour quoy je fais ⧺ au lieu de ⧺, et ⧺ au lieu de ⧺ ou ⧺ je dis qu'on découvre par ce moyen à la premiere veue l'origine et composition de tous ces signes, mais qu'outre cette commodité il y a même quelque necessité de faire de la sorte pour eviter l'equivocation ou confusion de deux signes de differente signification, car posons que le signe ⧺ doive entrer dans la composition d'un autre; si on en faisoit < alors > ⧺ en haussant simplement le trait d'embas on ne le discerneroit pas du signe ⧺ < quand il entreroit aussi dans une composition > par ce que en le haussant simplement, nous aurions eu < aussi > ⧺ au lieu de ⧺ donc voila deux ⧺ < de differente signification > l'un fait de ⧺ < c'est à dire du contraire à ⧺ c'est à dire à + ou ∓ : l'autre fait de ⧺, c'est à dire > de + ou ± c'est à dire du + et du contraire à ∓ : ce qui n'est pas le même.

Quand je dis < par exemple > que ⧺ vaut + ou ∓, et que ⧺ vaut + ou ± cela se doit entendre avec une relation entre ces deux signes ambigus composez; de sorte que si dans l'application de l'ambiguité ou generalité à un cas particulier, ⧺ est expliqué par —, alors ⧺ sera expliqué par + et vice versa < car entre ces trois equations susdites < de la 5me figure > il n'y a pas une, ou AB aussi bien BC tout a la fois soient affectées par — >. Mais si ⧺ est expliqué par +, il n'est pas necessaire que ⧺ soit expliqué par — par

PHIL., V, 10, f

Example of ambiguity signs, 2nd system.
Couturat 1903 (1961) p. 131

B-05 B-11 B-06 B-03

au lieu de ⁺‡; et ⁺ǂ au lieu de ⁺‡. Et à fin aussi qu'on voye la raison de la distance que je laisse entre le trait haussé, et les premiers, et pour quoy je fais ⁺ǂ au lieu de ⁺ǂ, et ⁺ǂ au lieu de ⁺ǂ ou ⁺ǂ je dis qu'on découvre par ce moyen à la premiere veue l'origine et composition de tous ces signes, mais qu'outre cette commodité il y a même quelque necessité de faire de la sorte, pour eviter l'equivocation, ou confusion de deux signes de differente signification, car posons que le signe ⁺‡ doive entrer dans la composition d'un autre, si on en faisoit alors ⁺ǂ en haussant simplement le trait d'embas on ne le

B-11 B-17 B-18 B-05

...ns une composition par ce que en le haussant simplement, nous aurions eu aussi ⁺ǂ au lieu de ⁺ǂ donc voila deux ⁺ǂ de differente signification l'un fait de ⁺‡, c'est à dire du contraire à ⁺† c'est à dire à + ou ǂ: l'autre fait de ⁺ǂ, c'est a dire de + ou ‡ c'est à dire du + et du contraire à ǂ: ce qui n'est pas le même.

Quand je dis par exemple que ⁺† vaut + ou ǂ, et que ⁺ǂ vaut + ou ‡ cela se doit entendre avec une relation entre ces deux signes ambigus composez; de sorte que si dans l'application de l'ambiguité ou generalité à un cas particulier, ⁺† est expliqué par −, alors ⁺ǂ sera expliqué par + et *vice versa* car entre ces trois equations susdites de la 5[me] figure il n'y a pas une, ou AB aussi bien que BC, tout a la fois soient affectées par −. Mais si ⁺† est expliqué par +, il n'est pas necessaire que ⁺ǂ soit expliqué par − par ce que dans une de ces equations particulieres, AB, aussi bien que BC, sont affectées par +. Par consequent si l'un de ces deux signes composés est expliqué par + l'autre sera expliqué par ǂ et *vice versa* (: avec la caution pourtant, que nous y apporterons plus bas :) de sorte que l'ambiguité decomposée qu'elle est, deviendra simple. Et par ce que la liste des Equations particulieres

AC ⊓ + AB + BC
− } ǂ AB + } ‡ BC } qui peuvent estre entendues
+ } AB − } BC

sous la Generale ⁺† AB ⁺ǂ BC,

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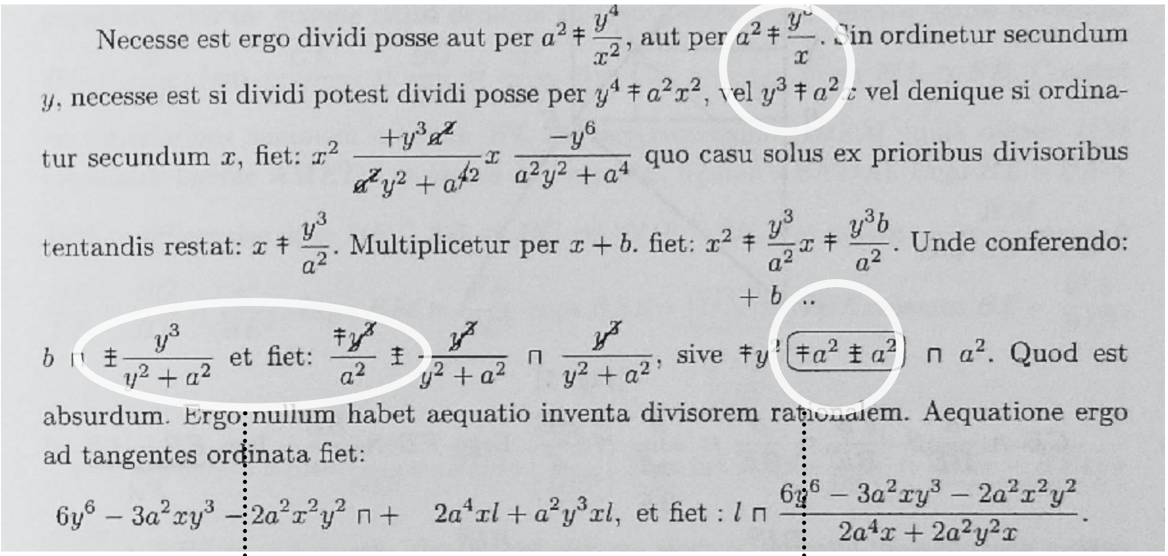
2 entre ... premiers *erg. L* 3 de ⁺ǂ ou ⁺ǂ *L ändert Hrsg.* 8–10 signe ⁺ǂ | qvand ... composition *erg.* | par ce qve *(1)* si on haussoit le signe *(2)* en ... eu | aussi *erg.* | ⁺ǂ au ... deux ⁺ǂ | de differente signification *erg.* | l'un *(a)* faisoit de *(aa)* ⁺ǂ, l'autre de + ou ‡ *(bb)* ⁺‡, l'autre de ⁺ǂ, c'est à dire de + ou ‡ *(b)* fait de ⁺‡, c'est à dire *(aa)* de + ou ǂ *(bb)* du contraire *L* 13 *(1)* On voit par la, a *(2)* Qvand je dis | par exemple *erg.* | *L* 14 f. dans *(1)* l'explication *(2)* l'application *L* 16 f. car ... susdites | de la 5[me] figure *erg.* | il ... bien | qve *erg. Hrsg.* | BC ... par − *erg. L*

Ambiguity signs, 2nd system. LAA VII-7 p. 125

A-01



Necesse est ergo dividi posse aut per $a^2 \mp \frac{y^4}{x^2}$, aut per $a^2 \mp \frac{y^3}{x}$. Sin ordinetur secundum y, necesse est si dividi potest dividi posse per $y^4 \mp a^2x^2$, vel $y^3 \mp a^2x$ vel denique si ordinatur secundum x, fiet: $x^2 \; \frac{+y^3a^2}{a^2y^2+a^4}x \; \frac{-y^6}{a^2y^2+a^4}$ quo casu solus ex prioribus divisoribus tentandis restat: $x \mp \frac{y^3}{a^2}$. Multiplicetur per $x+b$. fiet: $x^2 \mp \frac{y^3}{a^2}x \mp \frac{y^3b}{a^2}$. Unde conferendo:
$+b$ ..

$b \sqcap \pm \frac{y^3}{y^2+a^2}$ et fiet: $\frac{\mp y^3}{a^2} \pm \frac{y^3}{y^2+a^2} \sqcap \frac{y^3}{y^2+a^2}$, sive $\mp y^3 (\mp a^2 \pm a^2) \sqcap a^2$. Quod est absurdum. Ergo nullum habet aequatio inventa divisorem rationalem. Aequatione ergo ad tangentes ordinata fiet:

$6y^6 - 3a^2xy^3 - 2a^2x^2y^2 \sqcap + \; 2a^4xl + a^2y^3xl$, et fiet : $l \sqcap \frac{6y^6 - 3a^2xy^3 - 2a^2x^2y^2}{2a^4x + 2a^2y^2x}$.

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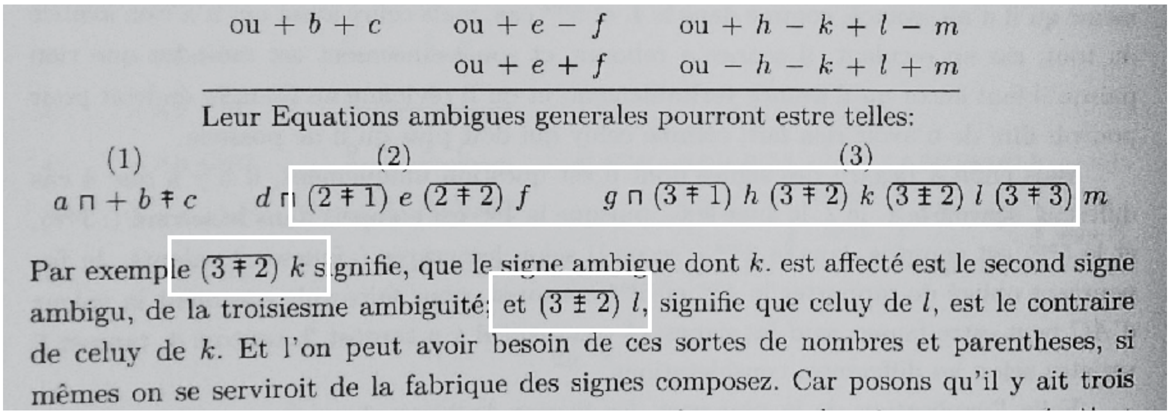
Ambiguity signs, 2nd system. LAA VII-3 p. 567



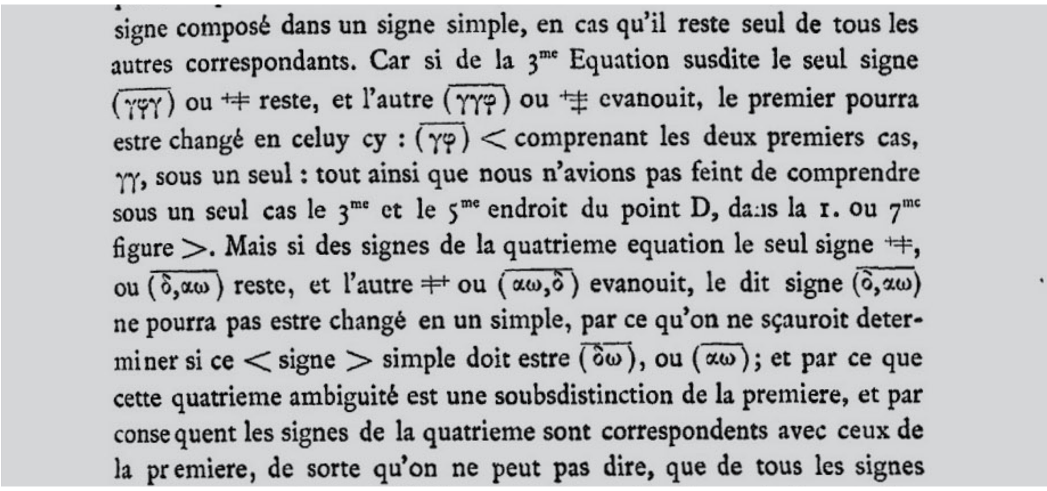
ou $+ b + c$ ou $+ e - f$ ou $+ h - k + l - m$
ou $+ e + f$ ou $- h - k + l + m$

Leur Equations ambigues generales pourront estre telles:

(1) $a \sqcap + b \mp c$ (2) $d \sqcap (2 \mp 1)\, e\, (2 \mp 2)\, f$ (3) $g \sqcap (3 \mp 1)\, h\, (3 \mp 2)\, k\, (3 \pm 2)\, l\, (3 \mp 3)\, m$

Par exemple $(3 \mp 2)\, k$ signifie, que le signe ambigue dont k. est affecté est le second signe ambigu, de la troisiesme ambiguité; et $(3 \pm 2)\, l$, signifie que celuy de l, est le contraire de celuy de k. Et l'on peut avoir besoin de ces sortes de nombres et parentheses, si mêmes on se serviroit de la fabrique des signes composez. Car posons qu'il y ait trois

Ambiguity signs, 3rd system. This notation uses ⁀ LEFT VIRGULA PARANTHESIS and ⁀ RIGHT VIRGULA PARANTHESIS. – LAA VII-7 p. 134



signe composé dans un signe simple, en cas qu'il reste seul de tous les autres correspondants. Car si de la 3^me^ Equation susdite le seul signe (γγγ) ou ⁺∓ reste, et l'autre (γγφ) ou ⁺∓ evanouit, le premier pourra estre changé en celuy cy : (γφ) < comprenant les deux premiers cas, γγ, sous un seul : tout ainsi que nous n'avions pas feint de comprendre sous un seul cas le 3^me^ et le 5^me^ endroit du point D, dans la 1. ou 7^me^ figure >. Mais si des signes de la quatrieme equation le seul signe ⁺∓, ou (δ,αω) reste, et l'autre ∓⁺ ou (αω,δ) evanouit, le dit signe (δ,αω) ne pourra pas estre changé en un simple, par ce qu'on ne sçauroit determiner si ce < signe > simple doit estre (δω), ou (αω); et par ce que cette quatrieme ambiguité est une soubsdistinction de la premiere, et par consequent les signes de la quatrieme sont correspondents avec ceux de la premiere, de sorte qu'on ne peut pas dire, que de tous les signes

⁀ LEFT VIRGULA PARANTHESIS, ⁀ RIGHT VIRGULA PARANTHESIS.
This sample shows the use of the 4th system of ambiguity notation, for which Leibniz used Greek letters. – Couturat 1903 (1961) p. 141

A-01 A-02

EINLEITUNG

ambigua für vier Fälle, $(\mp)\mp$ und $(\mp)\not\equiv$, aus. (Der Grund dafür
ist, dass in Leibniz' Ansatz der gegebene Punkt im Problem

⁀) RIGHT VIRGULA PARANTHESIS – LAA VII-7 p. XXIX
These special paranthesis characters form a part of system 2 and system 3. They are to connect to either side and fit to virgula characters such as OVERLINE (203E), COMBINING OVERLINE (0305) or COMBINING DOUBLE MACRON (035E).

selbe Zahl stehen, *signes heterogenes* unterschiedliche. Handel
Zahlen um eine 1, lässt Leibniz sie oft einfach weg. Diese Rege
$(\overline{3 \mp 2})$, $(\overline{3 \mp 2})$ oder $(\overline{2\mp})$. Die Ziffer rechts des Grundzeichens
inhaltlichen Aufschluss über die Ambiguität, sondern es werde

(⁀ LEFT VIRGULA PARANTHESIS, ⁀) RIGHT VIRGULA PARANTHESIS
LAA VII-7 p. XXX

Verwechslung der Mehrfachvorzeichen mit Koeffizienten oder
$(\overline{\alpha\omega})$ und $(\overline{\beta\psi})$ sind also voneinander unabhängige einfache D
chen $(\overline{\omega\alpha})$ und $(\overline{\psi\beta})$ ihre Negationen. Die zusammengesetzten Z
N. 10 durch ein Komma, welches zwei Fälle, einer darunter dop
eindeutig, voneinander abgrenzt, etwa $(\overline{\alpha, \alpha\omega})$. In seiner späte
Komma. Die Notation mit Komma spiegelt zwei Hierarchieeber

(⁀ LEFT VIRGULA PARANTHESIS, ⁀) RIGHT VIRGULA PARANTHESIS
This notation with Greek letters forms the 4th system of ambiguity notation. LAA VII-7 p. XXXI

ihre Vorzeichen unterschiedenen Gleichungen hervorgeht, in denen außer + und − auch das Zeichen $(\overline{3 \mp 2})$ oder sein Gegenstück auftreten. Auch in der *Méthode de l'universalité* II (N. 11) gibt Leibniz eine Einführung in das dritte System, streicht dann jedoch den entsprechenden Abschnitt. In der Praxis setzt er dieses System niemals ein. Beispiele:

[P]osons le cas qu'il y ait trois equations ambigües dans nostre calcul, sçavoir:

Equat. 1 Equat. 2 Equat. 3

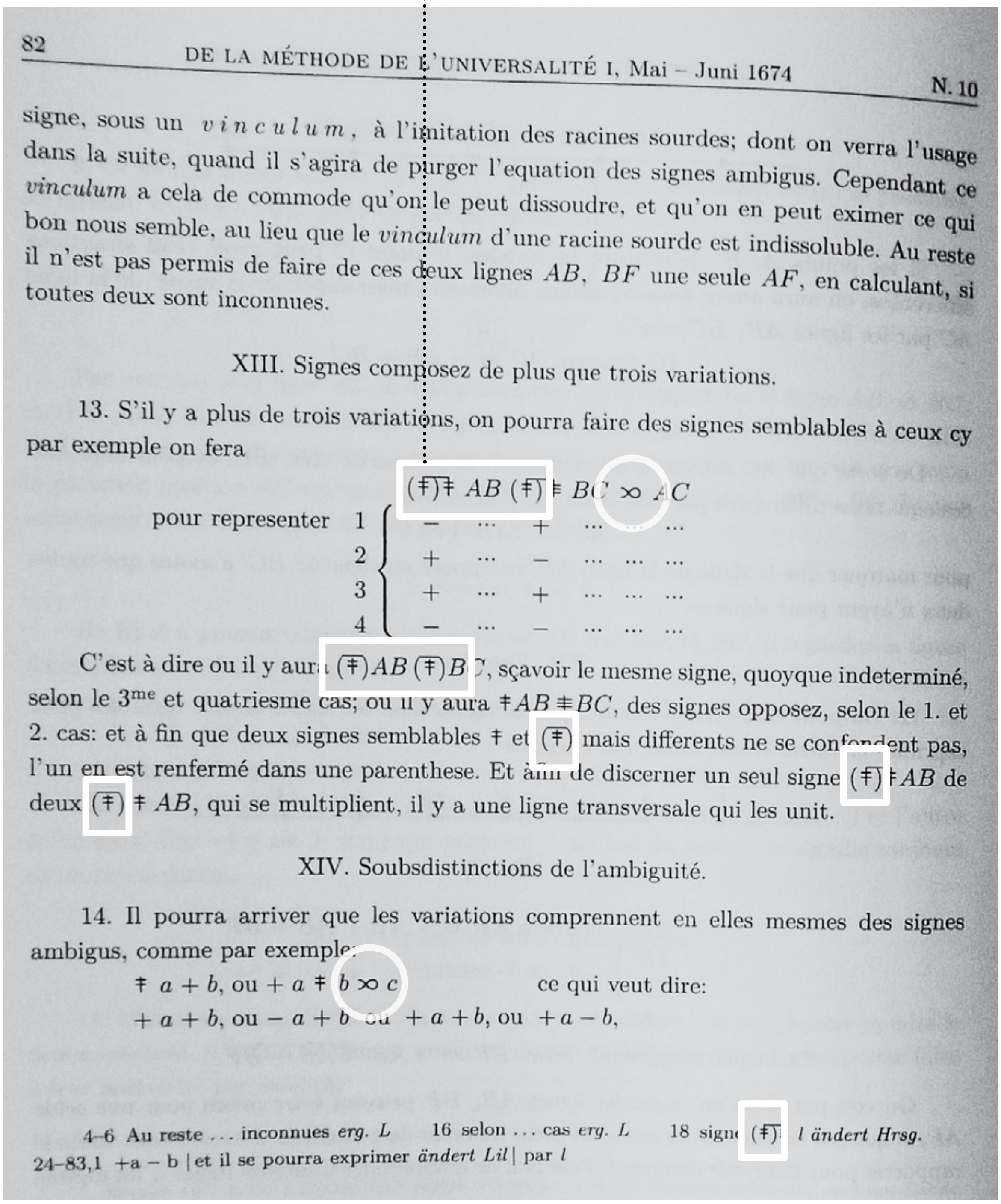
$$a \propto \begin{cases} +b-c \\ +\cdots+\cdots \end{cases} \quad \textit{item} \quad d \propto \begin{cases} -e+f \\ +\cdots-\cdots \\ +\cdots+\cdots \end{cases} \quad g \propto \left\{ \begin{matrix} -i+k-l-m \\ +i-k+l-m \\ -i-k+l+m \end{matrix} \right\}$$

Leur expression pourra estre telle:

$$a \propto b\,(\overline{\mp})\,c \quad d \propto (\overline{2\mp})\,e\,(\overline{2 \mp 2})\,f \quad g \propto (\overline{3\mp})\,i\,(\overline{3 \mp 2})\,k\,(\overline{3 \mp 2})\,l\,(\overline{3 \mp 3})\,m$$

(⁀ LEFT VIRGULA PARANTHESIS, ⁀) RIGHT VIRGULA PARANTHESIS
LAA VII-7 p. XLII

A-01



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signe, sous un *vinculum*, à l'imitation des racines sourdes; dont on verra l'usage dans la suite, quand il s'agira de purger l'equation des signes ambigus. Cependant ce *vinculum* a cela de commode qu'on le peut dissoudre, et qu'on en peut eximer ce qui bon nous semble, au lieu que le *vinculum* d'une racine sourde est indissoluble. Au reste il n'est pas permis de faire de ces deux lignes AB, BF une seule AF, en calculant, si toutes deux sont inconnues.

XIII. Signes composez de plus que trois variations.

13. S'il y a plus de trois variations, on pourra faire des signes semblables à ceux cy par exemple on fera

(∓)∓ AB (∓)∓ BC ∞ AC

pour representer							
1	−	···	+	···	···	···	
2	+	···	−	···	···	···	
3	+	···	+	···	···	···	
4	−	···	−	···	···	···	

C'est à dire ou il y aura (∓)AB (∓)BC, sçavoir le mesme signe, quoyque indeterminé, selon le 3me et quatriesme cas; ou il y aura ∓AB ∓BC, des signes opposez, selon le 1. et 2. cas: et à fin que deux signes semblables ∓ et (∓) mais differents ne se confondent pas, l'un en est renfermé dans une parenthese. Et afin de discerner un seul signe (∓)∓AB de deux (∓) ∓ AB, qui se multiplient, il y a une ligne transversale qui les unit.

XIV. Soubsdistinctions de l'ambiguité.

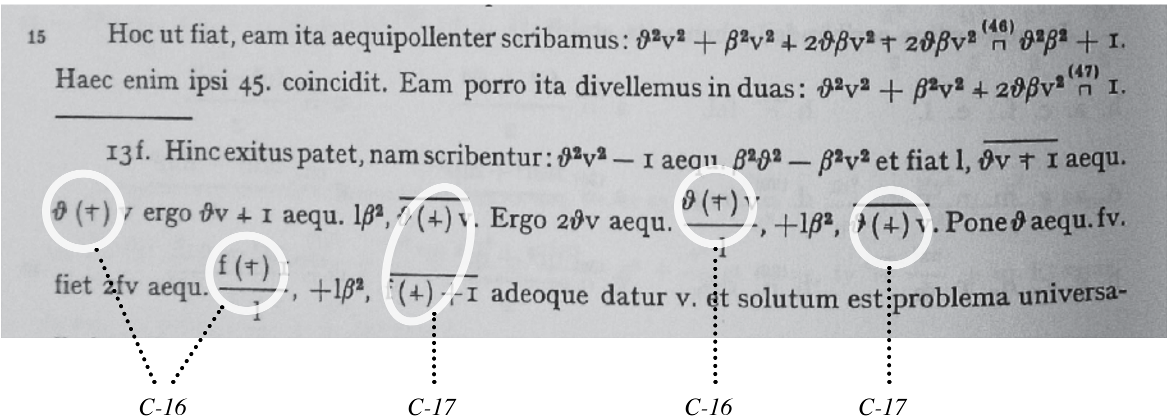
14. Il pourra arriver que les variations comprennent en elles mesmes des signes ambigus, comme par exemple:

∓ $a + b$, ou $+ a$ ∓ b ∞ c ce qui veut dire:

$+ a + b$, ou $- a + b$, ou $+ a + b$, ou $+ a - b$,

4–6 Au reste ... inconnues *erg. L* 16 selon ... cas *erg. L* 18 signe (∓) *l ändert Hrsg.*
24–83,1 +a − b | et il se pourra exprimer *ändert Lil* | par *l*

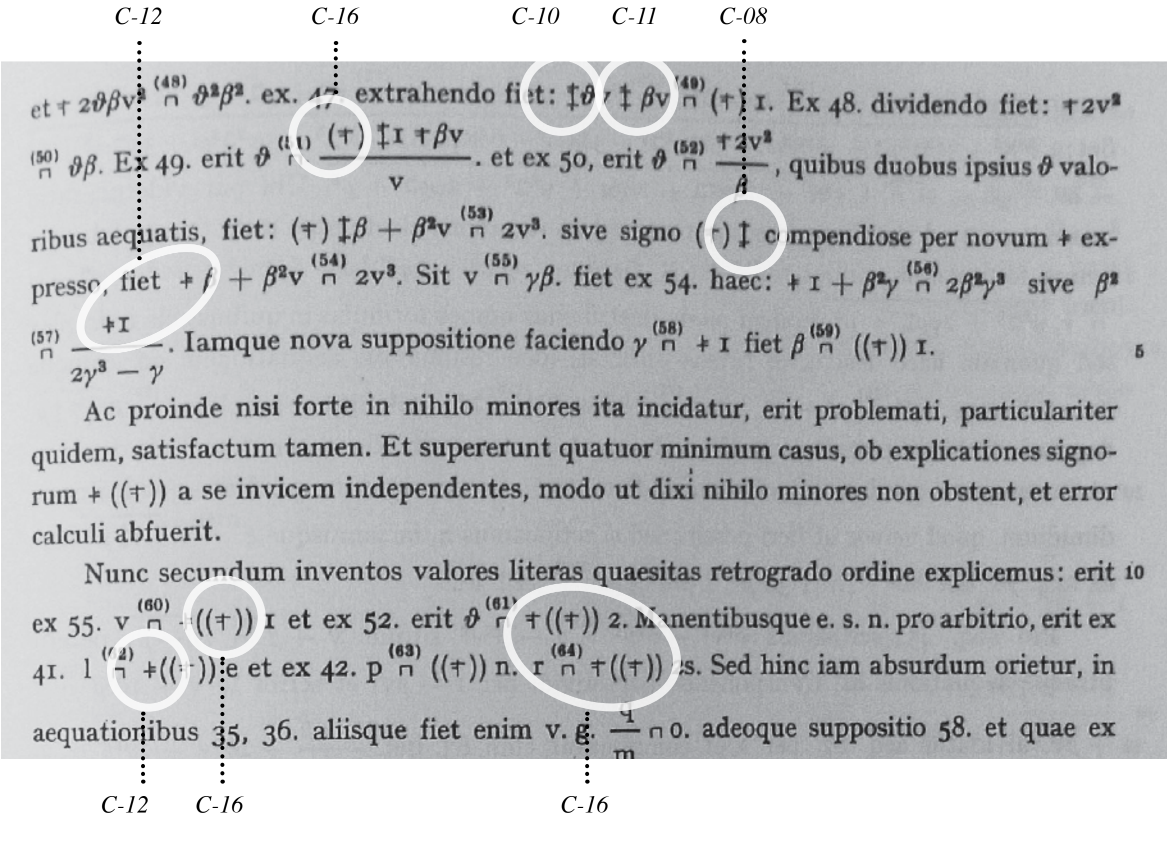
Ambiguity signs, 3rd system. This system uses
⁀ LEFT VIRGULA PARANTHESIS, ⁀ RIGHT VIRGULA PARANTHESIS.
This page also features ∞ CARTESIAN EQUAL SIGN.
LAA VII-7 p. XXXIII



Hoc ut fiat, eam ita aequipollenter scribamus: $\vartheta^2 v^2 + \beta^2 v^2 \pm 2\vartheta\beta v^2 \mp 2\vartheta\beta v^2 \overset{(46)}{\sqcap} \vartheta^2\beta^2 + 1$. Haec enim ipsi 45. coincidit. Eam porro ita divellemus in duas: $\vartheta^2 v^2 + \beta^2 v^2 \pm 2\vartheta\beta v^2 \overset{(47)}{\sqcap} 1$.

13f. Hinc exitus patet, nam scribentur: $\vartheta^2 v^2 - 1$ aequ. $\beta^2\vartheta^2 - \beta^2 v^2$ et fiat l, $\overline{\vartheta v \mp 1}$ aequ. $\vartheta (\mp) \gamma$ ergo $\vartheta v \pm 1$ aequ. $l\beta^2, \overline{\vartheta (\pm) v}$. Ergo $2\vartheta v$ aequ. $\frac{\vartheta (\mp) v}{1}$, $+l\beta^2, \overline{\vartheta (\pm) v}$. Pone ϑ aequ. fv. fiet 2fv aequ. $\frac{f (\mp)}{1}$, $+l\beta^2, \overline{f (\pm) \mp 1}$ adeoque datur v. et solutum est problema universa-

Ambiguity signs, 5th system. LAA VII-1 p. 618

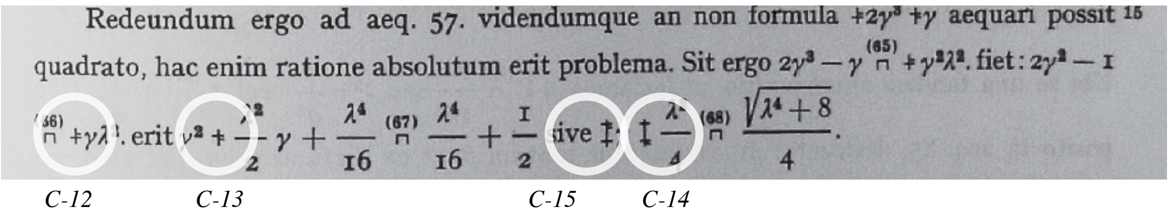


et $\mp 2\vartheta\beta v^2 \overset{(48)}{\sqcap} \vartheta^2\beta^2$. ex. 47. extrahendo fiet: $\ddagger\vartheta, \ddagger\beta v \overset{(49)}{\sqcap} (\mp) 1$. Ex 48. dividendo fiet: $\mp 2v^2 \overset{(50)}{\sqcap} \vartheta\beta$. Ex 49. erit $\vartheta \overset{(51)}{\sqcap} \frac{(\mp) \ddagger 1 \mp \beta v}{v}$. et ex 50, erit $\vartheta \overset{(52)}{\sqcap} \frac{\mp 2v^2}{\beta}$, quibus duobus ipsius ϑ valoribus aequatis, fiet: $(\mp) \ddagger\beta + \beta^2 v \overset{(53)}{\sqcap} 2v^3$. sive signo $(\mp)\ddagger$ compendiose per novum $\pm$ expresso, fiet $\pm\beta + \beta^2 v \overset{(54)}{\sqcap} 2v^3$. Sit $v \overset{(55)}{\sqcap} \gamma\beta$. fiet ex 54. haec: $\pm 1 + \beta^2\gamma \overset{(56)}{\sqcap} 2\beta^2\gamma^3$ sive $\beta^2 \overset{(57)}{\sqcap} \frac{\pm 1}{2\gamma^3 - \gamma}$. Iamque nova suppositione faciendo $\gamma \overset{(58)}{\sqcap} \pm 1$ fiet $\beta \overset{(59)}{\sqcap} ((\mp))\, 1$.

Ac proinde nisi forte in nihilo minores ita incidatur, erit problemati, particulariter quidem, satisfactum tamen. Et supererunt quatuor minimum casus, ob explicationes signorum $\pm$ $((\mp))$ a se invicem independentes, modo ut dixi nihilo minores non obstent, et error calculi abfuerit.

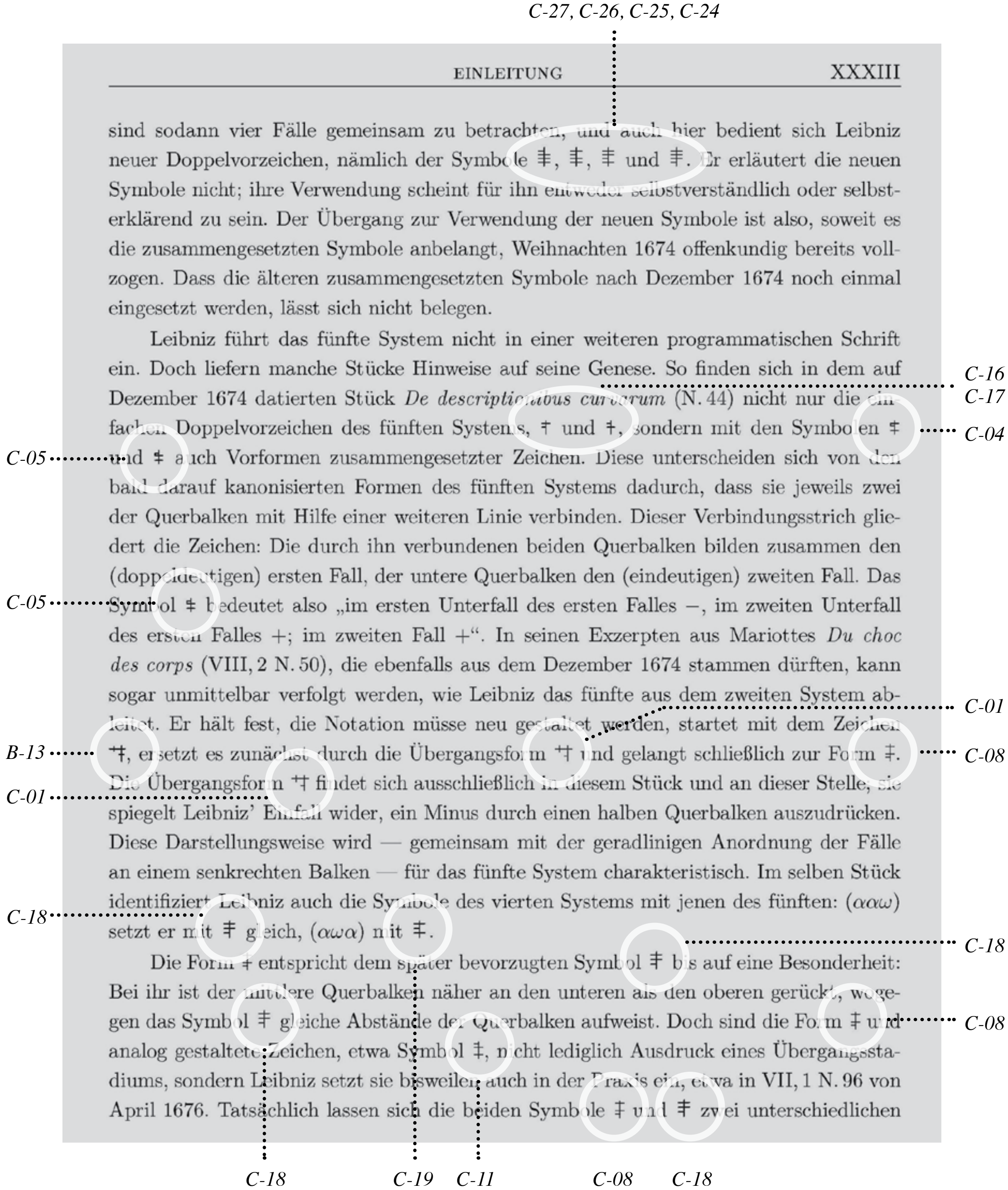
Nunc secundum inventos valores literas quaesitas retrogrado ordine explicemus: erit ex 55. $v \overset{(60)}{\sqcap} ((\mp))\, 1$ et ex 52. erit $\vartheta \overset{(61)}{\sqcap} \mp((\mp))\, 2$. Manentibusque e. s. n. pro arbitrio, erit ex 41. $l \overset{(62)}{\sqcap} \pm((\mp)) e$ et ex 42. $p \overset{(63)}{\sqcap} ((\mp))\, n$. $r \overset{(64)}{\sqcap} \mp((\mp))\, s$. Sed hinc iam absurdum orietur, in aequationibus 35, 36. aliisque fiet enim v. g. $\frac{q}{m} \sqcap 0$. adeoque suppositio 58. et quae ex

Ambiguity signs, 5th system. LAA VII-1 p. 619



Redeundum ergo ad aeq. 57. videndumque an non formula $\pm 2\gamma^3 \mp \gamma$ aequari possit quadrato, hac enim ratione absolutum erit problema. Sit ergo $2\gamma^3 - \gamma \overset{(65)}{\sqcap} \pm\gamma^2\lambda^2$. fiet: $2\gamma^2 - 1 \overset{(66)}{\sqcap} \pm\gamma\lambda^2$. erit $\gamma^2 \mp \frac{\lambda^2}{2}\gamma + \frac{\lambda^4}{16} \overset{(67)}{\sqcap} \frac{\lambda^4}{16} + \frac{1}{2}$ sive $\ddagger\gamma \ddagger \frac{\lambda^2}{4} \overset{(68)}{\sqcap} \frac{\sqrt{\lambda^4 + 8}}{4}$.

Ambiguity signs, 5th system. LAA VII-1 p. 619



EINLEITUNG XXXIII

sind sodann vier Fälle gemeinsam zu betrachten, und auch hier bedient sich Leibniz neuer Doppelvorzeichen, nämlich der Symbole ǂ, ǂ, ǂ und ǂ. Er erläutert die neuen Symbole nicht; ihre Verwendung scheint für ihn entweder selbstverständlich oder selbsterklärend zu sein. Der Übergang zur Verwendung der neuen Symbole ist also, soweit es die zusammengesetzten Symbole anbelangt, Weihnachten 1674 offenkundig bereits vollzogen. Dass die älteren zusammengesetzten Symbole nach Dezember 1674 noch einmal eingesetzt werden, lässt sich nicht belegen.

Leibniz führt das fünfte System nicht in einer weiteren programmatischen Schrift ein. Doch liefern manche Stücke Hinweise auf seine Genese. So finden sich in dem auf Dezember 1674 datierten Stück *De descriptionibus curvarum* (N. 44) nicht nur die einfachen Doppelvorzeichen des fünften Systems, † und ⊹, sondern mit den Symbolen ‡ und ‡ auch Vorformen zusammengesetzter Zeichen. Diese unterscheiden sich von den bald darauf kanonisierten Formen des fünften Systems dadurch, dass sie jeweils zwei der Querbalken mit Hilfe einer weiteren Linie verbinden. Dieser Verbindungsstrich gliedert die Zeichen: Die durch ihn verbundenen beiden Querbalken bilden zusammen den (doppeldeutigen) ersten Fall, der untere Querbalken den (eindeutigen) zweiten Fall. Das Symbol ‡ bedeutet also „im ersten Unterfall des ersten Falles −, im zweiten Unterfall des ersten Falles +; im zweiten Fall +“. In seinen Exzerpten aus Mariottes *Du choc des corps* (VIII, 2 N. 50), die ebenfalls aus dem Dezember 1674 stammen dürften, kann sogar unmittelbar verfolgt werden, wie Leibniz das fünfte aus dem zweiten System ableitet. Er hält fest, die Notation müsse neu gestaltet werden, startet mit dem Zeichen ⁺†, ersetzt es zunächst durch die Übergangsform ⁺† und gelangt schließlich zur Form ‡. Die Übergangsform ⁺† findet sich ausschließlich in diesem Stück und an dieser Stelle, sie spiegelt Leibniz’ Einfall wider, ein Minus durch einen halben Querbalken auszudrücken. Diese Darstellungsweise wird — gemeinsam mit der geradlinigen Anordnung der Fälle an einem senkrechten Balken — für das fünfte System charakteristisch. Im selben Stück identifiziert Leibniz auch die Symbole des vierten Systems mit jenen des fünften: $(\alpha\alpha\omega)$ setzt er mit ǂ gleich, $(\alpha\omega\alpha)$ mit ǂ.

Die Form ǂ entspricht dem später bevorzugten Symbol ǂ bis auf eine Besonderheit: Bei ihr ist der mittlere Querbalken näher an den unteren als den oberen gerückt, wogegen das Symbol ǂ gleiche Abstände der Querbalken aufweist. Doch sind die Form ‡ und analog gestaltete Zeichen, etwa Symbol ‡, nicht lediglich Ausdruck eines Übergangsstadiums, sondern Leibniz setzt sie bisweilen auch in der Praxis ein, etwa in VII, 1 N. 96 von April 1676. Tatsächlich lassen sich die beiden Symbole ‡ und ǂ zwei unterschiedlichen

Ambiguity signs, 2nd and 5th system. LAA VII-7 p. XXXIII

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(31) Ponamus jam contra directricem esse non AD, sed AE, constantem WL, quam vocabimus λ. Crementum ordinatarum EG, esse GW; ipsam $EH \sqcap l$. primum investigemus hoc modo: $2ax \ddagger \frac{2a}{q}x^2 \sqcap 2yl$. sive $l \sqcap \frac{ax \ddagger \frac{a}{q}x^2}{y} \sqcap \frac{2ax \ddagger \frac{a}{q}x^2 - ax}{y}$ sive $\frac{y^2 - ax}{y}$. Jam ut x inveniatur, erit $x^2 \ddagger \frac{2q\phi}{\phi}x + q^2 \sqcap q^2 \ddagger y^2$, adeoque fiet $\mp x \mp q \sqcap \sqrt{,}\, q^2 \mp y^2$, et $x \sqcap \mp q \mp \sqrt{q^2 \mp y^2}$ adeoque $l \sqcap \frac{y^2 \mp qa \mp a\sqrt{q^2 \mp y^2}}{y} \sqcap EH$. Ergo GW erit $\sqcap \frac{\lambda_{,,} \frown y^2 \mp qa \mp a\sqrt{q^2 \mp y^2}}{y_{,,} \frown \mp q \mp \sqrt{q^2 \mp y^2}}$; et $\frac{GB \frown WL^2}{GW} \sqcap \frac{y \frown \lambda^2, \frown y, \frown \mp q \mp \sqrt{q^2 \mp y^2}}{\lambda_{,,} \frown y^2 \mp qa \mp a\sqrt{q^2 \mp y^2}}$, cujus seriei itidem habetur summa, ex datis omnibus $\sqrt{q^2 \mp y^2}$.

Quae theoremata vel ideo annotanda duxi, quod semel elapsa non facile rursus in mentem venirent, et non nisi per multas ambages deprehensa sint. Et haec quidem de Trianguli characteristici usu ad dimensiones curvilineorum nunc sufficiant.

Ambiguity signs C-21, 5th system. LAA VII-5 p. 191

imatur, etc. $CL \sqcap BL \sqcap \sqrt{2ax - x^2}$ fiet EL. Nimirum si D sit intra A et T, seu quando $TD \sqcap +TA - AD$, erit $+EC + CL$, quando D intra A et B, tunc cadit E inter C. et L. et erit $EL \sqcap -EC + CL$. et $TD \sqcap TA + AD$: Quando D ultra B tunc TD etiam $TA + AD$. sed $EL \sqcap +EC - CL$. Quando D ultra T, seu quando $TD \sqcap -TA + AD$ tunc $EL \sqcap EC + CL$. Ut ergo digeramus erunt situs quatuor ipsius D, varietatem afferentes, $(1)D, (2)D, (3)D, (4)D$.

$(1)D$, dat:	$TD \sqcap -TA + AD$	$EL \sqcap +EC + CL$
$(2)D$ ...	$TD \sqcap +TA - AD$	$EL \sqcap +EC + CL$
$(3)D$ ...	$TD \sqcap +TA + AD$	$EL \sqcap -EC + CL$
$(4)D$ ...	$TD \sqcap +TA + AD$	$EL \sqcap +EC - CL$

Generaliter ergo TD ita exprimemus:

$$TD \sqcap \mp TA \mp AD, \quad EL \sqcap \mp EC \mp CL.$$

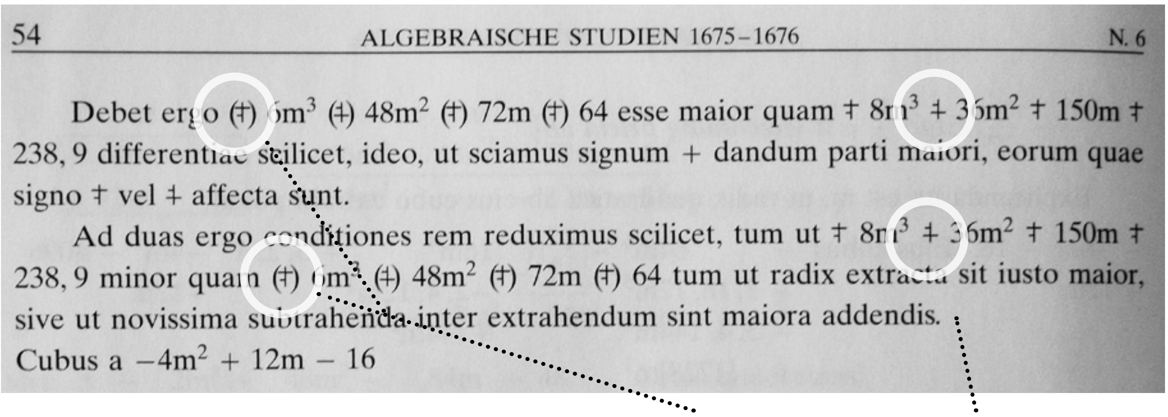
Ergo hoc modo $DE \sqcap \frac{\mp ax \mp af \mp xf}{a}$ et $EL \sqcap \mp \frac{f\sqrt{2ax - x^2}}{a} \mp \sqrt{2ax - x^2}$.

Ponendo jam $DE \sqcap y$, fiet: $\frac{ay \mp af}{\mp a \mp f} \sqcap x$, et $x^2 \sqcap \frac{a^2y^2 \mp 2a^2fy + a^2f^2}{a^2 \mp 2af + f^2}$.

Unde $EL \sqcap z \sqcap \frac{\mp f \mp a}{a} \frown \sqrt{2ax - x^2}$.

C-25, C-24 *C-31* *C-26* *C-28* *C-24*

Ambiguity signs, 5th system. LAA VII-5 p. 191



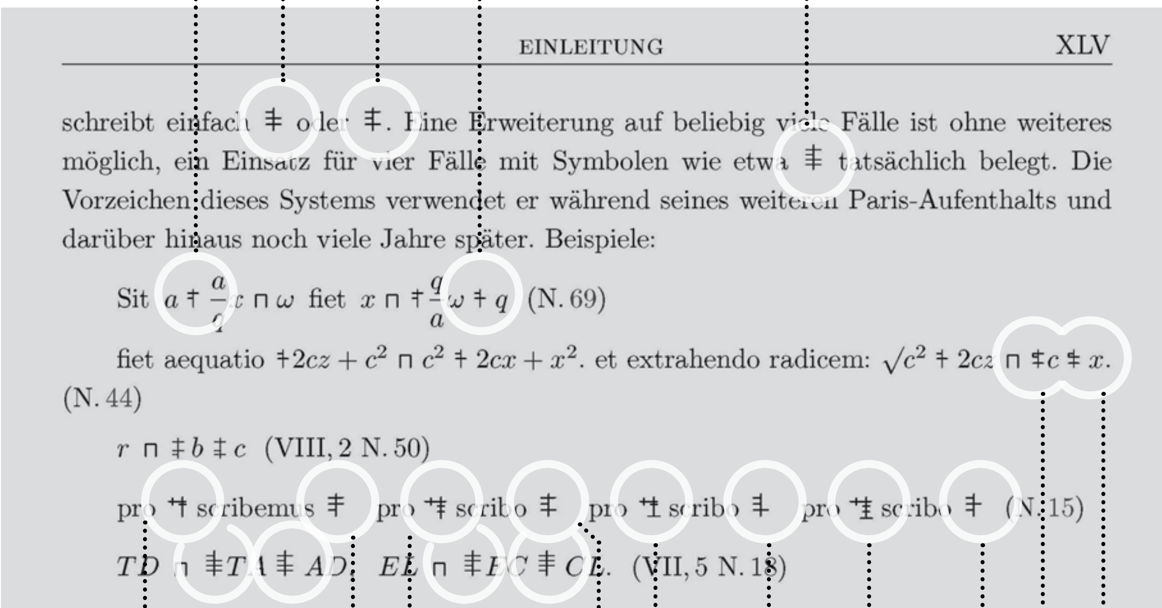
54 ALGEBRAISCHE STUDIEN 1675–1676 N. 6

Debet ergo (∓) 6m³ (±) 48m² (∓) 72m (∓) 64 esse maior quam ∓ 8m³ ± 36m² ∓ 150m ∓ 238, 9 differentiae scilicet, ideo, ut sciamus signum + dandum parti maiori, eorum quae signo ∓ vel ± affecta sunt.

Ad duas ergo conditiones rem reduximus scilicet, tum ut ∓ 8m³ ± 36m² ∓ 150m ∓ 238, 9 minor quam (∓) 6m³ (±) 48m² (∓) 72m (∓) 64 tum ut radix extracta sit iusto maior, sive ut novissima subtrahenda inter extrahendum sint maiora addendis.

Cubus a $-4m^2 + 12m - 16$

Ambiguity signs, 5th system. LAA VII-2 p. 54



EINLEITUNG XLV

schreibt einfach ∓ oder ∓. Eine Erweiterung auf beliebig viele Fälle ist ohne weiteres möglich, ein Einsatz für vier Fälle mit Symbolen wie etwa ∓ tatsächlich belegt. Die Vorzeichen dieses Systems verwendet er während seines weiteren Paris-Aufenthalts und darüber hinaus noch viele Jahre später. Beispiele:

Sit $a \mp \frac{a}{q} c \sqcap \omega$ fiet $x \sqcap \mp \frac{q}{a} \omega \mp q$ (N. 69)

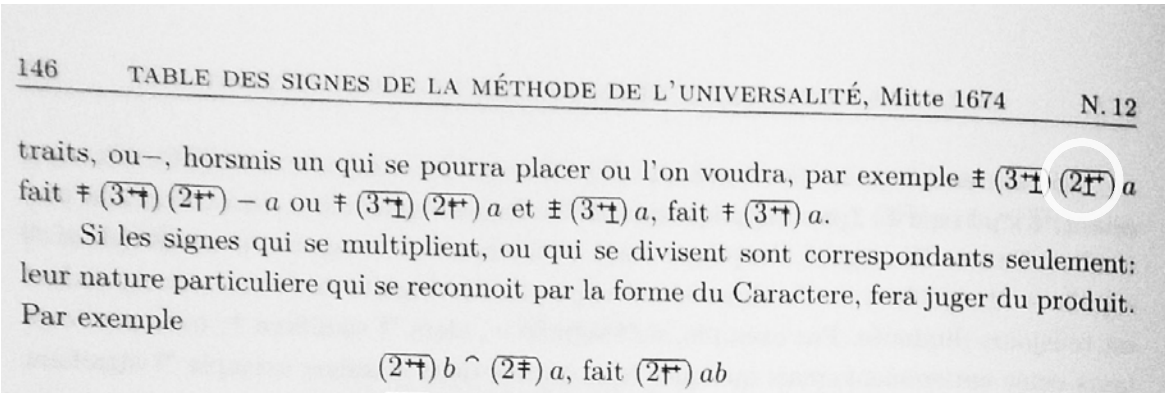
fiet aequatio $\mp 2cz + c^2 \sqcap c^2 \mp 2cx + x^2$. et extrahendo radicem: $\sqrt{c^2 \mp 2cz} \sqcap \mp c \mp x$. (N. 44)

$r \sqcap \ddagger b \ddagger c$ (VIII, 2 N. 50)

pro ∓ scribemus ∓ pro ∓ scribo ∓ pro ∓ scribo ∓ pro ∓ scribo ∓ (N. 15)

$TD \sqcap \mp TA \mp AD$; $EL \sqcap \mp EC \mp CL$. (VII, 5 N. 18)

Example of ambiguity signs, 2nd and 5th system. LAA VII-7 p. XLV



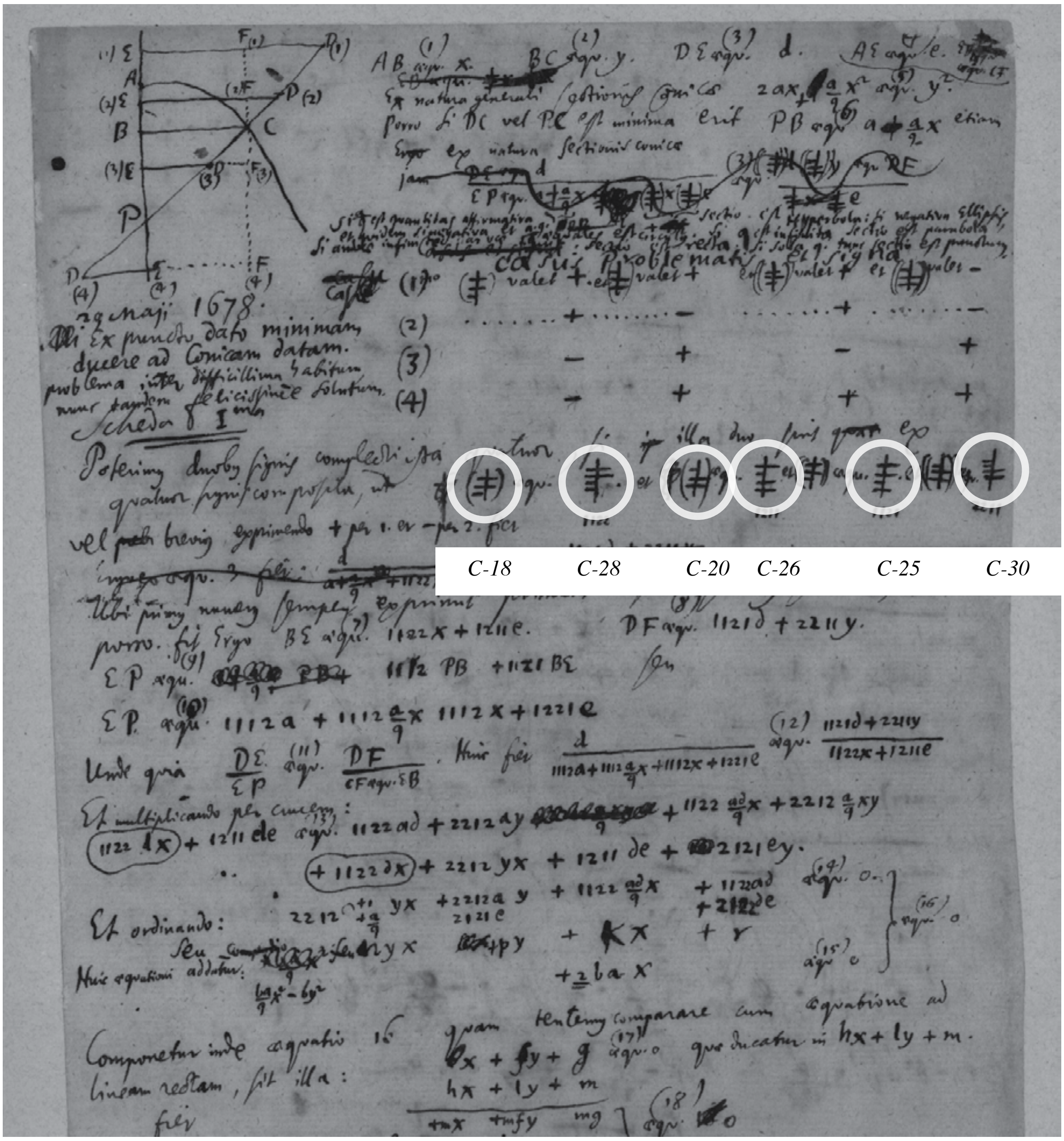
146 TABLE DES SIGNES DE LA MÉTHODE DE L'UNIVERSALITÉ, Mitte 1674 N. 12

traits, ou–, horsmis un qui se pourra placer ou l'on voudra, par exemple ± (3∓) (2∓) a fait ∓ (3∓) (2∓) – a ou ∓ (3∓) (2∓) a et ± (3∓) a, fait ∓ (3∓) a.

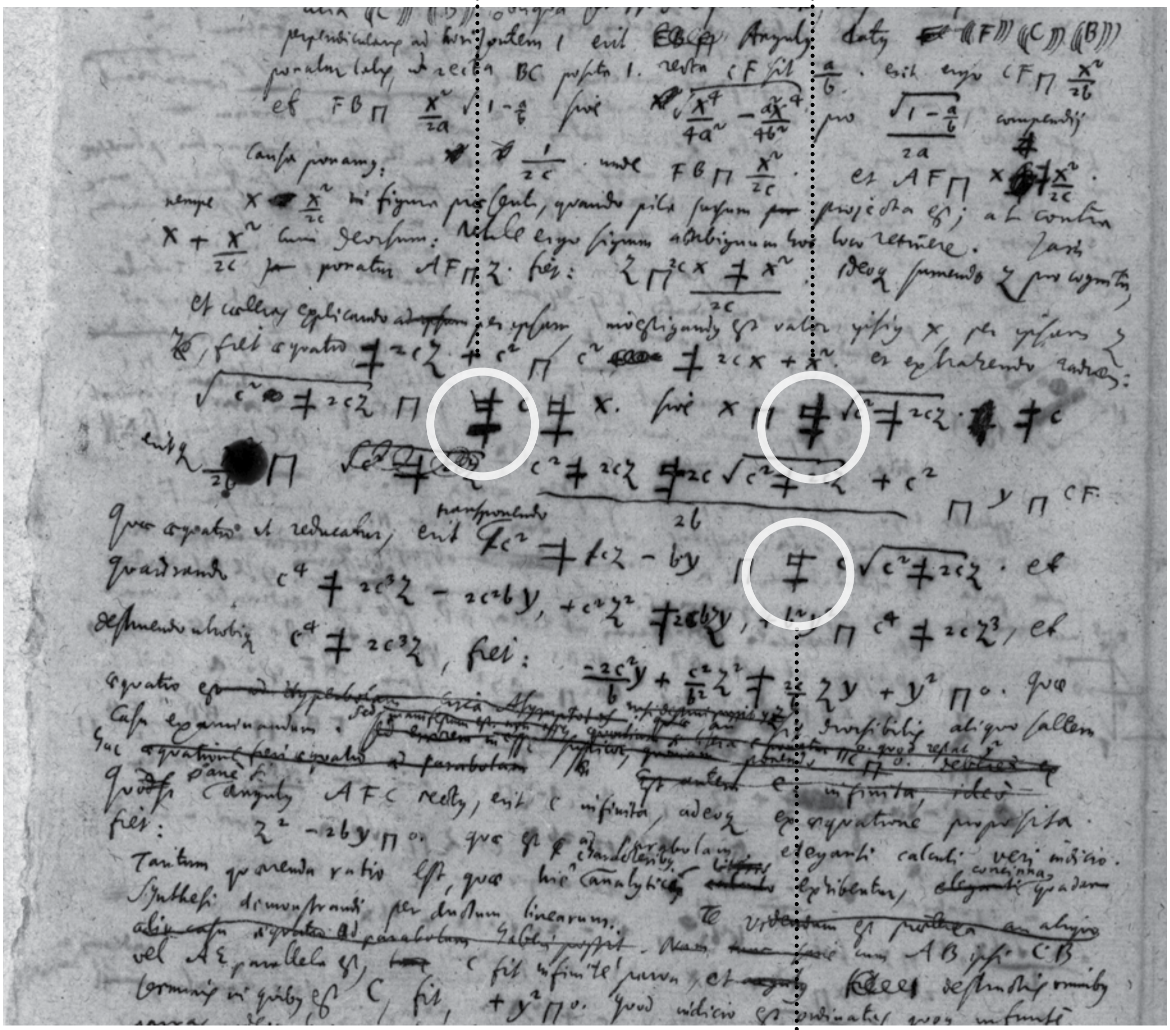
Si les signes qui se multiplient, ou qui se divisent sont correspondants seulement: leur nature particuliere qui se reconnoit par la forme du Caractere, fera juger du produit. Par exemple

(2∓) b ⌒ (2∓) a, fait (2∓) ab

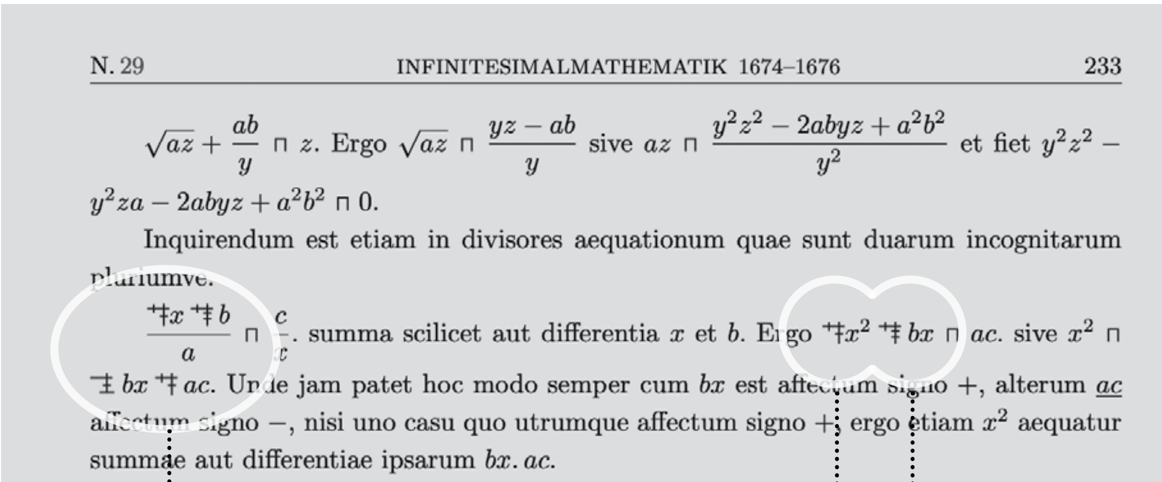
± AMBIGUITY SIGN B-10
LAA VII-7 p. 146



Ambiguity signs of the 5th system, as seen in one of Leibniz's manuscripts (LH 35 XII 1, 217v).
The edition of this manuscript is in preparation.



Ambiguity signs, 5th system.
LH 35 XII 1 fol. 227v


N. 29 INFINITESIMALMATHEMATIK 1674–1676 233

$\sqrt{az} + \frac{ab}{y} \sqcap z$. Ergo $\sqrt{az} \sqcap \frac{yz - ab}{y}$ sive $az \sqcap \frac{y^2z^2 - 2abyz + a^2b^2}{y^2}$ et fiet $y^2z^2 - y^2za - 2abyz + a^2b^2 \sqcap 0$.

Inquirendum est etiam in divisores aequationum quae sunt duarum incognitarum pluriumve.

$\frac{\pm x \pm b}{a} \sqcap \frac{c}{c}$. summa scilicet aut differentia x et b. Ergo $\pm x^2 \pm bx \sqcap ac$. sive $x^2 \sqcap \mp bx \pm ac$. Unde jam patet hoc modo semper cum bx est affectum signo $+$, alterum $\underline{ac}$ affectum signo $-$, nisi uno casu quo utrumque affectum signo $+$, ergo etiam x^2 aequatur summae aut differentiae ipsarum $bx.\ ac$.

Ambiguity signs, 5th system.
LAA VII-5 p. 233

B-05 B-11 B-18 B-15 B-05 B-14

au lieu de ⁺‡; et ⁺‡ au lieu de ⁺‡. Et à fin aussi qu'on voye la raison de la distance que je laisse entre le trait haussé, et les premiers, et pour quoy je fais ⁺‡ au lieu de ⁺‡, et ⁺‡ au lieu de ⁺‡ ou ⁺‡ je dis qu'on découvre par ce moyen à la premiere veue l'origine et composition de tous ces signes, mais qu'outre cette commodité il y a même quelque necessité de faire de la sorte, pour eviter l'equivocation, ou confusion de deux signes de differente signification, car posons que le signe ⁺‡ doive entrer dans la composition d'un autre, si on en faisoit alors ⁺‡ en haussant simplement le trait d'embas on ne le discerneroit pas du signe ⁺‡ quand il entreroit aussi dans une composition par ce que en le haussant simplement, nous aurions eu aussi ⁺‡ au lieu de ⁺‡ donc voila deux ⁺‡ de differente signification l'un fait de ⁺‡, c'est à dire du contraire à ⁺† c'est à dire à + ou ∓: l'autre fait de ⁺‡, c'est a dire de + ou ± c'est à dire du + et du contraire à ∓: ce qui n'est pas le même.

B-17

Quand je dis par exemple que ⁺† vaut + ou ∓, et que ⁺‡ vaut + ou ± cela se doit entendre avec une relation entre ces deux signes ambigus composez; de sorte que si dans l'application de l'ambiguité ou generalité à un cas particulier, ⁺† est expliqué par −, alors ⁺‡ sera expliqué par + et *vice versa* car entre ces trois equations susdites de la 5^me^ figure il n'y a pas une, ou AB aussi bien que BC, tout a la fois soient affectées par −. Mais si ⁺† est expliqué par +, il n'est pas necessaire que ⁺‡ soit expliqué par − par ce que dans une de ces equations particulieres, AB, aussi bien que BC, sont affectées par +. Par consequent si l'un de ces deux signes composés est expliqué par + l'autre sera expliqué par ∓ et *vice versa* (: avec la caution pourtant, que nous y apporterons plus bas :) de

Ambiguity signs, 5th system.
LAA VII-7 p. 125

IV. Ambig.

$a \sqcap$ (3⁺†) b (3⁺‡) c signifie $a \sqcap +b \;\; +c$, c'est à dire, $a \sqcap +b+c$
(3∓){ +b / −b } (3±){ −c / +c }   ou (3∓) b (3±) c

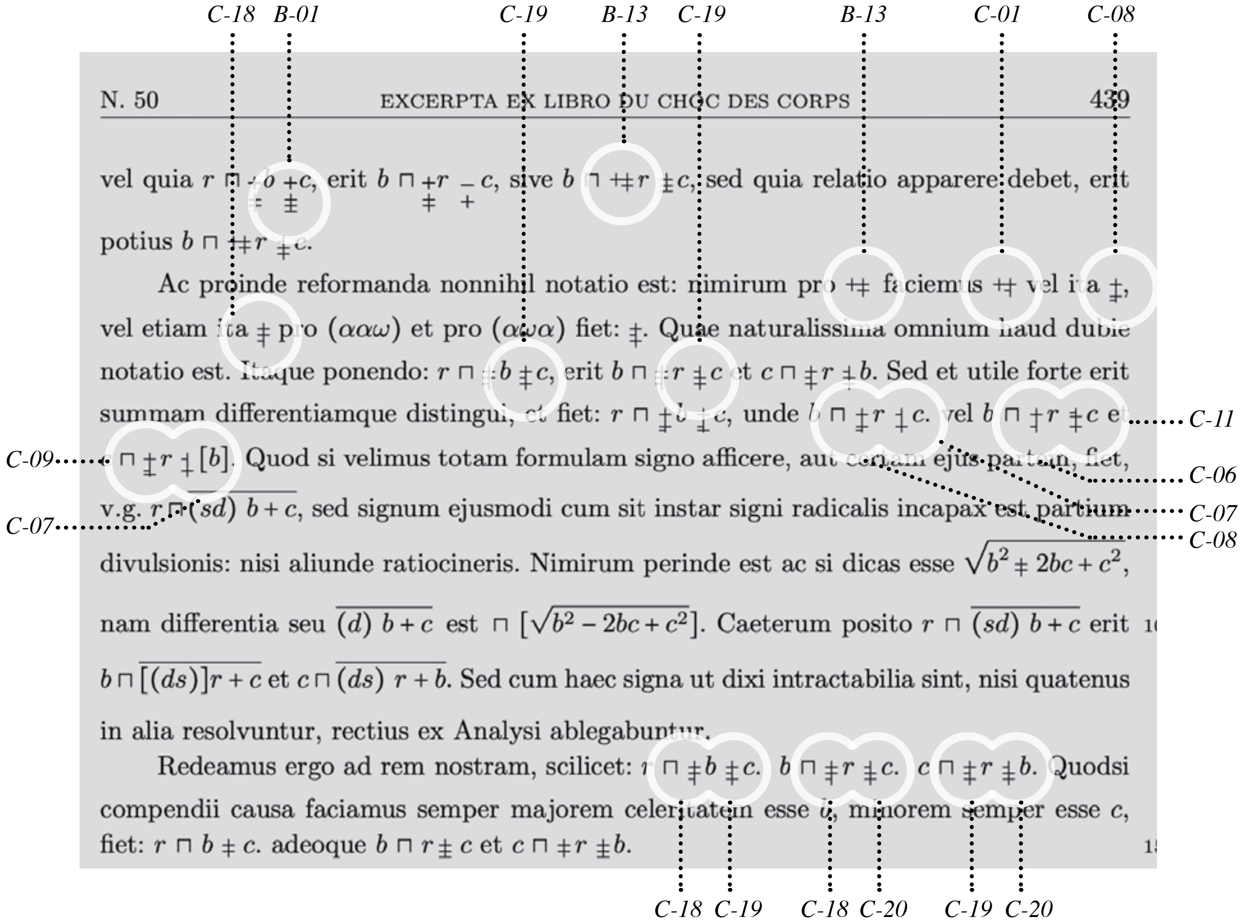
a estant ou la somme, ou la difference de b. c. cela fait voir clairement la raison de la fabrique des signes, et il faut remarquer seulement que de + ou (3±), on a fait tout expres (3⁺‡) au lieu de (3⁺‡) par ce que (3⁺‡) signifie le signe opposé à (3⁺†).

Si nous eussions eu

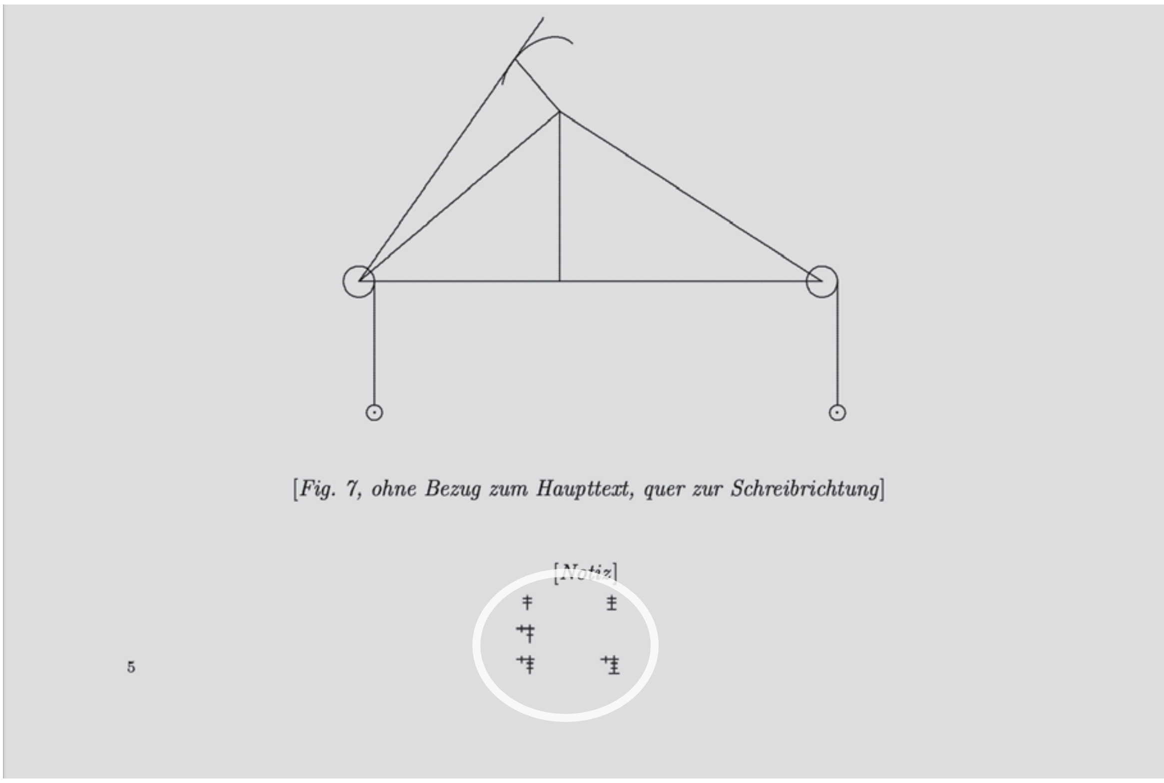
$f \sqcap +b \;\; +c \;\; (3∓)\{+e, -e\}$
(3∓){ +b / −b } (3±){ −c / +c } −e, cela auroit fait

$f \sqcap$ (3⁺†)b (3⁺‡)c (3∓⁻)e

LAA VII-7 p. 144 B-09



Example of ambiguity signs, 2nd and 5th system. LAA VIII-2 p. 438



B-02, B-06, B-03; LAA VII-3 p. 360

246 A HISTORY OF MATHEMATICAL NOTATIONS

by De Witt.[1] Wallis[2] wrote ♉ for + or −, and ☊ for the contrary. The sign ☊ was used in a restricted way, by James Bernoulli;[3] he says, "☊ significat + in pr. e − in post. hypoth.," i.e., the symbol stood for + according to the first hypothesis, and for −, according to the second hypothesis. He used this same symbol in his *Ars conjectandi* (1713), page 264. Van Schooten wrote also ♉ for ∓. It should be added that ♉ appears also in the older printed Greek books as a ligature or combination of two Greek letters, the omicron *ο* and the upsilon *υ*. The ♉ appears also as an astronomical symbol for the constellation Taurus.

Da Cunha[4] introduced ±' and ±', or ±' and ∓', to mean that the upper signs shall be taken simultaneously in both or the lower signs shall be taken simultaneously in both. Oliver, Wait, and Jones[5] denoted positive or negative N by $^{\pm}N$.

211. The symbol $[a]$ was introduced by Kronecker[6] to represent

♉ PLUSMINUS SIGN, ☊ MINUSPLUS SIGN; Cajori I p. 246. In this paragraph Cajori explains the different usage of this two symbols for "+ or –" and "- or +" by van Schooten , Bernoulli and Wallis. A variety of symbols was used during the 17th century for denoting plus-minus. Leibniz used the same symbols in a different context in order to denote *congruence*, hence the proposed character name in this proposal.
Despite of what Cajori writes here about the similar looking characters *omicron-upsilon* and the astrological *Taurus* symbol, the ♉ should not be mixed up with neither of them. See page 102 for this peculiar character.

☊ MINUSPLUS SIGN
Descartes, Geometria, p. 330

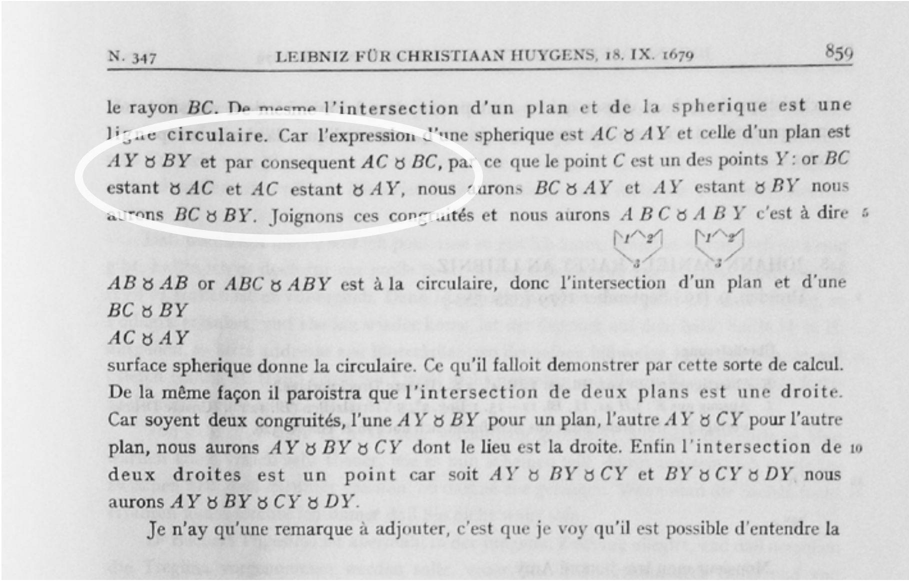
binomium *a* ☊ √*bc*,

Where the First Term hath the Sign + (because made by Multiplying + into +;) The Second Term is wanting (because − *ya*³ and + *ya*³ destroy each other:) In the Third Term, *yy* hath − (because made of + *y* into − *y*;) and *b*, *d*, have the same Terms as in the Quadraticks, (which Sign, be it + or −, we here design by ♉, and its contrary by ☊:) In the Fourth Term, *c* hath the same Sign as before (because Multiplied into + *y*;) but *d* the contrary to what it had (because Multiplyed into − *y*.) And thus far it holds constantly, whatever be the Signs of *p*, *q*, *r*.

♉ PLUSMINUS SIGN, ☊ MINUSPLUS SIGN
Wallis, Algebra, p. 210

(☊ *significat* ∓ *in pr.* & − *in post. hypoth.*

☊ MINUSPLUS SIGN
Acta eruditorum 1701, p. 214

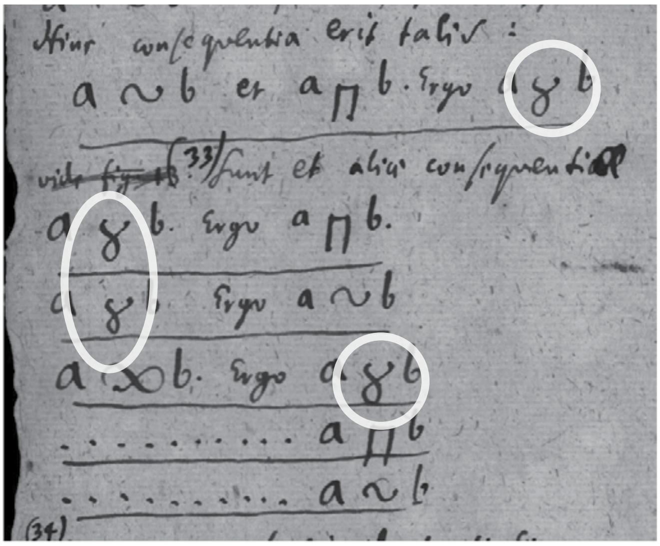


N. 347 LEIBNIZ FÜR CHRISTIAAN HUYGENS, 18. IX. 1679 859

le rayon *BC*. De mesme l'intersection d'un plan et de la spherique est une ligne circulaire. Car l'expression d'une spherique est *AC* ♉ *AY* et celle d'un plan est *AY* ♉ *BY* et par consequent *AC* ♉ *BC*, par ce que le point *C* est un des points *Y* : or *BC* estant ♉ *AC* et *AC* estant ♉ *AY*, nous aurons *BC* ♉ *AY* et *AY* estant ♉ *BY* nous aurons *BC* ♉ *BY*. Joignons ces congruités et nous aurons *A B C* ♉ *A B Y* c'est à dire *AB* ♉ *AB* or *ABC* ♉ *ABY* est à la circulaire, donc l'intersection d'un plan et d'une
BC ♉ *BY*
AC ♉ *AY*
surface spherique donne la circulaire. Ce qu'il falloit demonstrer par cette sorte de calcul. De la même façon il paroistra que l'intersection de deux plans est une droite. Car soyent deux congruités, l'une *AY* ♉ *BY* pour un plan, l'autre *AY* ♉ *CY* pour l'autre plan, nous aurons *AY* ♉ *BY* ♉ *CY* dont le lieu est la droite. Enfin l'intersection de deux droites est un point car soit *AY* ♉ *BY* ♉ *CY* et *BY* ♉ *CY* ♉ *DY* nous aurons *AY* ♉ *BY* ♉ *CY* ♉ *DY*.

Je n'ay qu'une remarque à adjouter, c'est que je voy qu'il est possible d'entendre la

♉ PLUSMINUS SIGN,
is used here by Leibniz as a symbol for "congruence" instead; LAA III-2 p. 859.



♉ PLUSMINUS SIGN,
is used here by Leibniz as a symbol for "congruence" instead; manuscript LH 35 I 11 fol. 9r

4.d) Geometrical signs

A B
C

[2] AC ⊓ [2] AB + [2] CB − 2 ▭ACB
[3] AC ⊓ [3] AB − [3] CB − 3 ⬚ACBA

Sit linea AB secta alicubi in C. Demonstravit Euclides, quadratum ab AB aequari quadrato ab AC, + quad. a CB, + bis rectang. ACB. Et idem demonstravit, quadratum ab AC alterutra partium aequari, quadrato ab AB, + quadr. a CB, − rectang. ABC. Inventor regularum Cardani demonstravit, cubum ab AB aequari cubo ab AC, + cub. a CB, + 3 rectang. solido ACBA, sive ter rectang. solido, comprehenso sub rectis AC, CB, BA; et cubum ab AC aequari cubo ab AB, − cub. a CB, − 3 rectang. solido ACBA.

[5] AC ⊓ [5] AB − [5] CB −
− 5 ⬚ACBA in $\overline{[2]\ \text{AC} + ▭\text{ABC}}$

Haec tabula continuata pro omnibus aliis potestatibus altioribus similia theoremata concinnare docet; nimirum surdesolidum ab AC aequatur surdesolid. ab AB − surdes. a CB,

⬚ CUBUS 1
LAA III-1 p. 643

302 ARITHMETISCHE KREISQUADRATUR 1673–1676 N. 26

Als men de $\angle ACB$ wil 2 mahl in 2 gelijcke deel, deelen; om AF te vinden, soo kan men het dus oock doen[:]

Regel.

Gelijck als
$AC + BC$, sijn ⬚ staet tot also het tot het
$-\Box AB$, multipl. in BC ⊤⊤ $\Box AB$, multipl. in AC ⊤⊤ $\Box AC$ ⊤⊤ $\Box AF$.

⬚ CUBUS 2. This figure shows also the use of PROPORTION 2. – LAA VII-6 P. 302

173. Deeply influenced by geometrical considerations was Jean Buteon,[1] in his *Logistica quae et Arithmetica vulgo dicitur* (Lugduni, 1559). In the part of the book on algebra he rejects the words *res*, *census*, etc., and introduces in their place the Latin words for "line," "square," "cube," using the symbols ρ, ◇, ⬚. He employs also P and M, both as signs of operation and of quality. Calling the sides of an equation *continens* and *contentum*, respectively, he writes between them the sign [as long as the equation is not reduced to the simplest form and the *contentum*, therefore, not in its final form. Later the *contentum* is inclosed in the completed rectangle [ ]. Thus Buteon writes 3ρ M 7 [8 and then draws the inferences, 3ρ [15], 1ρ [5]. Again he writes $\frac{1}{4}$ ◇ [100, hence 1◇ [400], 1ρ [20]. In modern symbols: $3x-7=8$, $3x=15$, $x=5$; $\frac{1}{4}x^2=100$, $x^2=400$, $x=20$. Another example: $\frac{1}{8}$ ⬚ P 2 [218, $\frac{1}{8}$ ⬚ [216, 1 ⬚ [1,728], 1ρ [12]; in modern form $\frac{1}{8}x^3+2=218$, $\frac{1}{8}x^3=216$, $x^3=1{,}728$, $x=12$.

When more than one unknown quantity arises, they are repre-

⬚ CUBUS 2. Cajori vol. 1, p. 176

ducta est) tangat. Ex altero extremo B, recta BE radio AW perpendiculariter occurrat in E. Iungatur EG tum AM ipsi AW, et LM, ipsi AM perpendiculariter incidant. Aio si rectangulum AL multiplex secundum numerum δ, adimatur triangulo GWE, differentiam fore aream segmenti $BWCB$.

Ex his facile intelligi potest, numerum δ, esse u n i t a t e imo et s e m i s s e minorem. Nam si BCW sit arcus quadrantis, erit ▭ AL duplum ◺ AW, sequitur et ex data quadratura circuli totius dari quadraturam quarumlibet partium quae geometrice abscindi possint. Et rursus vel unica eius portione quae geometrice abscindi possit

◺ RIGHT TRIANGLE POINTING RIGHT *The Rectangle has codepoint 25AD.*
LAA VII-3 p. 275

$\frac{a^2[\sqrt{2}]}{a\sqrt{2}+x-\sqrt{2a^2+x^2}}$ ⊓ z. Contra si x. investigare velis, retenta z, fiet: $\sqrt{2a^2+x^2}$ ⊓ $a\sqrt{2}+x-\frac{a^2}{z}\sqrt{2}$. Unde $\boxed{2a^2}\boxed{+x^2}$ ⊓ $\boxed{2a^2}+2ax\sqrt{2}\boxed{+x^2}$, $\boxed{-\frac{2a^2\sqrt{2}\sqrt{2}}{z}}-\frac{4a^2}{z}-\frac{2a^2x\sqrt{2}}{z}+\frac{2a^4}{z^2}$ ⊓ 0. sive: $2axz^2\sqrt{2}-4a^2z-2a^2xz\sqrt{2}+a^4$ ⊓ 0. et x ⊓ $\frac{4a^2z-a^4}{2az^2\sqrt{2}-2a^2z\sqrt{2}}$. Iam pro z. pone $z-b$. fiet: $\frac{4a^2z-4a^2b-a^4}{2az^2-4azb\sqrt{2}+2ab^2-2a^2z\sqrt{2}+2a^2b\sqrt{2}}$. quarum duarum x. differentia utique est ff.

Iam spat. $\beta Ad\beta$ ⊓ ▭$A\lambda\beta$ − spat. $\beta\lambda\beta$. sed spatium $\beta\lambda\beta$ ⊓ spat. $\beta ff\beta$ − ▭ $ff\xi$ − ◺ $\beta\xi\pi$ + ◺$\pi\lambda\beta$. Ergo spat. $\beta Ad\beta$ ⊓ ▭ $A\lambda\beta$ − spat. $\beta ff\beta$ + ▭ $ff\xi$ + ◺$\beta\xi\pi$ − ◺$\pi\lambda\beta$.

◺ RIGHT TRIANGLE POINTING RIGHT
LAA VII-3 p. 506

N. 6₁ I. GEOMETRISCHE STUDIEN 1672–1676 63

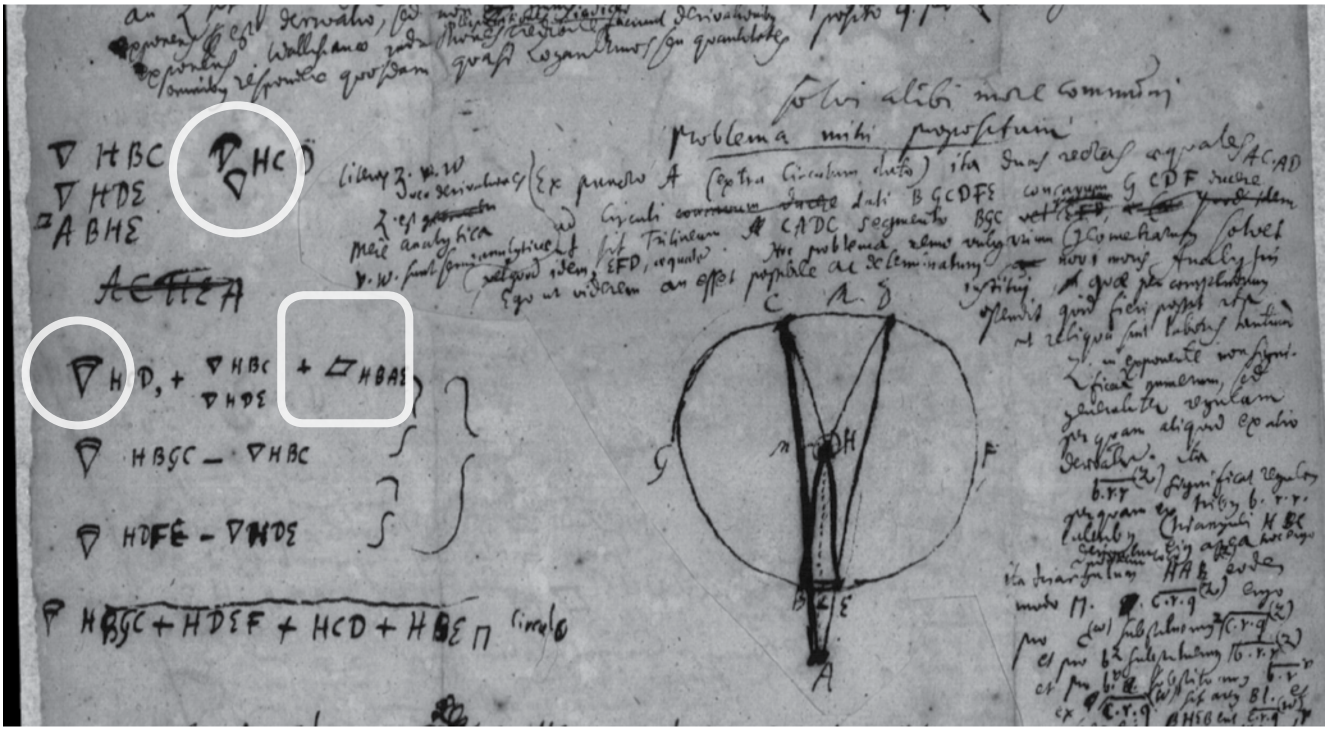
Ut est diameter ad circumferentiam, ita est semifigura circa suum axem voluta ad superficiem curvam.

$\frac{\text{rad. a}}{\text{circumf. b}} = \frac{□}{\text{sup. cycl.}} = \frac{◗}{\text{sup. hem.}}$. Ergo $\frac{\text{sup. cyl.}}{\text{sup. hem.}} = \frac{□}{◗}$.

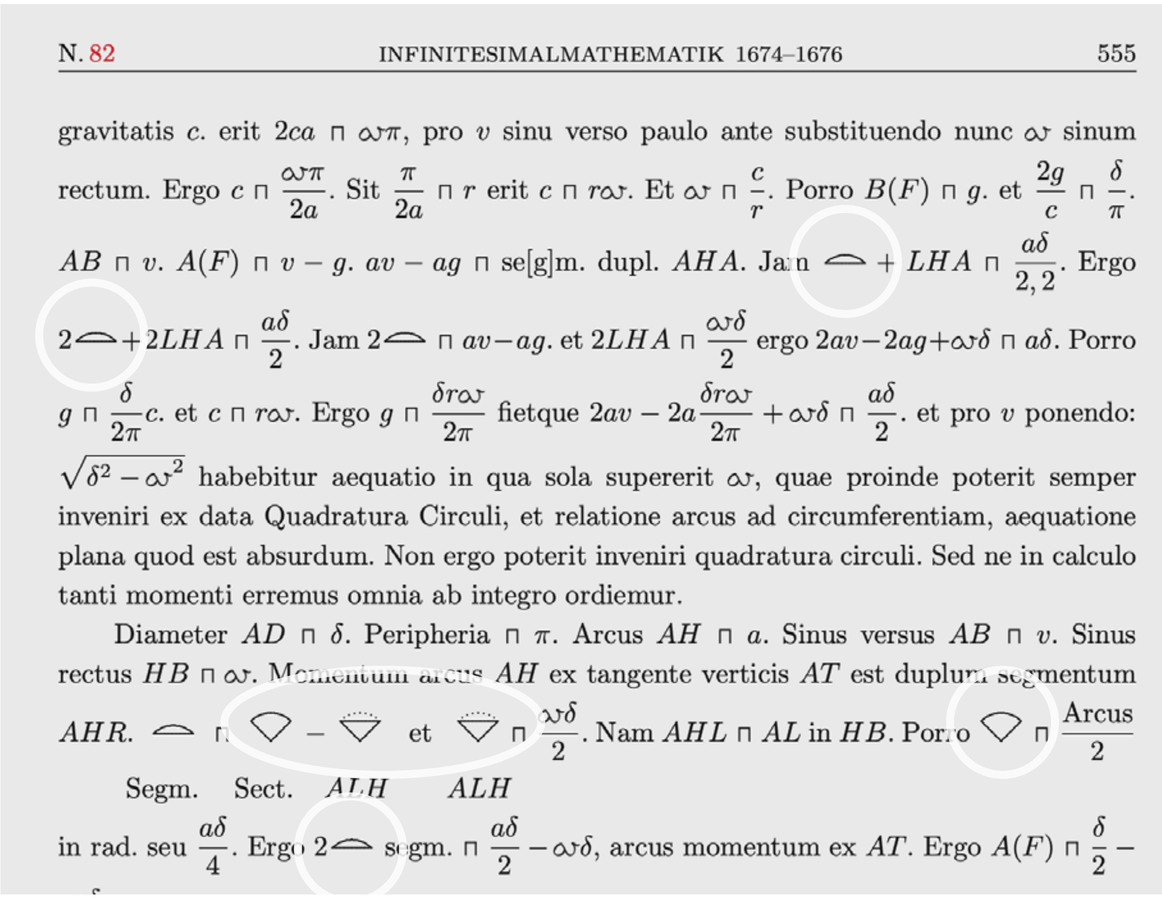
Ratio cyl. ad hemisph. est ut 3 ad 2. ergo ratio quadr. circumscr. vel quad. diam. ad circ. ut Rq 3. ad Rq 2.

Ergo diam. 1r. □. diam. 1r. erit Rq 3 —q Rq 2 —q 1 — Rq 2rqq circ. dividatur per 1r.

◗ HALF RIGHTHAND CIRCLE WITH DIAMETER
LAA VII-1 p. 63



▽ SMALL SECTOR, ▽ SMALL SECTOR WITH DOUBLE ARC, ▱ KITE SIGN
LH 35 I 14 fol. 88v. *The edition of this manuscript is currently in progress*.



gravitatis c. erit $2ca \sqcap \omega\pi$, pro v sinu verso paulo ante substituendo nunc ω sinum rectum. Ergo $c \sqcap \frac{\omega\pi}{2a}$. Sit $\frac{\pi}{2a} \sqcap r$ erit $c \sqcap r\omega$. Et $\omega \sqcap \frac{c}{r}$. Porro $B(F) \sqcap g$. et $\frac{2g}{c} \sqcap \frac{\delta}{\pi}$. $AB \sqcap v$. $A(F) \sqcap v - g$. $av - ag \sqcap$ se[g]m. dupl. AHA. Jam $\frown + LHA \sqcap \frac{a\delta}{2,2}$. Ergo $2\frown + 2LHA \sqcap \frac{a\delta}{2}$. Jam $2\frown \sqcap av - ag$. et $2LHA \sqcap \frac{\omega\delta}{2}$ ergo $2av - 2ag + \omega\delta \sqcap a\delta$. Porro $g \sqcap \frac{\delta}{2\pi}c$. et $c \sqcap r\omega$. Ergo $g \sqcap \frac{\delta r\omega}{2\pi}$ fietque $2av - 2a\frac{\delta r\omega}{2\pi} + \omega\delta \sqcap \frac{a\delta}{2}$. et pro v ponendo: $\sqrt{\delta^2 - \omega^2}$ habebitur aequatio in qua sola supererit ω, quae proinde poterit semper inveniri ex data Quadratura Circuli, et relatione arcus ad circumferentiam, aequatione plana quod est absurdum. Non ergo poterit inveniri quadratura circuli. Sed ne in calculo tanti momenti erremus omnia ab integro ordiemur.

Diameter $AD \sqcap \delta$. Peripheria $\sqcap \pi$. Arcus $AH \sqcap a$. Sinus versus $AB \sqcap v$. Sinus rectus $HB \sqcap \omega$. Momentum arcus AH ex tangente verticis AT est duplum segmentum AHR. $\frown \sqcap \triangledown - \triangledown$ et $\triangledown \sqcap \frac{\omega\delta}{2}$. Nam $AHL \sqcap AL$ in HB. Porro $\triangledown \sqcap \frac{\text{Arcus}}{2}$
Segm. Sect. ALH ALH
in rad. seu $\frac{a\delta}{4}$. Ergo $2\frown$ segm. $\sqcap \frac{a\delta}{2} - \omega\delta$, arcus momentum ex AT. Ergo $A(F) \sqcap \frac{\delta}{2} -$

⌓ SMALL SEGMENT, ▽ SMALL SECTOR and ▽ SMALL SECTOR TRIANGLE
LAA VII-5 p. 555

parabolicum, nam aequatio talis $y^2 = ax - a^2$. est parabolica, ut patet. Iam si ponatur $y^2 = x^2$. non ideo minus aequatio parabolica erit, seu cuius locus est parabola. Id ergo videmur obtinuisse, ut hoc pacto quadratura circuli devenerit problema solidum solubile, et construi possit, quemadmodum problemata solida omnia. Sed in eo malum est, quod una tantum est cognita a^2. Si quaedam b. aequationem ingrederetur, tunc solvi posset problema ope parabolae, deberet nimirum fieri aequatio talis posito $y = x$.

$$y^2 = ax - b^2. \quad \text{vel} \quad x^2 = [ay] - b^2.$$

haberemus solutionem saltem per parabolam, seu locum solidum. Quare si quis exhibere posset segmentum circuli aequale cuidam sectori cuius arcus est radix segmenti demto quodam quadrato cuius radix est alia a radio. Sed his non opus, sufficit prior illa aequatio:

$$\frac{x^2}{\alpha} = \frac{bx}{\beta} - b^2.$$

1

▽ ⌓ ⦵

$a^2 + x^2 = ax$

x a

▽ SMALL SECTOR WITH CHORD – LAA VII-4 p. 192

Si esset corpus quod pro aetate ☽ mutaret pondus, daret motum perpetuum. Fiat talis rota ◕ ubi nigrum sit alterius formae ☽ non subditae et tota rota, ita in axe librata ut utraque forma in naturali statu aequalis sit ponderis, haud dubie perpetuo movebitur juxta motum ☽.

◕ CIRCLE WITH HALF MOON OBLIQUE
LAA VII-8 (preliminary edition)

Si esset corpus quod pro ætate ☽ mutaret pondus, daret motum perpetuum. Fiat talis rota ◕ ubi nigrum sit alterius formæ ☽ non subditæ ex totâ rotâ, ita in axe librata ut utraque forma in naturali

◕ CIRCLE WITH HALF MOON OBLIQUE
Foucher de Careil (ed.): Œvres inédites de Descartes, vol. I p. 34; 1859

25. DE SERIE AD SEGMENTUM CIRCULI
[Herbst 1673]

Überlieferung: *L* Konzept: LH 35 II 1 Bl. 248–249. 1 Bog. 2°. 4 S.
Cc 2, Nr. 554.

Datierungsgründe: Das Wasserzeichen des Papiers ist für den Zeitraum August 1673 bis Juni 1674 belegt. Das Stück setzt die Entdeckung der Kreisreihe voraus; es ist vermutlich kurz danach entstanden, da es direkte Bezüge zur bisher frühesten bekannten Abhandlung zur Kreisreihe, der *Dissertatio de arithmetico circuli tetragonismo* (Cc 2, Nr. 563 u. 1233 A), aufweist. Außerdem enthält N. 25 einen Verweis auf *De quadratura circuli et hyperb.* (Cc 2, Nr. 1237), das auf demselben Bogen steht wie N. 22 und nach diesem geschrieben ist. N. 25 ist also nach N. 22 enstanden.

[*Teil 1*]

Inventum est a me:

Prop. 1. Si dato quodam circuli segmento ⌓ ; cuius arcus non sit quadrante maior, radius ponatur esse $\underline{a}$, tangens semiarcus $\underline{b}$, sinus versus vero arcus integri $\underline{c}$, tunc seriei in infinitum productae $\frac{b^3}{3a} - \frac{b^5}{5a^3} + \frac{b^7}{7a^5} - \frac{b^9}{9a^7}$ etc. etc., summam, aequalem fore ipsi $\frac{bc}{2} - ⌓$; seu residuo post segmentum datum ex semirectangulo tangentis semiarcus in sinum versum arcus ductu facto, subtractum.

Unde ante omnia consequentia ducitur eiusmodi:

Prop. 2. Posito radio $a, = 1$, erit: $\frac{bc}{2} - ⌓ = \frac{b^3}{3} - \frac{b^5}{5} + \frac{b^7}{7} - \frac{b^9}{9}$ etc.

14 tunc *(1)* differentiam inter $\frac{bc}{2} - ⌓$, *(a)* fore *(b)* erit = *(2)* seriem in infinitum productam *(3)*

Primum ergo posito $a = 1$, iam supra prop. 2. ostensum est, fieri: $\frac{bc}{2} - ⌓ = \frac{b^3}{3} - \frac{b^5}{5} + \frac{b^7}{7} - \frac{b^9}{9}$ etc. ex $\frac{bc}{2} - ⌓ = \frac{b^3}{3a} - \frac{b^5}{5a^3} + \frac{b^7}{7a^5} - \frac{b^9}{9a^7}$ etc.

At posito $a = 1$, et praeterea $b = \frac{a}{\gamma}$, seu sumta serie p r o p. 6. quae erat: $\frac{bc}{2} - ⌓ = \frac{a^2}{3\gamma^3} - \frac{a^2}{5\gamma^5} + \frac{a^2}{7\gamma^5} - \frac{a^2}{9\gamma^9}$, fiet aequatio haec:

Prop. 8. $\frac{bc}{2} - ⌓ = \frac{1}{3\gamma^3} - \frac{1}{5\gamma^5} + \frac{1}{7\gamma^7} - \frac{1}{9\gamma^9}$, etc.

At ex $\frac{bc}{2} - ⌓ = \frac{a^2\lambda^3}{3} - \frac{a^2\lambda^5}{5}$ etc., fiet aequatio haec:

Prop. 9. $\frac{bc}{2} - ⌓ = \frac{\lambda^3}{3} - \frac{\lambda^5}{5} + \frac{\lambda^7}{7} - \frac{\lambda^9}{9}$ etc. posito $a = 1$, et $\lambda = \frac{b}{a} = \frac{1}{\gamma}$.

⌓ SMALL SEGMENT, LAA VII-3 p. 282, 286

et ponendo $w^3 - v^3 \sqcap \pi^3$. et $-\mu^9 + \omega^9$^[w^3] $\sqcap \upsilon^{12}$.
et $-3\omega^3w^3 + 3\mu^3v^3 \sqcap \beta^6$. et $3\omega^6w^3 - 3\mu^6v^3 \sqcap \gamma^9$. et fiet:

⦷ $\pi^3x^9 + \beta^6x^6 + \gamma^9x^3 \sqcap \upsilon^{12}$.

Atque ita sublatae sunt irrationales duae, nempe v. et w. iam ipsarum r. et s. tollenda est alterutra. Iam conferendo aequationes ⦷ et ⊙ tolletur x, nec restabit incognita aut

$a^4h^4x + a^5h^3l$

Unde ex aeq. ⦷ fiet aeq.

$$⊕ \left\{ \begin{array}{lllll} -3\pi^3a^4h^2lx^2 & + & \pi^3a^4h^4x & - & \pi^3a^6l^3 \\ & - & 3\pi^3a^5hl^2. & + & \pi^3a^5h^3l \\ +\ \beta^6a^2h^2.. & + & 2\beta^6a^3hl. & + & \beta^6a^4l^2 \\ & - & \gamma^9ah. & - & \gamma^9a^2l \\ & & & - & \upsilon^{12} \end{array} \right\} \sqcap 0.$$

Ubi notandum $w^3 - v^3$ seu π^3, valere $-2a^2\sqrt{\frac{1}{4}l^2 + \frac{1}{27}\frac{h^3}{a}}$ et ω^3 seu $v^3 + w^3$ valere $-a^2l$.

et $\lambda^3 \sqcap 6a^2\sqrt{\frac{1}{4}l^2 + \frac{1}{27a}\frac{h^3}{a}}$. et $\mu^3 \sqcap a^2l - 6a^2\sqrt{\frac{1}{4}l^2 + \frac{1}{27a}h^3}$. et

$$\beta^6 \sqcap \boxed{+\,3a^2lw^3 + 3a^2lv^3}\ \boxed{-\,6a^2\sqrt{\frac{1}{4}l^2 + \frac{1}{27a}h^3}v^3}$$

$$-\,3a^4l^2 \qquad + 3a^4\sqrt{\frac{1}{4}l^2 + \frac{1}{27a}h^3} - \frac{6a^4}{4}l^2 - \frac{6a^3}{27}h^3.$$

Unde terminus x^2 aequationis ⊕ fiet

$$+\,6a^6h^2l\sqrt{\frac{1}{4}l^2 + \frac{1}{27a}h^3} - 3a^6h^2l^2 - \frac{6a^3}{27}h^3.$$

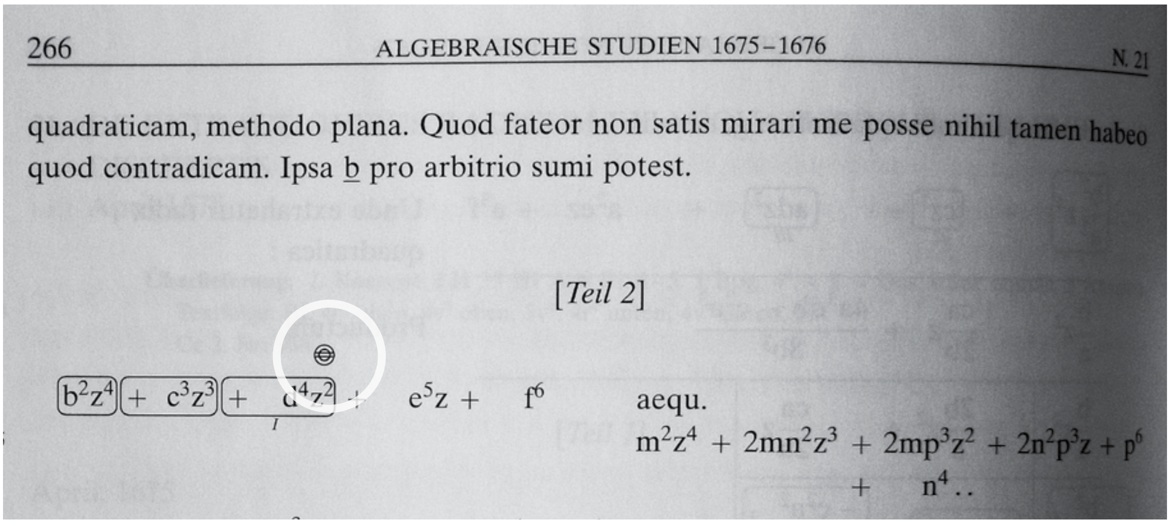
qui utique non est ut metuebam nihilo aequalis. Nisi sit in calculo error, nam metuo ne omnes termini aequationis ⊕ sint nihilo aequales, quod ultimum est effugium quo se tuetur natura rerum proteiformis.

Imo iam iudico necessariam esse hanc destructionem, erroremque haud dubie in calculo admissum, quia calculus aequationis ⦷ et ⊕ oritur ex sola aequatione $x \sqcap v + w$. quae eadem est cum aequatione $x^3 \ * \ + alx + a^2l \sqcap 0$. et omissa a nobis mentio ipsius m, dum ⦶ aequationem per x + m. divisimus. Itaque nihil hinc nisi identicum duci potuit. Ergo non aequatio ɤ. sed ⦶ adhibenda fuit. Et praeterea resumendus est calculus certo erroneus.

This paragraph also contains ɤ ALCHEMICAL SYMBOL FOR ALUMEN-PISCES.

Compendii causa potuisset methodo qua initio huius paginae usi sumus aequatio $x \sqcap v + r$. resolvi donec ipsarum v. et r. tollatur asymmetria, inde orta aequatio ⦷ poterit multiplicari per x + m. sed nonne sufficit in aequatione ⦷ pro x substitui eius valorem ex aeq. ⦶, ita arbitror fieri compendiosissime. Optimum ergo credi resumi methodum paginae praecedentis, ut ope aequationis $x \sqcap v + r$. tollatur primum asymmetria ex v. et w, et corrigatur calculus paginae praecedentis, qui fuit erroneus; deinde ut in aequatione producta ab hac asymmetria libera, tollatur x. ope aequationis ⦶, restabit aequatio in qua nullae erunt incognitae, et duae tantum asymmetriae, r. et s.

⦷ CIRCLE WITH DOUBLE VERTICAL LINE, ⊕ CIRCLE WITH DOUBLE VERTICAL AND HORIZONTAL LINE – LAA VII-2 p. 256–259

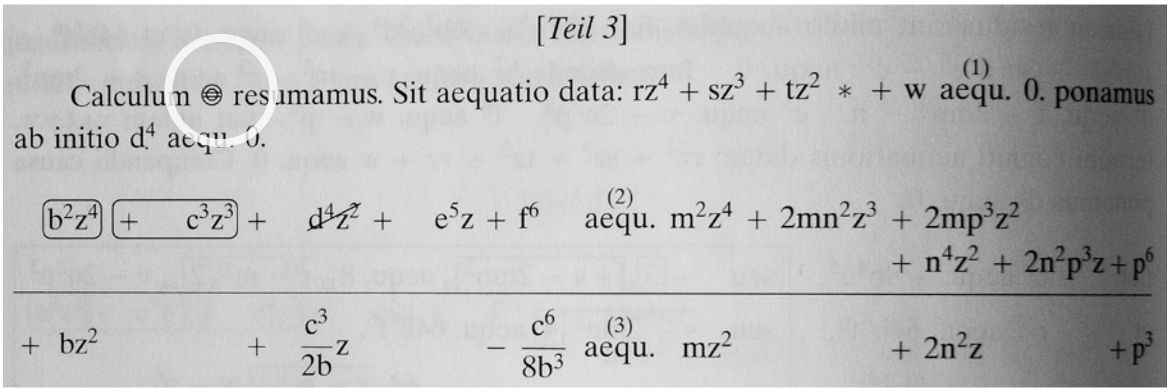

266 ALGEBRAISCHE STUDIEN 1675–1676 N. 21

quadraticam, methodo plana. Quod fateor non satis mirari me posse nihil tamen habeo quod contradicam. Ipsa b pro arbitrio sumi potest.

[*Teil 2*]

$$b^2z^4 + c^3z^3 + d^4z^2 + e^5z + f^6 \quad \text{aequ.} \quad m^2z^4 + 2mn^2z^3 + 2mp^3z^2 + 2n^2p^3z + p^6 + n^4 \ldots$$

⊜ DOUBLE CIRCLE WITH DOUBLE HORIZONTAL LINE
LAA VII-2 p. 266

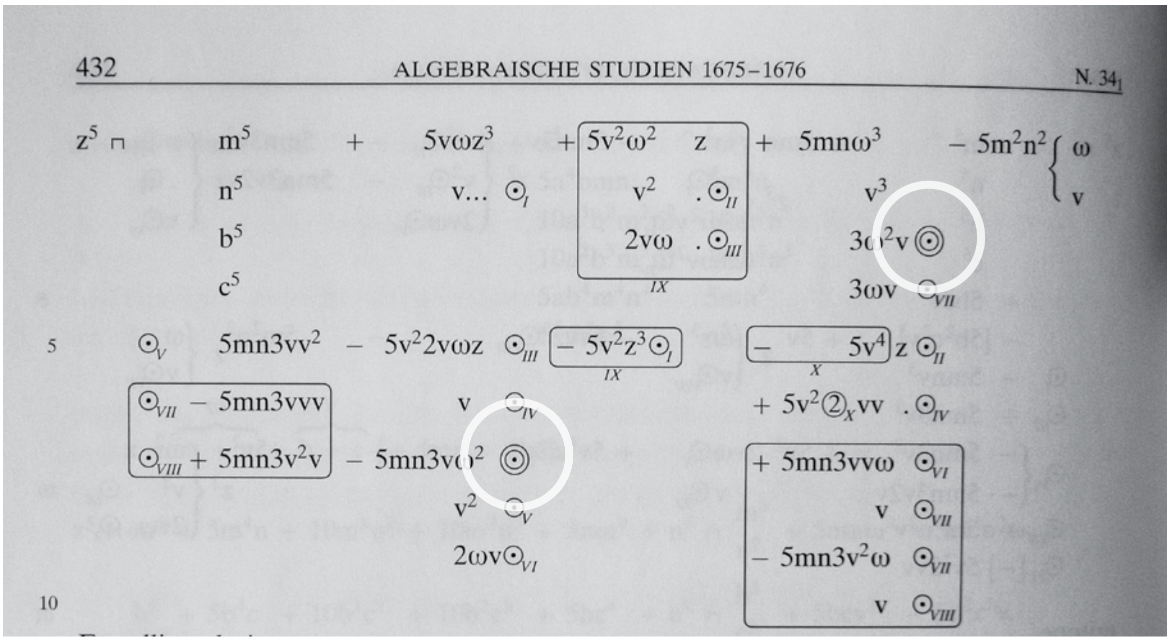

[*Teil 3*]

Calculum ⊜ resumamus. Sit aequatio data: $rz^4 + sz^3 + tz^2 * + w$ aequ. 0. ponamus ab initio d^4 aequ. 0.

$$b^2z^4 + c^3z^3 + d^4z^2 + e^5z + f^6 \quad \text{aequ.} \quad m^2z^4 + 2mn^2z^3 + 2mp^3z^2 + n^4z^2 + 2n^2p^3z + p^6$$

$$+ bz^2 + \frac{c^3}{2b}z - \frac{c^6}{8b^3} \quad \text{aequ.} \quad mz^2 + 2n^2z + p^3$$

⊜ DOUBLE CIRCLE WITH DOUBLE HORIZONTAL LINE
LAA VII-2 p. 268


432 ALGEBRAISCHE STUDIEN 1675–1676 N. 34

z^5 ⊓ m^5 + $5v\omega z^3$ + $5v^2\omega^2$ z + $5mn\omega^3$ − $5m^2n^2$ { ω, v }

n^5 v... $\odot_I$ v^2 . $\odot_{II}$ v^3

b^5 $2v\omega$. $\odot_{III}$ $3\omega^2v$ ◎

c^5 $3\omega v$ $\odot_{VII}$

$\odot_V$ $5mn3vv^2$ − $5v^22v\omega z$ $\odot_{III}$ − $5v^2z^3\odot_I$ − $5v^4$ z $\odot_{II}$

$\odot_{VII}$ − $5mn3vvv$ v $\odot_{IV}$ + $5v^2②_X vv$. $\odot_{IV}$

$\odot_{VIII}$ + $5mn3v^2v$ − $5mn3v\omega^2$ ◎ + $5mn3vv\omega$ $\odot_{VI}$

v^2 $\odot_V$ v $\odot_{VII}$

$2\omega v\odot_{VI}$ − $5mn3v^2\omega$ $\odot_{VII}$

v $\odot_{VIII}$

◉ DOUBLE CIRCLE WITH DOT
LAA VII-2 p. 432

ITALIAN: F. GHALIGAI

(1521, 1548, 1552)

139. Ghaligai's *Pratica d'arithmetica*[1] appeared in earlier editions, which we have not seen, in 1521 and 1548. The three editions do not differ from one another according to Riccardi's *Biblioteca matematica italiana* (I, 500–502). Ghaligai writes (fol. 71*B*): x = *cosa* = c^{o}, x^2 = *censo* = □, x^3 = *cubo* = ◫, x^5 = *relato* = ⊟, x^7 = *pronico* = ⊞, x^{11} = *tronico* = ⊞, x^{13} = *dromico* = ⊞. He uses the m^{o} for "minus" and the $\tilde{p}$ and *e* for "plus," but frequently writes in full *piu* and *meno*.

[1] *Pratica d'arithmetica di Francesco Ghaligai Fiorentino* (Nuouamente Riuista, & con somma Diligenza Ristampata. In Firenze. M.D.LII).

◫ HORIZONTAL DOUBLE SQUARE, ⊟ VERTICAL DOUBLE SQUARE, ⊞ THREE-PART BIG SQUARE 1, ⊞ THREE-PART BIG SQUARE 2, ⊞ FOUR-PART BIG SQUARE

Cajori I. p. 112

For the simple square one would use the character 25FB or 25A1.

plicare el ⊞ nel □, o uero della c° nel □ di □, el ⊞ di □ del ◫ quadrato, o uero del □ nel □ di □, o si dello ⊟ nella c°, el ⊞ del ◫ nel □ di □, o ue/ro del □ nel ⊟, o si della c° nel ◫ di □, & cosi in infinito puoi segaire,

n°	Numero	1
c°	Cosa	2
□	Censo	4
◫	Cubo	8
□ di □	□ di □	16
⊟	Relato	32
⊞ di □	◫ di □	64
⊞	Pronico	128
□ di □ di □	□ di □ di □	256
◫ di ◫	◫ di ◫	512
⊟ di □	⊟ di □	1024
⊞	Tronico	2048
◫ di □ di □	◫ di □ di □	4096
⊞	Dromico	8192
⊞ di □	⊞ di □	16384
◫. ⊟	◫. ⊟	32768

◫ HORIZONTAL DOUBLE SQUARE, ⊟ VERTICAL DOUBLE SQUARE, ⊞ THREE-PART BIG SQUARE 1, ⊞ THREE-PART BIG SQUARE 2, ⊞ FOUR-PART BIG SQUARE

Francesco Ghaligai, Pratica d'Arithmetica, 1552 (after Cajori)

13 Zu Fig. 3: Nach Aussage (4) soll D ein beliebiger Punkt auf dem Quadranten AO sein. Leibniz hat in seiner Handzeichnung den Bogen AD jedoch gleich 60° gewählt, wodurch die Allgemeinheit verloren gegangen ist. Leibniz hat dies, wie die Zusätze neben der Figur zeigen, später bemerkt. Er hat aber keine neue Zeichnung angefertigt, sondern hat sich damit begnügt, den allgemeinen Fall mittels Einzeichnen der Linie $F\omega\varphi$, der Verlagerung der Linie $A\beta\alpha$ sowie vieler zusätzlicher Winkelmarkierungen darzustellen. Hierbei bedeuten ∡ = 25°; ⦣ = 50°; ⦤ = 65° und ⦥ = 40°. — Die Handzeichnung ist bis auf einige wenige Winkelangaben korrekt. 14 AN: s. dazu N. 29 S. 523 Z. 22 – S. 524 Z. 8 . 15 modo: Eine ähnlich unbestimmte Haltung bezüglich der Existenz des Höhenschnittpunkts im Dreieck nimmt Leibniz *LSB* VII, 1 N. 2 S. 4 ein.

$$\left.\begin{Bmatrix} 90 \\ 25 \\ 50 \end{Bmatrix}\right\} \text{Ang.} \qquad \begin{array}{lr} 90-25=65=25+40 & 65 \\ 90-50=40 & 65 \\ 65+40=105 & 50 \\ 180-105=75 & \overline{180} \end{array}$$

NB. recta DB continuata non cadit in ϖ punctum medium rectae CF nisi ∡ sit = ⦣ nam angulus $EF\varpi$ est ∡ ob ▽CEF. et idem foret ⦣ ob ▽$D\varpi F$.

[*Teil 2*]

Determinatio punctorum, sive quantitas linearum in fig. 3.

(1) Ex centro B radio BA describatur circulus.

(2) Ducatur diameter ABC producta utcunque versus $C\gamma$.

(3) et ex puncto A ducatur tangens sive ad diametrum perpendicularis AH.

∡ ANGLE 1, ⦣ ANGLE 2, ⦤ ANGLE 3, ⦥ ANGLE 4
LAA VII-4 p. 409, 410

In circulo AB ducta applicata seu sinu CD iunctisque chordis AD. DB erit ▽[lo] ADB simile ADC. quia ∨ACD = ∨ADB. rectus recto et ∨DAB = ∨DAC. ergo ∨ADC = ∨DBA. Eodem modo ▽[lum] DCB simile utrique.

Ergo $\dfrac{AB}{AD} = \dfrac{AD}{AC}$. Ergo $AB \frown AC = AD \frown AD$. seu rectangulum sub diametro et sinu verso aequatur quadrato chordae.

∨[lus] HID (vel DHI) = ∨[lo] HLS. supplenti dimidii anguli dati ALD nempe ALH ad quadrantem.

Ang. ADB rect. = AGD rect. ∨$ADC = CBD$. $AG = AC$. $DC = GD$. $AH = HD$. et quia $AK = GD$. ergo $GH = IK = IC$. Porro ∨CIK = ∨AHD. item ∨$CIK = AID$.

∨ ANGLE VERTICAL
LAA VII-4 P. 377, 385

nec adhibitae sunt irrationales. Imo non nisi unicum exemplum datum est; quod attulit Mercator. Methodus mea revocandi ad progressiones geometricas, commodior est altera Mercatoris per divisionem; quia, ita series qualescunque propositae etiam irregulares satis nec ordine procedentes, ad figuram convenientem, revocantur, qualis ista est: $\frac{b}{1} - \frac{b^3}{3} + \frac{b^2}{2}$ etc. Variae aliae coniunctiones institui possunt, ut ista:

$$\frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \frac{1}{8} + \frac{1}{9} - \frac{1}{10} + \frac{1}{11} - \frac{1}{12} \text{ [etc.]}$$

$$\frac{3}{4} - \frac{1}{6} + \frac{3}{40} - \frac{1}{42} + \frac{3}{108} - \frac{1}{110} \text{ etc.}$$

Et ita semper novae erui possunt figurae. Sumtis seriebus fractionum quadraticarum unitate deminutarum:

$$\frac{1}{3} + \frac{1}{8} + \frac{1}{15} + \frac{1}{24} + \frac{1}{35} + \frac{1}{48} + \frac{1}{63} + \frac{1}{80} + \frac{1}{99} + \frac{1}{120} + \frac{1}{143} + \frac{1}{168} + \frac{1}{195} + \frac{1}{224} \text{ [etc.]}$$

· □ · □ · □ · □ · □ · □ · □

○ ○ ○ ○

◠ ◠ ◠ ◠

ꝏ ꝏ ꝏ

∝ ∝ ∝

Omnium terminorum punctatorum habetur summa; item omnium terminorum □ notatorum; ac proinde et totius seriei; sed termini circulo notati pendent ex quad. circuli, termini ◠ notati ex quad. hyperb.

Sed quid termini $\frac{1}{3}$ $\frac{1}{24}$ $\frac{1}{63}$ $\frac{1}{120}$ [etc.], sane sunt: $\frac{1}{1 \frown 3}$ $\frac{1}{4 \frown 6 \sqcap 3 \frown 8}$ $\frac{1}{7 \frown 9}$ $\frac{1}{10 \frown 12}$ [etc.]

◠ HYPERBOLE

LAA VII-3 p. 386, p. 388 – these samples shows the neccessary distinction between HYPERBOLE and ⌢ LEIBNIZIAN PRODUCT SIGN. The HYPERBOLE should be a character on the baseline, approximating the size of mathematical relation and operation characters.

$\frac{1}{3}$ $\frac{1}{35}$ $\frac{1}{99}$ [etc.] resoluta dant:

$\frac{1}{1 \frown 3}$ $\frac{1}{5 \frown 7}$ $\frac{1}{9 \frown 11}$ etc., cuius seriei origo est

$\frac{b}{1} - \frac{b^3}{3} + \frac{b^5}{5} - \frac{b^7}{7} + \frac{b^9}{9} - \frac{b^{11}}{11}$ etc. facta ex summis omnium:

$1 - y^2 + y^4 - y^6 + y^8 - y^{10}$ etc.

Idem plane evenit, examinatis duabus alteris ad hyperbolam seriebus, ∝ et ◠; ut non sit opus immorari. Videamus quid fiat, ademtis:

3 f. $\frac{y^2}{1+y^2}$. *(1)* Eodem modo sumatur series, alia per saltus tertianos, quam ita notavi ∝ . $\frac{1}{3}$ $\frac{1}{35}$ $\frac{1}{99}$ *(2)* Quoniam autem *(a)* constat seriem *(3)* series *L* 7–389,6 etc. *erg. Hrsg. fünfmal* 7 f. circuli. *(1)* Miror autem eandem ex una quam ex altera serie prodire figuram. Nam *(2)* $\frac{1}{3}$ *L* 9 etc., *(1)* unde

4.e) Alchemical symbols

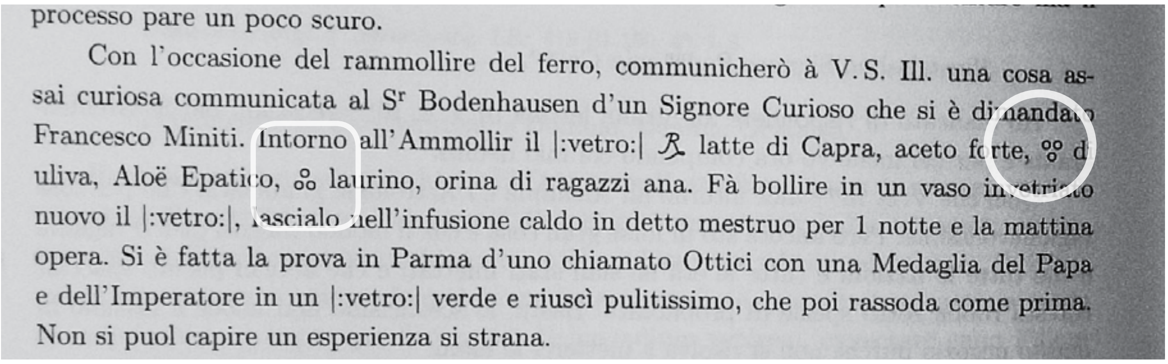
512 J. H. KORNMANN AN LEIBNIZ, 4. (14.) X. 1678 N. 212

m. h. H. ob nicht die destillation sine affuso liquore per descensum geschehe in diesem n° 3 wirdt keiner destillation sine affuso liquore gedacht, sondern die rectificatio 🜈 geschicht in dem 🝁 purificatiss. in ein weiswullen zeug gebunden wirdt, alß ein knopff, daran man ein faden last, thut den 🜈 in ein Zuckerglaß hanget dan daß 🝁ri hinein, so solvirt die phlegma daß 🝁ri, vnd fält gleichsam tropffenweiß wie ein regen auf den boden v. wirdt also der 🜈 von der phlegma geschieden, vnd man alßdan per Separatorium scheidet. N° 7 vnd 8 destilliret man sine affuso liquore aber nicht per descensum, sondern wie gebrauchlich vnd bestehet die kunst nur an den Zinnern, kupffern, vnd gläsern gefäßen, ist sehr curios nutzlich vnd leicht. N° 9 Menstruum Willisii ist nicht spiritus Zwelfferi ♀ris welcher von dem soluto nicht totaliter kan geschieden werden, sondern es ist sal 🜍ri purificatiss. wie Willis in tractatu de fermentatione dessen operation klärlich mit dem 🜔re entdecket, aber daß 🝁ri verschweiget. N° 10 Lilium Paracelsi verum wirdt allein aus ♄, oder auch mit Zusetzung anderer metallen alß ☉ so ist es universal, oder mit ♀, ♂, ♃, etc. so ist eß particular vnd gewissen membris vnd kranckheit appropriret, gemacht, habe eß auch gemacht aber

🝁 ALCHEMICAL SYMBOL FOR TARTAR-SALT
LAA III-2 p. 512

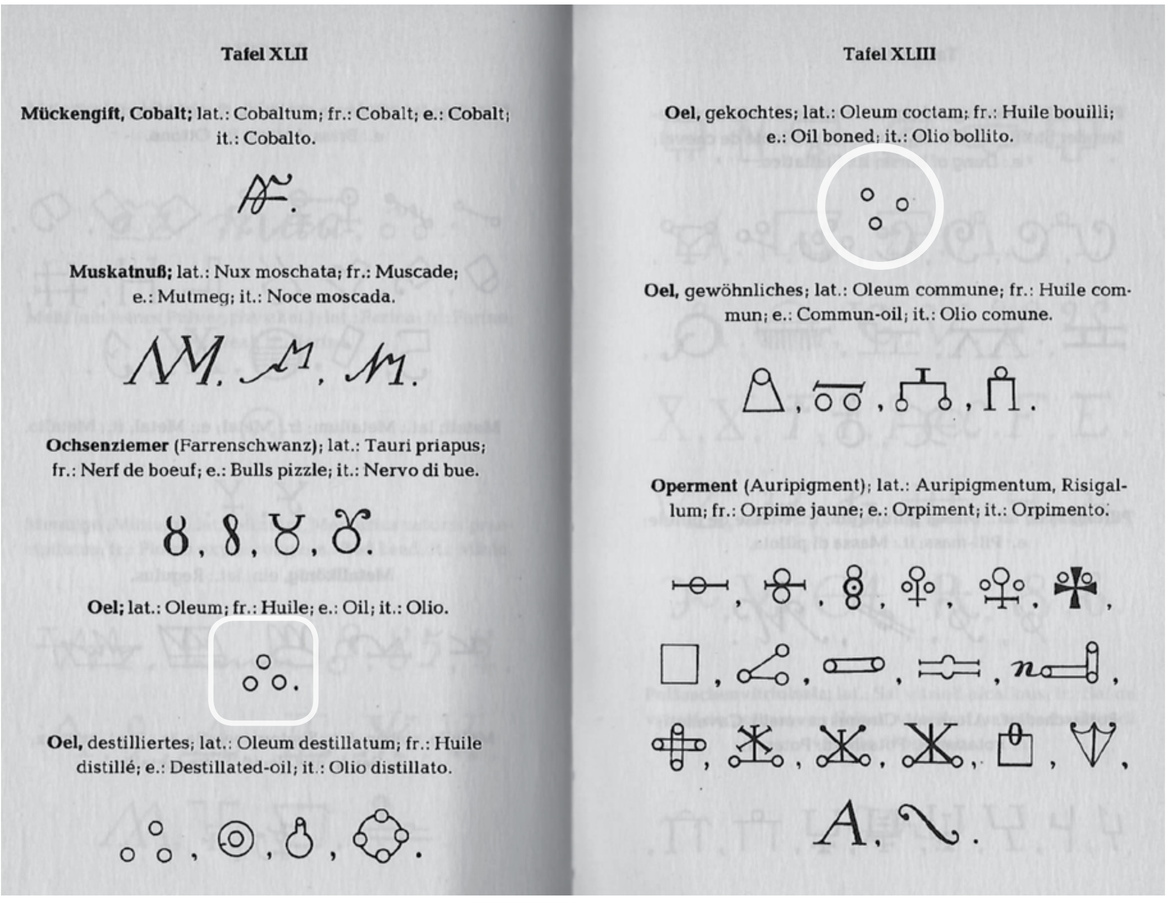
ohne diese Massa aber auf gemeine art weich vnd zu Kalck, dieses habe offters eigenhandig gemacht. N° 12. ☿ per se zu praecipitiren ist gantz leicht, wan man nur erst daß glaß hat woran daß gantze secret hanget. N° 24 bey der separation ☉ v. ☽ ex ♃ ist kein verlust sondern gewin, weil aber der ♃ ohne Arsenic vnd andere schädliche sachen nicht wohl zu zwingen oder sich capelliren läst, ist es nicht für Curiose leute so ihr zeit vnd gesundheit besser anzuwenden wissen, doch wan man erst ♃ in cineres bringt vnd selbige wieder in ein Corpus redigieret, ist solche operation nicht so schadlich, aber doch bey beyden modis grose rüche vnd zeitversäumnüß also mehr für grobe arbeitsamme leut so es wie ein handt

♃ ALCHEMICAL SYMBOL FOR MOON-JUPITER denotes *silver-bearing tin*.
LAA III-2 p. 514


processo pare un poco scuro.

Con l'occasione del rammollire del ferro, communicherò à V. S. Ill. una cosa assai curiosa communicata al S[r] Bodenhausen d'un Signore Curioso che si è dimandato Francesco Miniti. Intorno all'Ammollir il |:vetro:| latte di Capra, aceto forte, d'uliva, Aloë Epatico, laurino, orina di ragazzi ana. Fà bollire in un vaso invetriato nuovo il |:vetro:|, lascialo nell'infusione caldo in detto mestruo per 1 notte e la mattina opera. Si è fatta la prova in Parma d'uno chiamato Ottici con una Medaglia del Papa e dell'Imperatore in un |:vetro:| verde e riuscì pulitissimo, che poi rassoda come prima. Non si puol capire un esperienza si strana.

ALCHEMICAL SYMBOL FOR OIL BOILED. Graphically this is the common oil symbol, rotated 180 degrees. As "boiled oil" it bears a different meaning than the ordinary oil smbol and both characters can occur in text alongside each other. LAA III-8 p. 248


Tafel XLII

Mückengift, Cobalt; lat.: Cobaltum; fr.: Cobalt; e.: Cobalt; it.: Cobalto.

Muskatnuß; lat.: Nux moschata; fr.: Muscade; e.: Mutmeg; it.: Noce moscada.

Ochsenziemer (Farrenschwanz); lat.: Tauri priapus; fr.: Nerf de boeuf; e.: Bulls pizzle; it.: Nervo di bue.

Oel; lat.: Oleum; fr.: Huile; e.: Oil; it.: Olio.

Oel, destilliertes; lat.: Oleum destillatum; fr.: Huile distillé; e.: Destillated-oil; it.: Olio distillato.

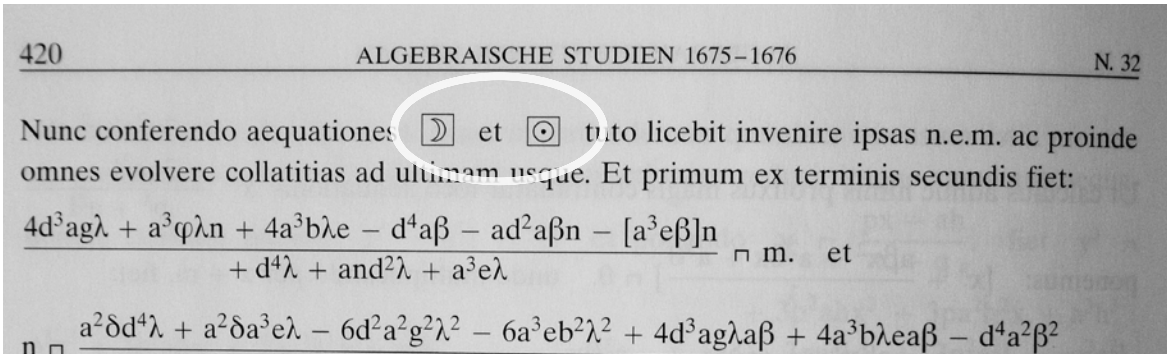
Tafel XLIII

Oel, gekochtes; lat.: Oleum coctam; fr.: Huile bouilli; e.: Oil boned; it.: Olio bollito.

Oel, gewöhnliches; lat.: Oleum commune; fr.: Huile commun; e.: Commun-oil; it.: Olio comune.

Operment (Auripigment); lat.: Auripigmentum, Risigallum; fr.: Orpime jaune; e.: Orpiment; it.: Orpimento.

ALCHEMICAL SYMBOL FOR OIL BOILED (on the right) is clearly distinguished from the common symbol for oil (on the left) in Geßmanns book on alchemical symbols.



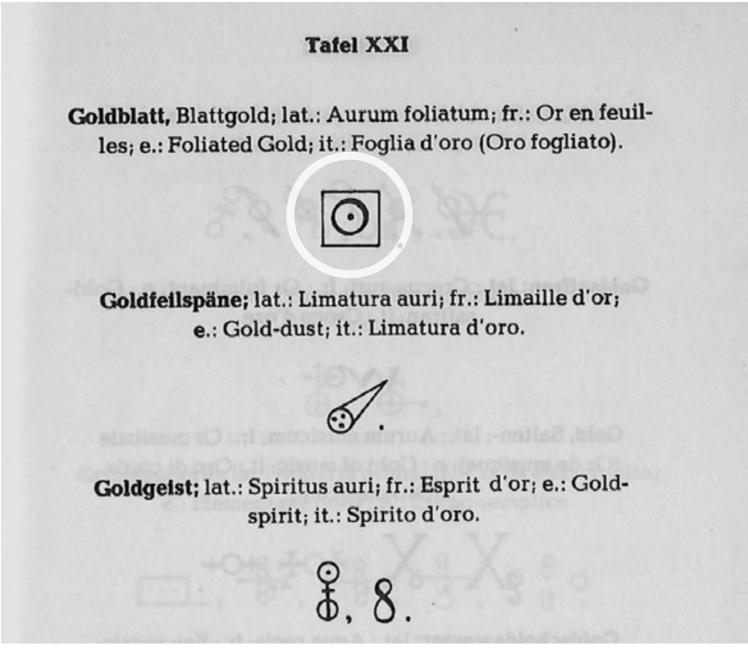
420 ALGEBRAISCHE STUDIEN 1675–1676 N. 32

Nunc conferendo aequationes ☽⃞ et ☉⃞ tuto licebit invenire ipsas n.e.m. ac proinde omnes evolvere collatitias ad ultimam usque. Et primum ex terminis secundis fiet:

$$\frac{4d^3ag\lambda + a^3\varphi\lambda n + 4a^3b\lambda e - d^4a\beta - ad^2a\beta n - [a^3e\beta]n}{+ d^4\lambda + and^2\lambda + a^3e\lambda} \sqcap m. \text{ et}$$

$$n \sqcap \frac{a^2\delta d^4\lambda + a^2\delta a^3e\lambda - 6d^2a^2g^2\lambda^2 - 6a^3eb^2\lambda^2 + 4d^3ag\lambda a\beta + 4a^3b\lambda ea\beta - d^4a^2\beta^2}{}$$

☉⃞ ALCHEMICAL SYMBOL ENCLOSED SUN, denotes *foliated gold;*
☽⃞ ALCHEMICAL SYMBOL ENCLOSED MOON, denotes *foliated silver.*
LAA VII-2 p. 420

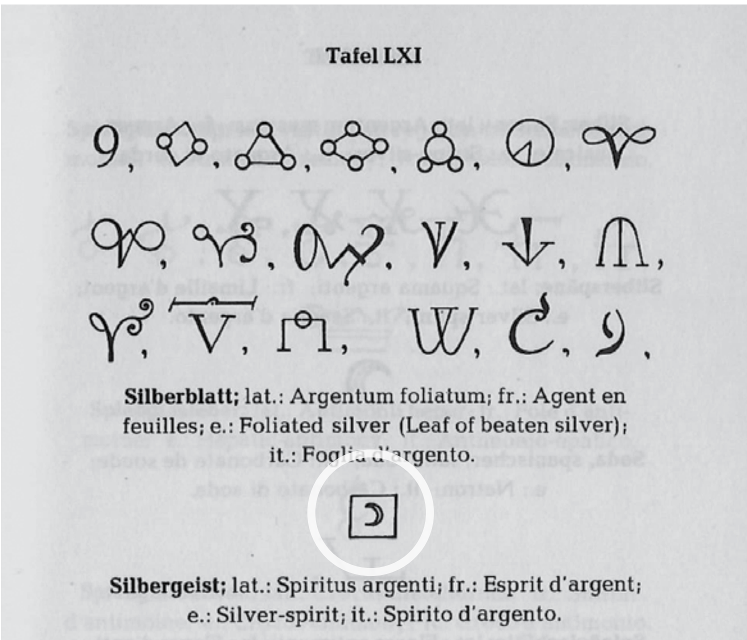


Tafel XXI

Goldblatt, Blattgold; lat.: Aurum foliatum; fr.: Or en feuilles; e.: Foliated Gold; it.: Foglia d'oro (Oro fogliato).

Goldfeilspäne; lat.: Limatura auri; fr.: Limaille d'or; e.: Gold-dust; it.: Limatura d'oro.

Goldgeist; lat.: Spiritus auri; fr.: Esprit d'or; e.: Gold-spirit; it.: Spirito d'oro.



Tafel LXI

Silberblatt; lat.: Argentum foliatum; fr.: Agent en feuilles; e.: Foliated silver (Leaf of beaten silver); it.: Foglia d'argento.

Silbergeist; lat.: Spiritus argenti; fr.: Esprit d'argent; e.: Silver-spirit; it.: Spirito d'argento.

☉⃞ ALCHEMICAL SYMBOL ENCLOSED SUN,
☽⃞ ALCHEMICAL SYMBOL ENCLOSED MOON,
as shown in Geßmann's concise book on alchemical symbols (1964)

Habita iam affectione sub quadrato, unica tantum superest affectio sub latere. Habuimus autem iam pagina praecedente 60mnvωx, et hic + 105mnvωx; et [symbol] valet − 135mnvωx, quae simul faciunt: + 30mnvωx. Quae supersunt nunc tandem absolvamus: iunctis ergo inter se aggregatis + ♃. + ☿. − ♄. − ♂. + ♀. quaeramus primum m^2n^2x, et similia, fiet:

♃ $10m^2n^2\frac{m}{n}$

☿ $30m^2n^2\frac{v}{\omega}$ (sive $10m^2n^2\frac{v}{\omega}$ + 20mnv, mn + 20mnω, mn)

♄ $-15m^2n^2x$ − 15mnv, mn − 15mnω, mn

♂ − 30mnω, mn
. . . . v, . .

♀ + $30\frac{mnv}{\omega}$, mn

[symbol] ALCHEMICAL SYMBOL FOR ALUMEN-PISCES.
LAA VII-2 p. 810

Urin mus nicht alle genommen sondern die faeces zuruck gelaßen werden, d
schaumen zustarck, und treiben das andere mit zum pot heraus.

Die abgerahmte materi kan man in einen glas stehen laßen, so sezet sich ein
zuckerkandi zuboden, daß kan man weg thun.

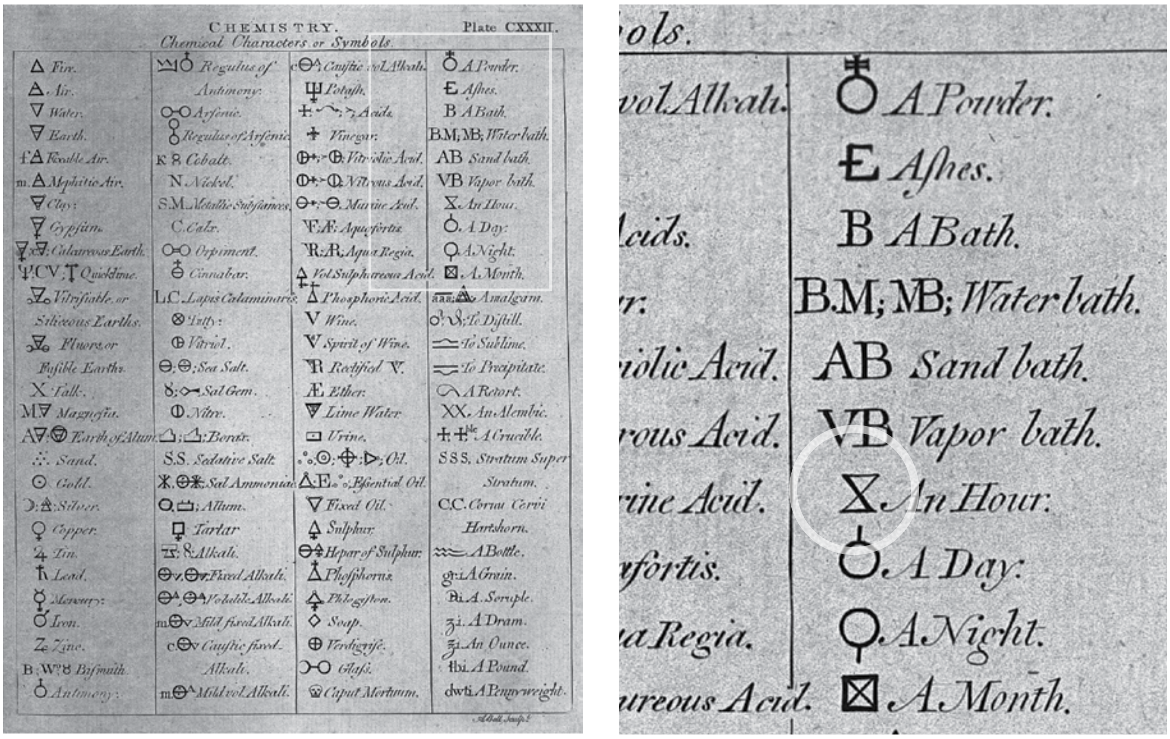
Hernach sie ubergetrieben, man kan sie auch wohl zu dem einmahl uberg
[symbol] thun, so darff man nicht selbst ubertreiben.

Das salz so sich anfangs vor zuckerCandi zuboden sezet, auch im uberdi
zuruck bleibt möchte wohl etwas guthes in sich haben, alleine es reißet alle retort
zwey, man müste es mit einer ♂sernen retorte probiren.

Die retorten alhier zu Hanover halten die spiriten wohl. Die Heßische Erde al
theüer, sie ist nothig die ofen inwendig damit zubestreichen, so schmelzen sie nich

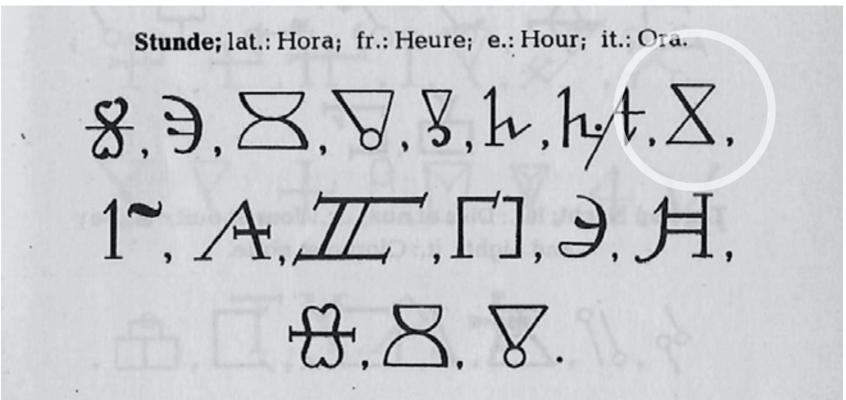
[symbol] ALCHEMICAL SYMBOL FOR REALGAR 3
LAA III-2 p. 825

The char. ⧖ ALCHEMICAL SYMBOL FOR HORA 2 appears in historical sources frequently in its most usual, straight X-like shape, either singularily or at first place. Because of the considerable form difference to the encoded 1F76E, the latter is not suitable to represent ‘the simple hora sign’.

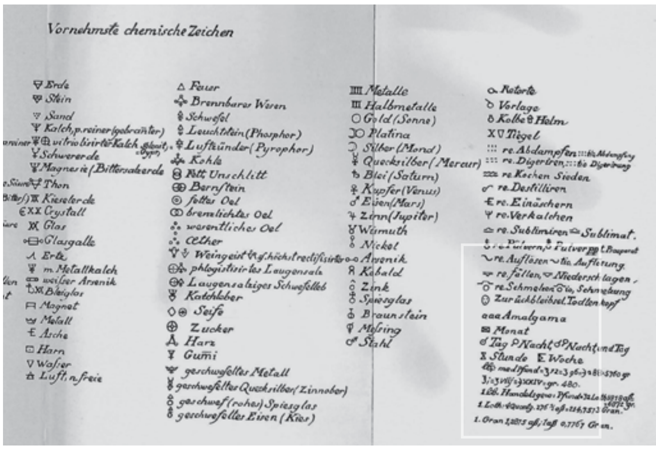


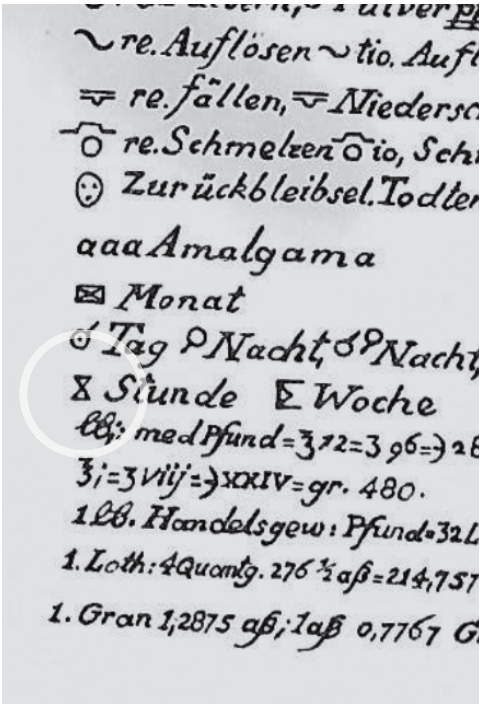
⧖ ALCHEMICAL SYMBOL FOR HORA 2, in an engraving of Andrew Bell (1726–1809); from: Welcome collection (https://wellcomecollection.org/works/s5vs9bfr)

A part from a copperplate by Basil Valentine, The Last Will and Testament of Basil Valentine, 1671. (source: Newton N3584 Alchemy Unicode Proposal---March 31 2009.pdf)



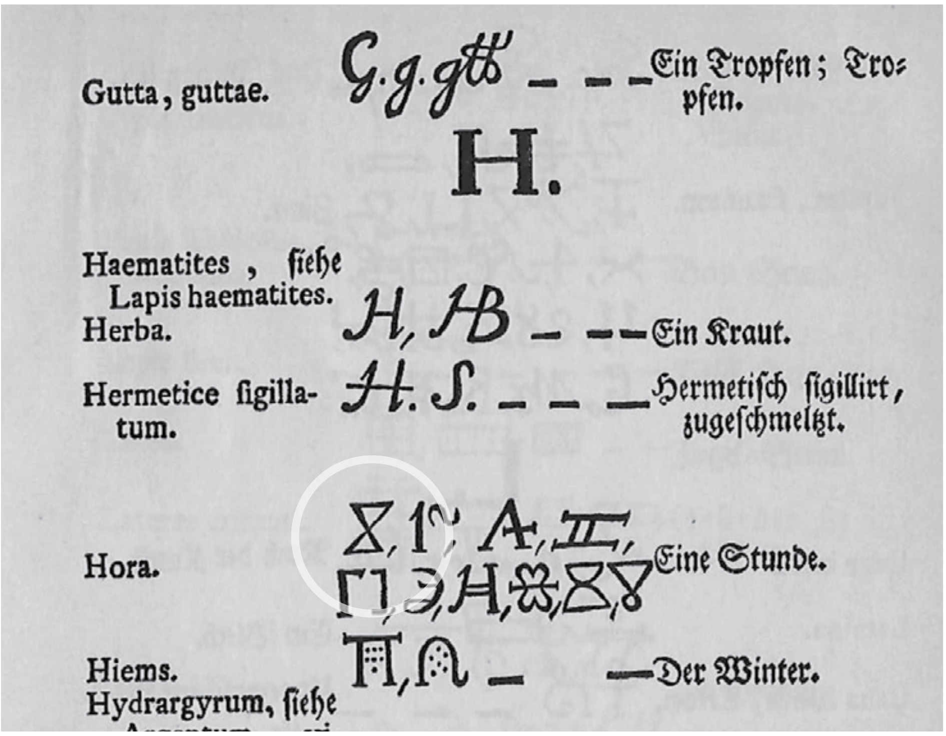
From Geßmann (1964)





ALCHEMICAL SYMBOL FOR HORA 2,
table from Karl Gottfried Hagen, Grundriß der Experimentalchemie (1786); after Schneider 1964.

Medicinisch-Chymisches und Alchemistisches Oraculum (Ulm 1755); after Schneider 1964.


Gutta, guttae. — Ein Tropfen; Tropfen.

H.

Haematites, siehe Lapis haematites.

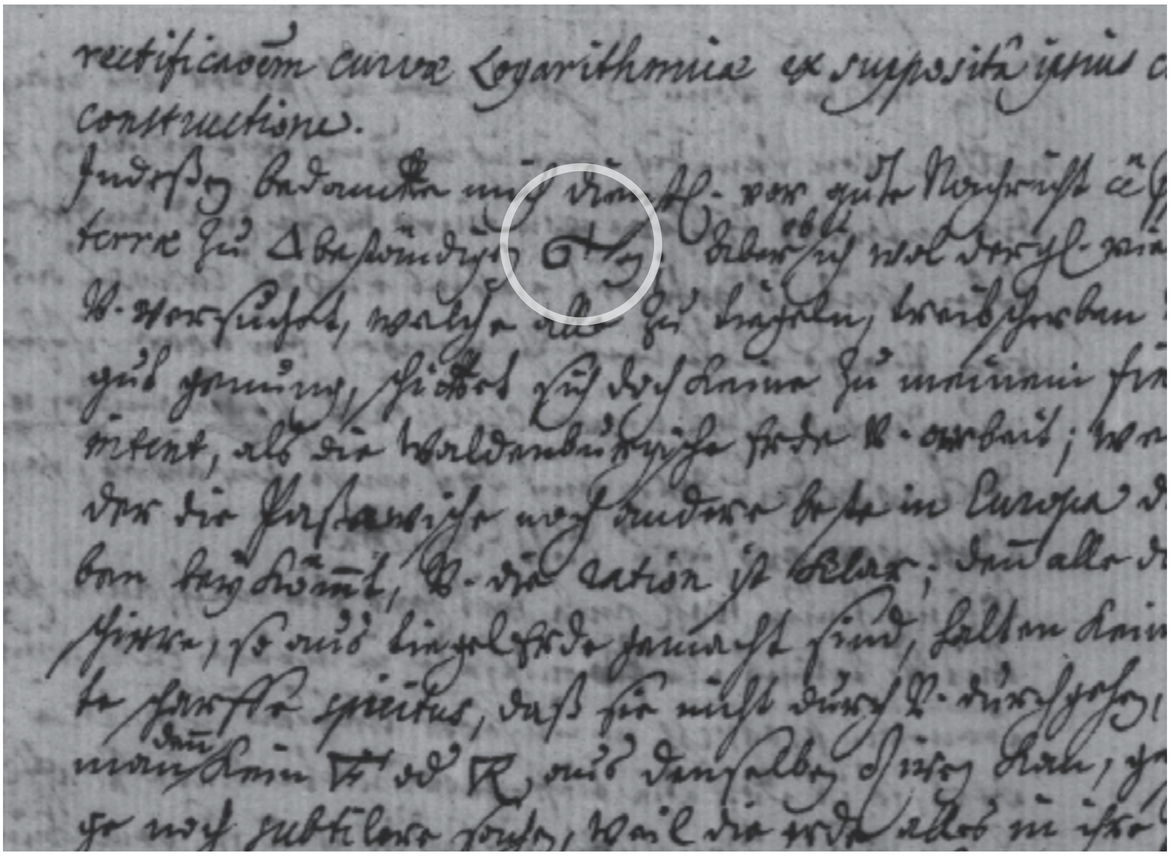
Herba. — Ein Kraut.

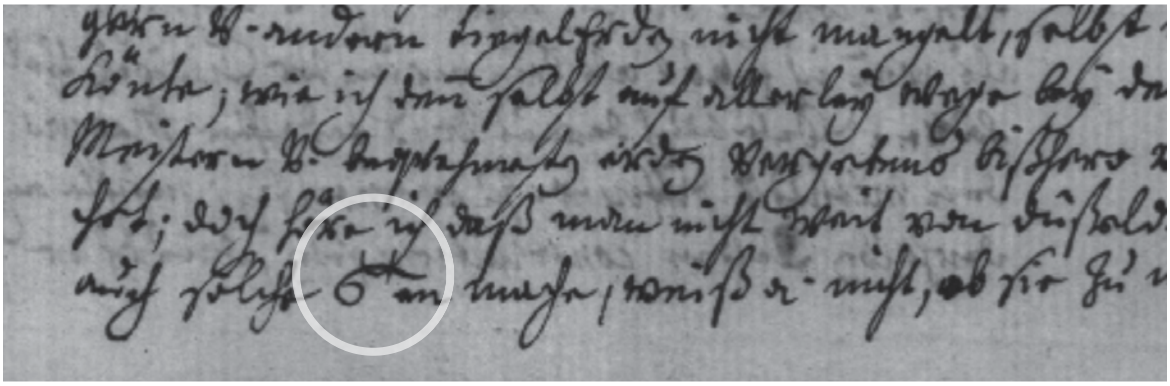
Hermetice sigillatum. — Hermetisch sigillirt, zugeschmelzt.

Hora. — Eine Stunde.

Hiems. — Der Winter.

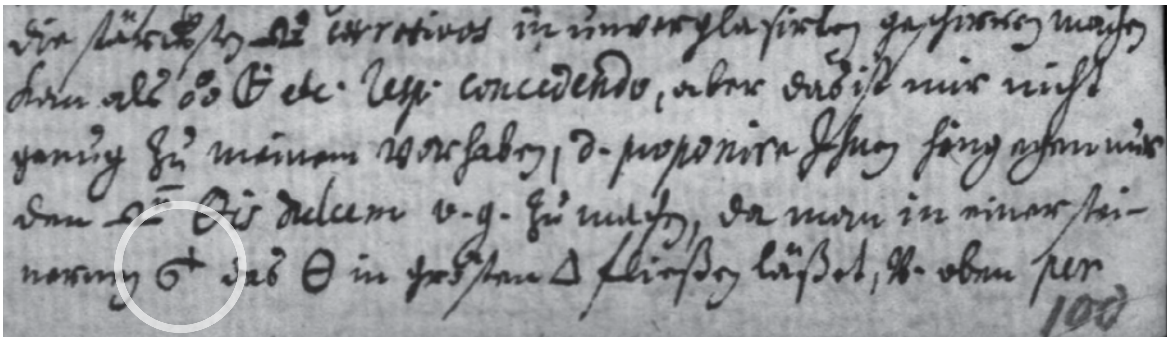
Hydrargyrum, siehe



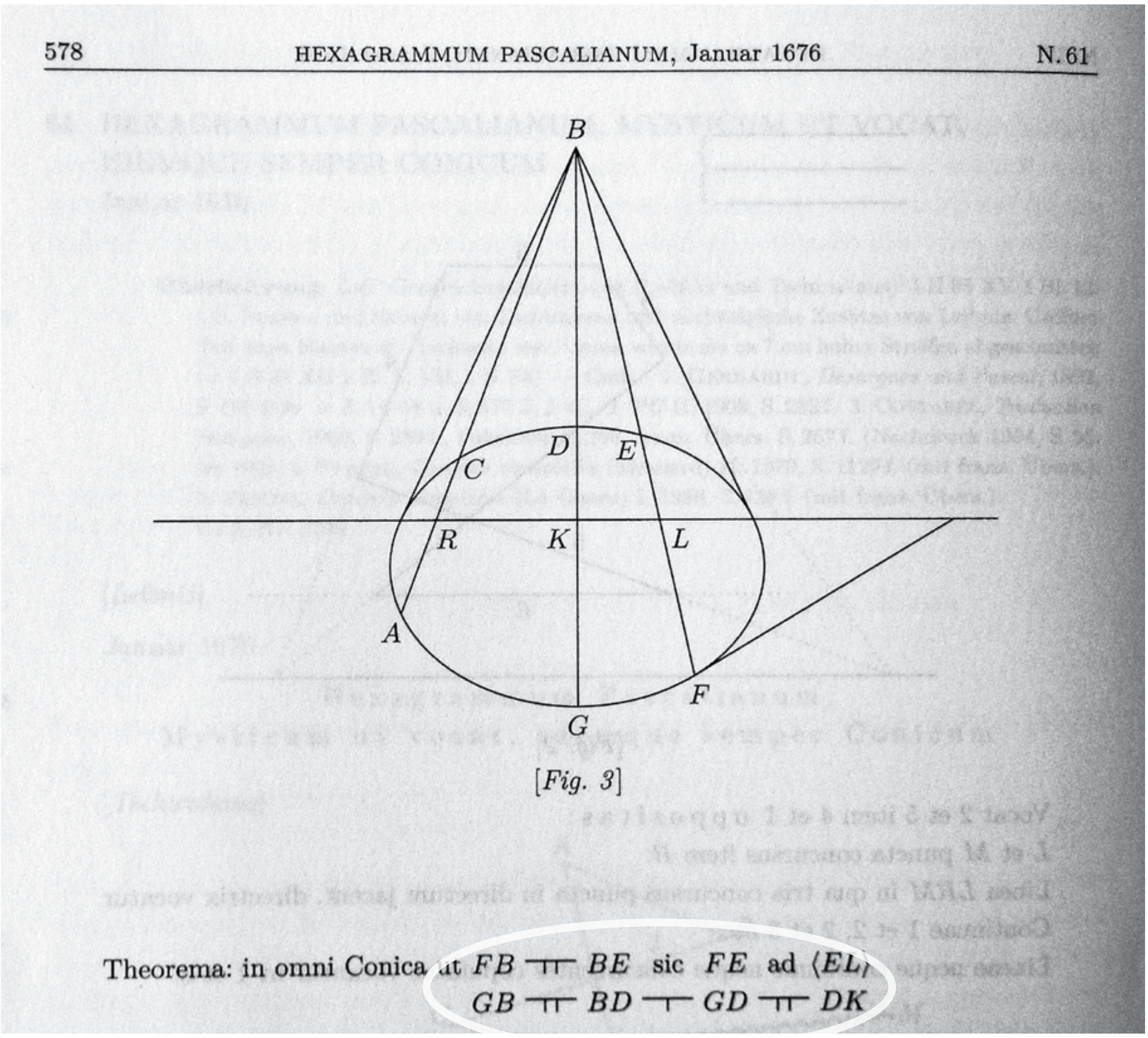


ALCHEMICAL SYMBOL FOR RETORT 2

We propose a new codepoint for this alchemical symbol “retorte”. The symbol 1F76D already exists with the meaning “retorte”, but it has a considerably different glyph shape and it would be a violation of editorial principles to use it instead. Among the alchemical symbols already encoded there are a few precedents for the practice to admit several characters with homonymous definition. – LBr 79 fol. 90v (top, middle), 100r (below)



4.f) Miscellaneous scientific signs


578 HEXAGRAMMUM PASCALIANUM, Januar 1676 N. 61

[*Fig. 3*]

Theorema: in omni Conica ut FB ⫪ BE sic FE ad $\langle EL\rangle$

GB ⫪ BD ⊤ GD ⫪ DK

⊤ PROPORTION 1, ⫪ PROPORTION 2
LAA VII-7 P. 578

850 ALGEBRAISCHE STUDIEN 1675–1676 N. 76

AB BL DC

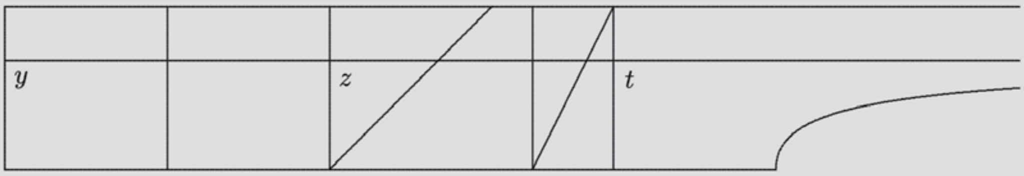
A͡B͡L C͡D͡L x ⊤⊤ t ⊤ c ⊤⊤ y – u

DF ϼ a [BG ϼ d. $d^2 - a^2$ ϼ $b^2 - c^2$]

[*Leibniz*]

ABDC semicirculus. AG ⊓ AF. BCG est recta. DCF est recta.
DF ⊓ a. BG ⊓ d. BC ⊓ b. BL ⊓ t. AD ⊓ y. AL ⊓ v.

⊤ PROPORTION 1, ⊤⊤ PROPORTION 2
LAA VII-2 p. 850



[*Fig. 6*]

z *y* – *z* ⊤⊤ *z* ⊤⊤ *z* ⊤⊤ $\frac{zz}{y-z}$ ϼ *t*
y ϼ *xx* ⊤⊤ *x* ⊤⊤ *x* ⊤⊤
z ϼ *a* – *x* *x* ⊤⊤ *x* ⊤⊤

⊤⊤ PROPORTION 2
This figure shows also the use of the TSCHIRNHAUS EQUAL SIGN ϼ.
LAA VII-6 p. 271

302 ARITHMETISCHE KREISQUADRATUR 1673–1676 N. 26

Als men de ∠*ACB* wil 2 mahl in 2 gelijcke deel, deelen; om *AF* te vinden, soo kan men het dus oock doen[:]

Regel.

Gelijck als
AC + *BC*, sijn ❒ staet tot also het tot het
–□*AB*, multipl. in *BC* ⊤⊤ □*AB*, multipl. in *AC* ⊤⊤ □*AC* ⊤⊤ □*AF*.

⊤⊤ PROPORTION 2
This figure shows also the use of ❒ CUBUS 2.
LAA VII-6 P. 302

Characteristica omnis consistit in formatione Expressionis et transitu ab Expressione ad expressionem.Expressio simplex est vel composita, quae formatur vel per appositionem, vel per coalitionem. Appositione fit f o r m u l a. Coalitione fit c h a r a c t e r novus. Sed pro Calculo non opus est coalitione, sed sufficit simplex appositio seu formula, et compendii causa assumtione arbitrarii characteris cujus significatio tantum nota est. Licet ad perfectionem characteristicae necessaria sit coalitio, ut ingredientia indicentur. In appositione rursus interveniunt ordo (quando ejus habetur ratio); et signa quibus variatur appositio.

Transitus ab expressione ad expressionem, significat una expressione posita poni posse aliam. Hinc dantur jam porro formulae transitum involventes, seu enuntiantes; et transitus ab enuntiatione ad enuntiationem seu consequentiae. Transitus species simplicissima est substitutio, et ex substitutionibus ipsa mutua substitutio seu aequipollentia. Generalis transitus est, ut positis *A* et *B* dicere liceat *AB*, nisi quid scilicet ex specialibus calculi regulis obstet; est inter generalia postulata. Sunt et generales enuntiationes, tales circa *est* et *non*; item inversio relationis, ut A^{be}⟜B^{eb}, *ergo* [B^{eb}⊸A^{be}]. Seu si *A* se habet aliquo modo ad *B*, tunc *B* determinato quodam modo priori contrario se habet ad *A*.

⟜ RIGHTHAND RELATION SIGN, ⊸ LEFTHAND RELATION SIGN

In expressions such as A ⟜ B, both signs are used for relations between some A and B. The relations are not further specified by a specific rule, with ⊸ being the inverse relation to ⟜.
LAA VI-4 p. 917, 988 (below)

Considerandum etiam est cum dicitur *sapiens*, quod concretum est, duo dici: Ens in recto, et abstractum sapientis in obliquo et quidem simplici obliquitate. Itaque si *A* ∞ *Ens* ⊸ *B*, sitque haec propositio per se manifesta, erit *A* concretum, *B* ejus abstractum.

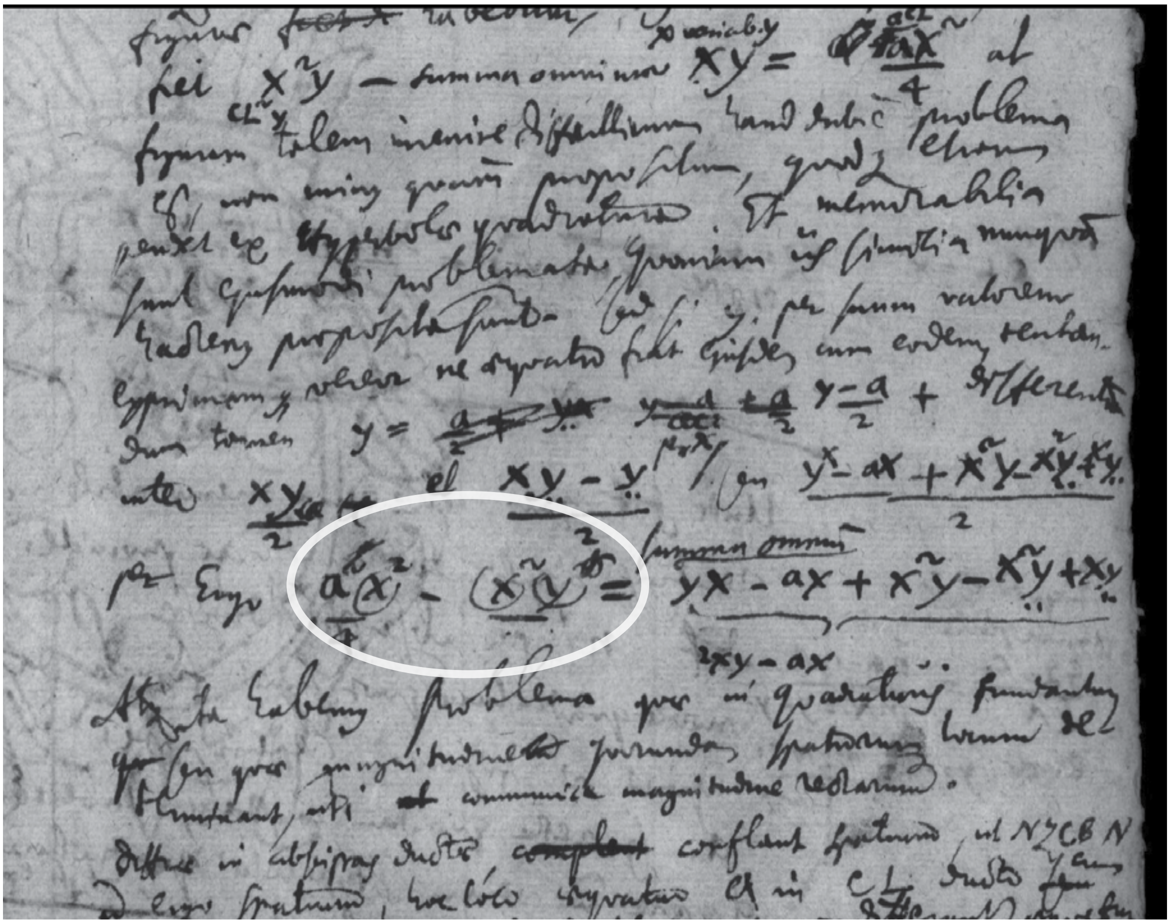
I m m e d i a t e nimirum p e r t i n e t *B* ad *A*, sapientia ad sapientem, hoc est si sapientia non est, etiam sapiens non erit, idque apparet non consequentia aliqua, sed ex ipso hujusmodi terminorum instituto. Et proinde dici potest sapientiam esse immediatam conditionem sapientis. Et *sapientem habere sapientiam*, propositio est per se nota, nec opus est ut ejus cognoscendae causa explicentur termini.

Praeterea Concretum et abstractum eadem omnia involvunt, et quidem eodem modo seu ordine. Et quia plus est dicere quam involvere (*dicere* enim est continere manifeste vel certe facili consequentia) recte asseretur utrumque etiam eadem dicere, cum enim

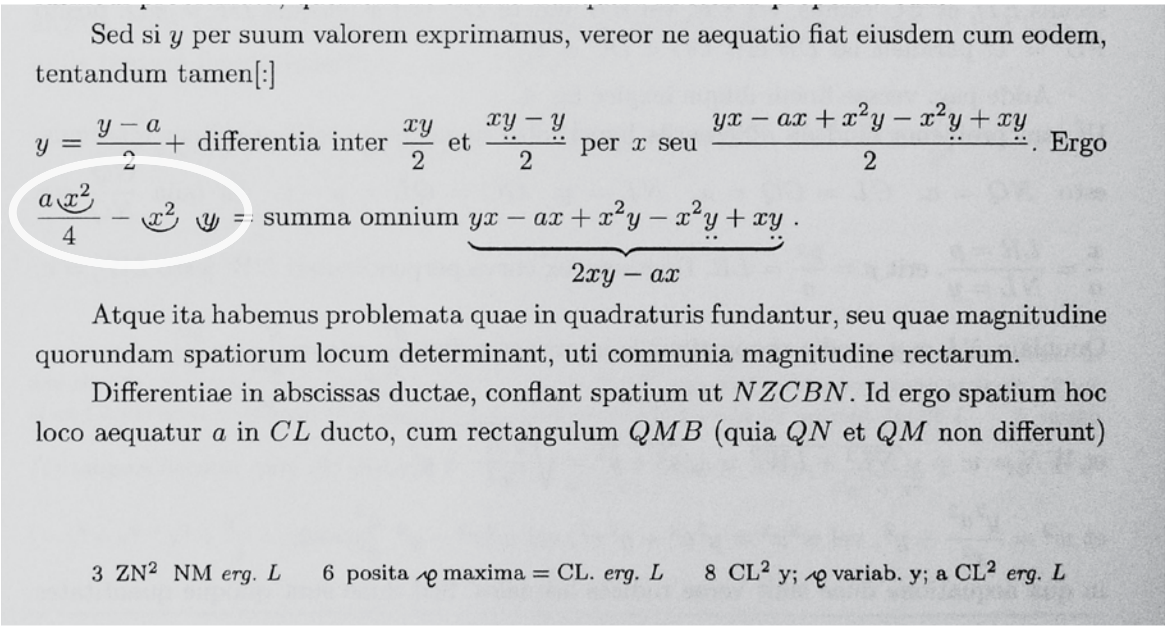
---

[1] *Am Rande*: Duplo calidius dicitur aliquid, si effectus similaris, per quem quid agnoscitur calidum sit duplus. Is effectus est rarefactio; vel si mavis Elastrum aëris auctum, ut si duplo vel triplo majus pondus sustineat.

3 effectus | potius *gestr.* | Sapientiae, *L* 6 f. est, (*1*) si sit (*2*) si in (*3*) in concretis esse posse duos terminos, (*4*) fieri . . . termini, *L* 8 f. sapientem; (*1*) in abstractis fieri posse ut duo inde fiant termini, nempe *sapientia* et *divitia* ut cum veteribus loquar. (*2*) et . . . ut | in *str. Hrsg.* | abstracta . . . Entia. *L* 10 idem (*1*) est sapiens et virtuosus, nec tamen alia res est virtus de qua agitur quam sapientia, nam ipsa (*2*) qui . . . etiam *L* 13 *Ens* (*1*) ~ B (*2*) | ⊸ B *erg.* | *L* 13 concretum, (*1*) Ens (*2*) *B* | ejus *erg.* | *L* 13 f. abstractum. (*1*) Nam ea natura est (*2*) Immediate requiritur (*3*) Ex his (*4*) I m m e d i a t e (*a*) etiam



◌᷂ COMBINING HALF CIRCLE BELOW
The shape of the character is typically at least a half circle, often it approximates 3/5 of a circle. Hence it is considerably different from COMBINING DOUBLE BREVE BELOW (035C).
LH XIII 35 3 fol. 250v



Sed si y per suum valorem exprimamus, vereor ne aequatio fiat eiusdem cum eodem, tentandum tamen[:]

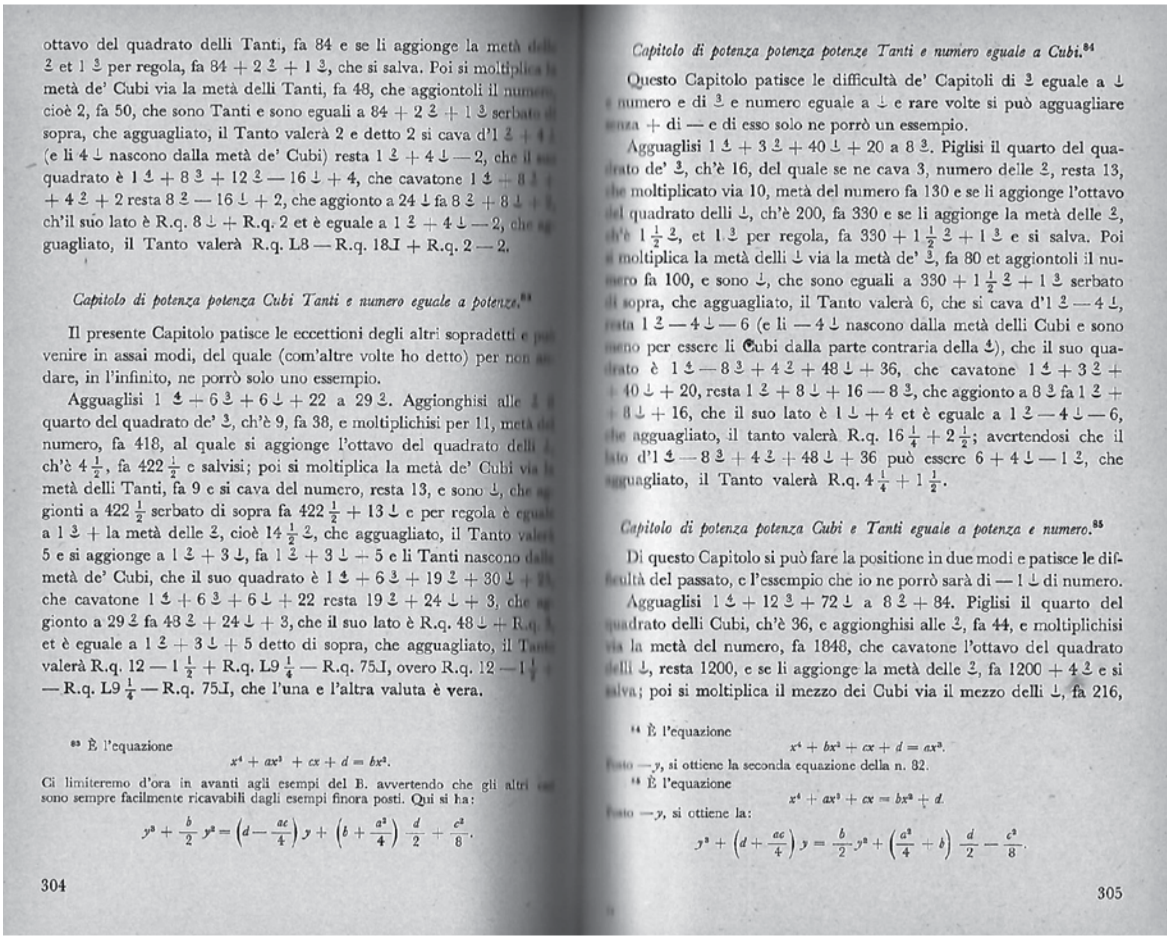
$y = \frac{y-a}{2} +$ differentia inter $\frac{xy}{2}$ et $\frac{xy-y}{2}$ per x seu $\frac{yx - ax + x^2y - x^2y + xy}{2}$. Ergo $\frac{ax^2}{4} - x^2\ y =$ summa omnium $\underbrace{yx - ax + x^2y - x^2y + xy}_{2xy-ax}$.

Atque ita habemus problemata quae in quadraturis fundantur, seu quae magnitudine quorundam spatiorum locum determinant, uti communia magnitudine rectarum.

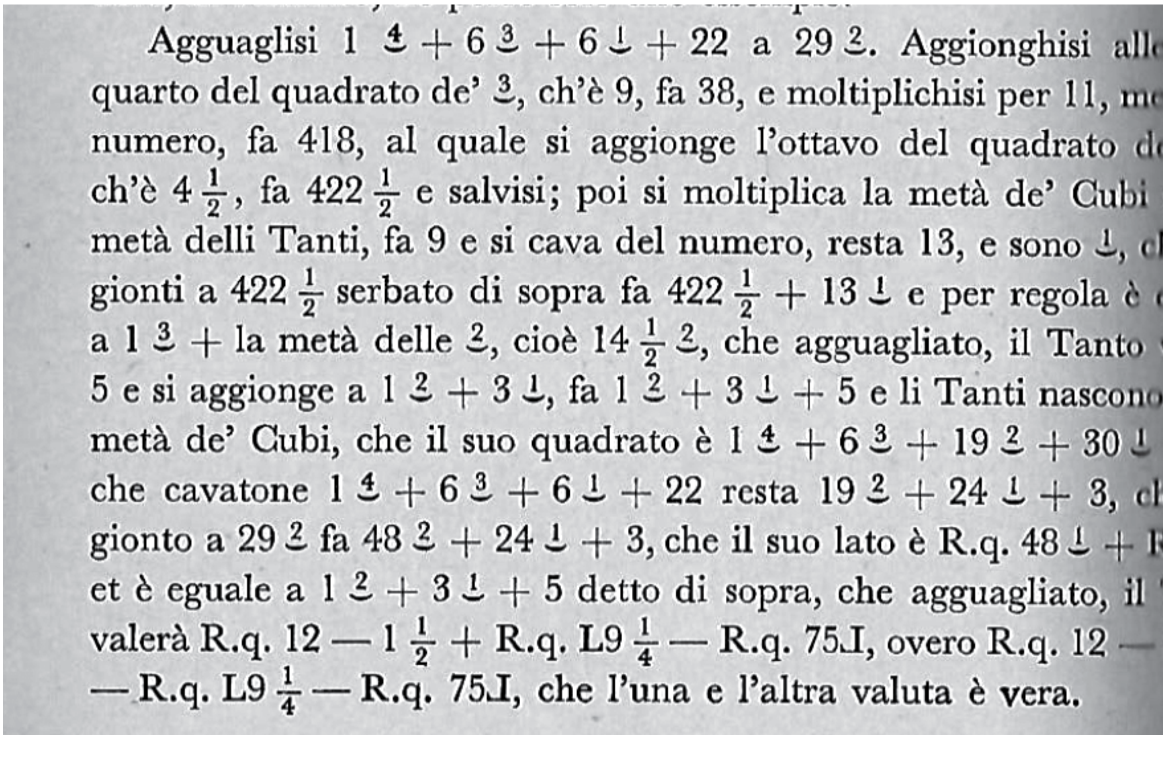
Differentiae in abscissas ductae, conflant spatium ut $NZCBN$. Id ergo spatium hoc loco aequatur a in CL ducto, cum rectangulum QMB (quia QN et QM non differunt)

3 ZN^2 NM *erg. L* 6 posita ꝭ maxima = CL. *erg. L* 8 CL^2 y; ꝭ variab. y; a CL^2 *erg. L*

◌᷂ COMBINING HALF CIRCLE BELOW
LAA VII-4 p. 824



◌ COMBINING BOMBELLI POWER MARK
A number combined with a small bow below as introduced by R. Bombelli denotes the n-th power of a quantity. Example from Bombelli (1966).



Agguaglisi 1 $\underset{\smile}{4}$ + 6 $\underset{\smile}{3}$ + 6 $\underset{\smile}{1}$ + 22 a 29 $\underset{\smile}{2}$. Aggionghisi all[e] quarto del quadrato de' $\underset{\smile}{3}$, ch'è 9, fa 38, e moltiplichisi per 11, m[età] numero, fa 418, al quale si aggionge l'ottavo del quadrato d[elli] ch'è 4 $\frac{1}{2}$, fa 422 $\frac{1}{2}$ e salvisi; poi si moltiplica la metà de' Cubi metà delli Tanti, fa 9 e si cava del numero, resta 13, e sono $\underset{\smile}{1}$, c[he ag]gionti a 422 $\frac{1}{2}$ serbato di sopra fa 422 $\frac{1}{2}$ + 13 $\underset{\smile}{1}$ e per regola è a 1 $\underset{\smile}{3}$ + la metà delle $\underset{\smile}{2}$, cioè 14 $\frac{1}{2}$ $\underset{\smile}{2}$, che agguagliato, il Tanto 5 e si aggionge a 1 $\underset{\smile}{2}$ + 3 $\underset{\smile}{1}$, fa 1 $\underset{\smile}{2}$ + 3 $\underset{\smile}{1}$ + 5 e li Tanti nascon[o] metà de' Cubi, che il suo quadrato è 1 $\underset{\smile}{4}$ + 6 $\underset{\smile}{3}$ + 19 $\underset{\smile}{2}$ + 30 $\underset{\smile}{1}$ che cavatone 1 $\underset{\smile}{4}$ + 6 $\underset{\smile}{3}$ + 6 $\underset{\smile}{1}$ + 22 resta 19 $\underset{\smile}{2}$ + 24 $\underset{\smile}{1}$ + 3, c[he ag]gionto a 29 $\underset{\smile}{2}$ fa 48 $\underset{\smile}{2}$ + 24 $\underset{\smile}{1}$ + 3, che il suo lato è R.q. 48 $\underset{\smile}{1}$ + et è eguale a 1 $\underset{\smile}{2}$ + 3 $\underset{\smile}{1}$ + 5 detto di sopra, che agguagliato, il valerà R.q. 12 — 1 $\frac{1}{2}$ + R.q. L9 $\frac{1}{4}$ — R.q. 75⅃, overo R.q. 12 — — R.q. L9 $\frac{1}{4}$ — R.q. 75⅃, che l'una e l'altra valuta è vera.

Dimostratione delle Rc. legate con il p i u d i m e n o, e m e n o d i m e n o, *in linea* (+ puto: *in linee* +).

Habbisi Rc.⌊4. p. di m. Rq.11⌋. p. Rc.⌊4. m. di m. Rq. 11⌋ e per trovare la sua linea aggiongasi 16. quadrato del 4. con 11. quadrato di Rq.11. fa 27. e di questo si pigli il lato cubo ch'è 3. e per regola si moltiplichi per 3. fa 9, e salvisi, poi per regola si moltiplica il 4. per 2. fa 8, e queste due [Rc.] legate sono nate dall'agguagliatione d' 1 $\overset{3}{\smile}$ a 9 $\overset{1}{\smile}$ p. 8. però faccisi la dimostratione in linea d' 1 $\overset{3}{\smile}$ eguale a 9 $\overset{1}{\smile}$ p. 8. cioè in superficie piana e si troverà che la longhezza del tanto sarà ancora la longhezza delle due Rc. legate proposte.

Subicit postea demonstrationem quae originem exhibet inventionis regularum Cardani per sectionem cubi. Sed notat ipse

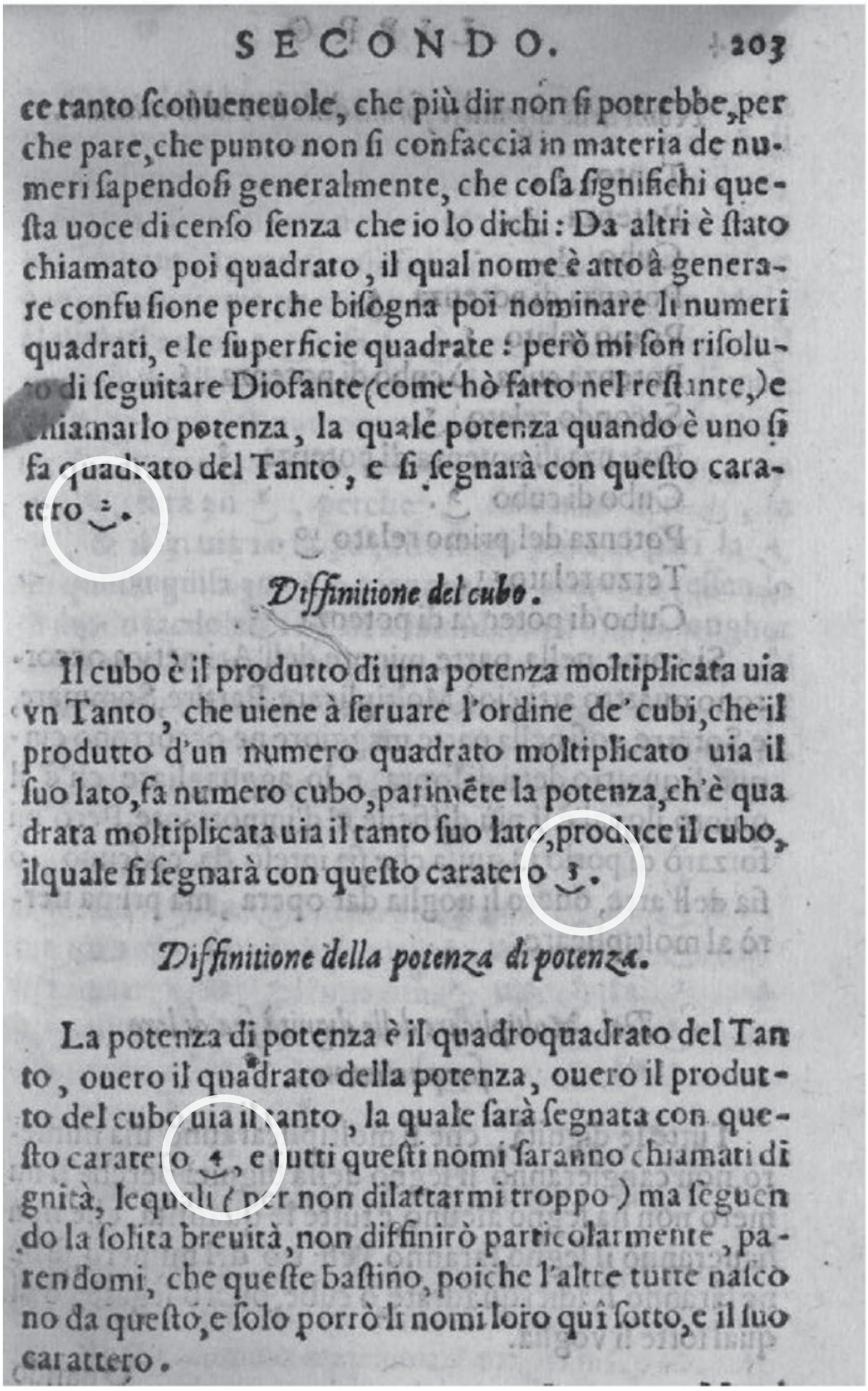
Si 1 $\overset{3}{\smile}$ eguale a 6 $\overset{1}{\smile}$ p. 4. e sia la q. la unità. Tirisi la m.e. e faccisi m.l. che sia pari alla q. cioè sia 1. e l.f. 6. cioè quanto è il numero delli tanti, e sopra detta l.f. si faccia un parallelogrammo che sia 4. di superficie, cioè quanto il numero, e sarà il parallelogrammo a.b.f. poi allonghisi la a.b. sino in d. ed' a.l. sino in r. poi habbiansi due squadri, delli quali l'uno si ponga con l'angolo sopra la linea r. e che l'uno delle braccia tocchi la estremità m, il qual squadro si alzi o abbassi tanto, che tirato dal angolo del squadro una linea, che tocchi la estremità f. che vada a toccare la b.d. in tal luogo, che mettendo un altro squadro con l'angolo al detto toccamento, e con l'uno delle braccia sopra la d.a. vadi a intersegare il braccio dell'altro squadro nella linea f.e. fatto questo dico che la linea, ch'è dal punto l. sino al angolo del squadro, è la valuta del tanto, e lo provo in questo modo. Prosupposto che si habbia alzato e abbassato lo squadro talmente, che in i. tirando la i.f. sino in c., e che il braccio dello squadro p. tagliassi con l'altro squadro in g. suso la linea g.e. fatto questo; dico la linea l.i. essere la valuta del tanto. Perche essendo la l.i. 1 $\overset{1}{\smile}$ et m.l.i (+ male credo impressum, lege: et m.l. 1. +) la l.g. sarà 1 $\overset{2}{\smile}$, perche tanto può la m.l. in l.m. (+ lege: in l.g. +) quanto l.i. in se stessa, essendo il angolo i. retto, il parallelogrammo i.l.g. sarà un cubo (+ vel y^3 +) et il parallelogrammo i.l.f. sarà 6 $\overset{1}{\smile}$, perche i.l. è 1 $\overset{1}{\smile}$, et l.f. 6. et il parallelogrammo h.f.g. sarà 4, perch'è pari al parallelogrammo a.l.f. ch'era 4, e essendo i.l.g. tutto insieme 6 $\overset{1}{\smile}$, e 4.; e per l'altra ragione è provato essere 1 $\overset{3}{\smile}$, dunque 1 $\overset{3}{\smile}$ sarà eguale à 6 $\overset{1}{\smile}$ p. 4., et la i.l. sarà 1 $\overset{1}{\smile}$, che per la agguagliatione insegnata la l.i. sarà Rq.3. p. 1. la l.g. sarà 4. p. Rq.12. la f.g. sarà

◌̮ COMBINING BOMBELLI POWER MARK

A number combined with a small bow below as introduced by R. Bombelli denotes the n-th power of a quantity. By this, Bombelli provides a different formalization of what is adressed by the use of cossic signs. Today, we write x, x^2, x^3, … instead.

Note that the vertical alinement of the raised figures with the mark is not ideal in this example.

LAA VII-2 p. 662, 663


SECONDO. 203

ce tanto sconueneuole, che più dir non si potrebbe, per che pare, che punto non si confaccia in materia de numeri sapendosi generalmente, che cosa significhi questa uoce di censo senza che io lo dichi: Da altri è stato chiamato poi quadrato, il qual nome è atto à generare confusione perche bisogna poi nominare li numeri quadrati, e le superficie quadrate: però mi son risoluto di seguitare Diofante (come hò fatto nel restante,) e chiamarlo potenza, la quale potenza quando è uno si fa quadrato del Tanto, e si segnarà con questo caratero $\overset{2}{\smile}$.

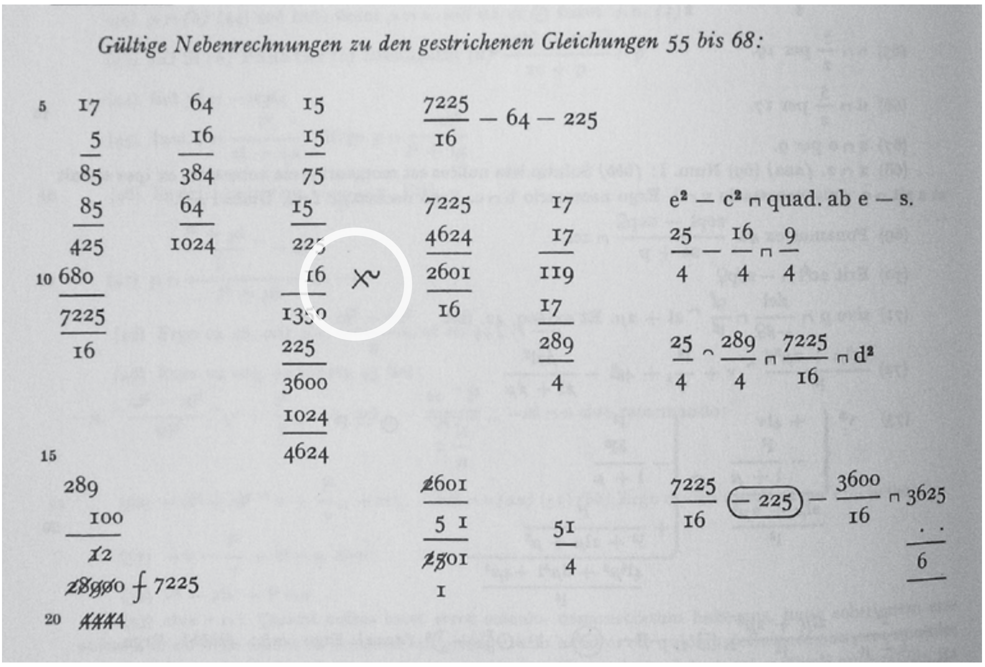
Diffinitione del cubo.

Il cubo è il produtto di una potenza moltiplicata uia vn Tanto, che uiene à seruare l'ordine de' cubi, che il produtto d'un numero quadrato moltiplicato uia il suo lato, fa numero cubo, parimēte la potenza, ch'è quadrata moltiplicata uia il tanto suo lato, produce il cubo, ilquale si segnarà con questo caratero $\overset{3}{\smile}$.

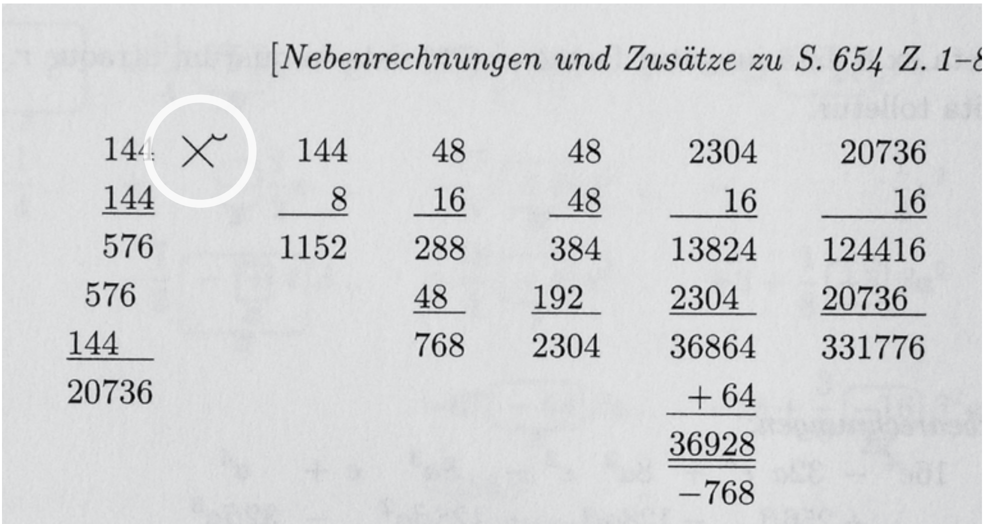
Diffinitione della potenza di potenza.

La potenza di potenza è il quadroquadrato del Tanto, ouero il quadrato della potenza, ouero il produtto del cubo uia il Tanto, la quale sarà segnata con questo caratero $\overset{4}{\smile}$, e tutti questi nomi saranno chiamati dignità, lequali (per non dilattarmi troppo) ma seguendo la solita breuità, non diffinirò particolarmente, parendomi, che queste bastino, poiche l'altre tutte nascono da questo, e solo porrò li nomi loro quì sotto, e il suo carattero.

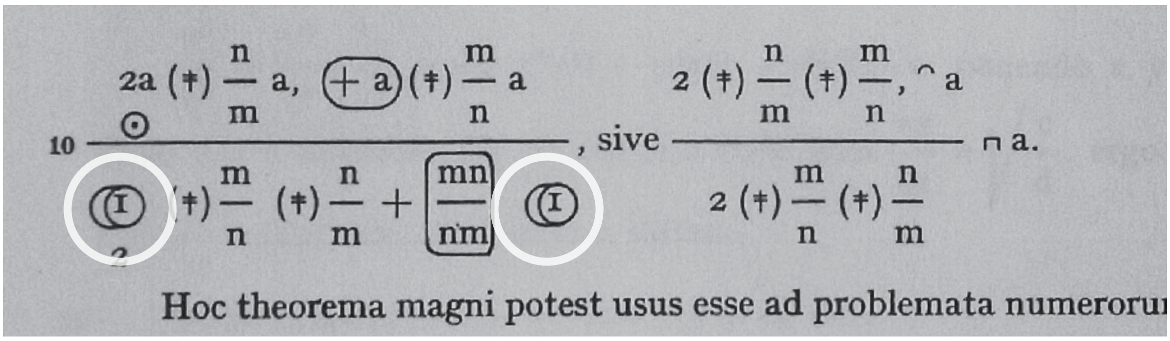
◌᷍ COMBINING BOMBELLI POWER MARK
R. Bombelli, L'Algebra. Bologna 1579, p. 203


Gültige Nebenrechnungen zu den gestrichenen Gleichungen 55 bis 68:

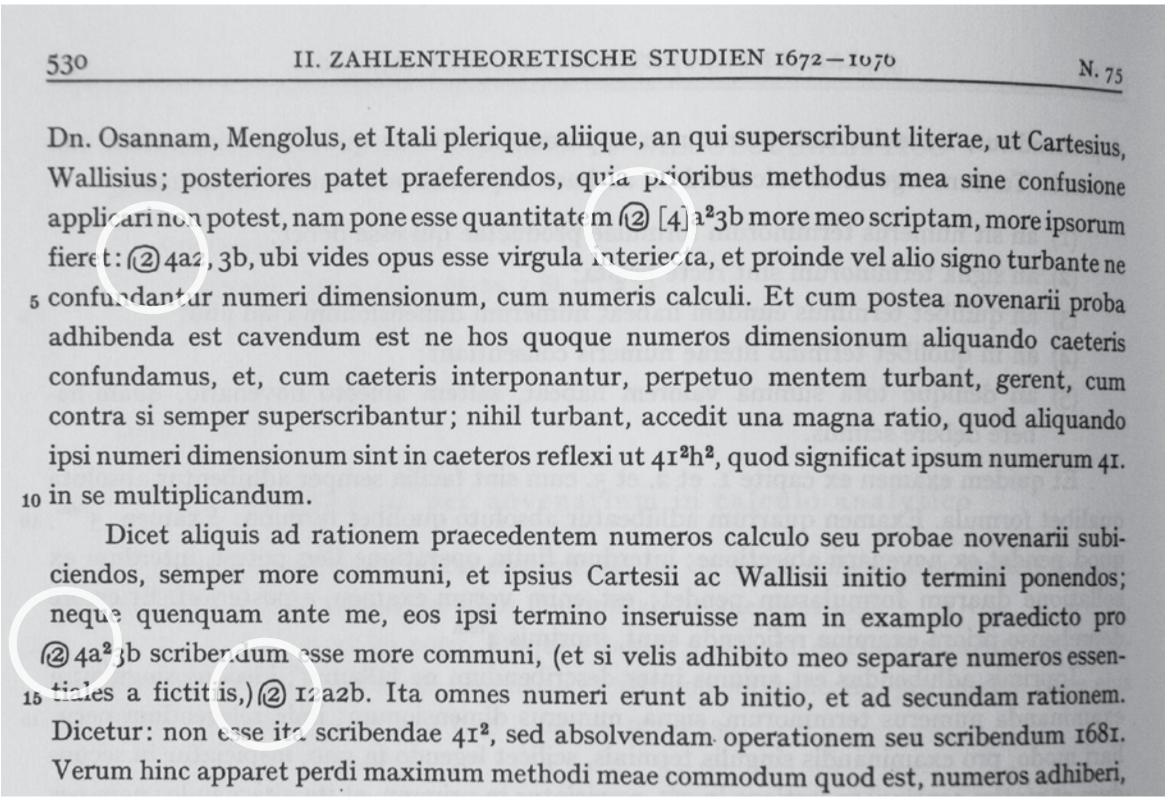
⤫ CASTING-OUT-NINES SIGN
LAA VII-1 p. 408; VII-3 p.660 (below)


[*Nebenrechnungen und Zusätze zu S. 654 Z. 1–8*

144	144	48	48	2304	20736
144	8	16	48	16	16
576	1152	288	384	13824	124416
576		48	192	2304	20736
144		768	2304	36864	331776
20736				+ 64	
				36928	
				−768	


Hoc theorema magni potest usus esse ad problemata numeroru

LUNATE ENCIRCLED DIGIT ONE
LAA VII-1 p. 472

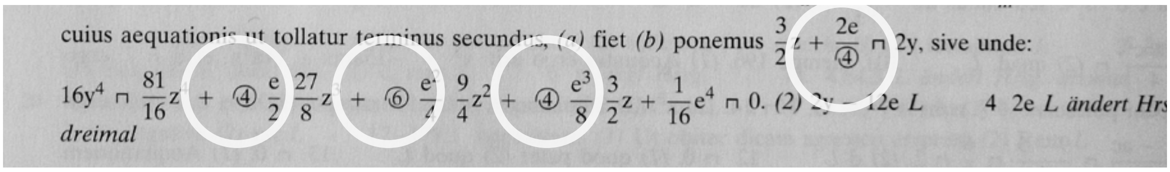


530 II. ZAHLENTHEORETISCHE STUDIEN 1672—1676 N. 75

Dn. Osannam, Mengolus, et Itali plerique, aliique, an qui superscribunt literae, ut Cartesius, Wallisius; posteriores patet praeferendos, quia prioribus methodus mea sine confusione applicari non potest, nam pone esse quantitatem ② [4] a²3b more meo scriptam, more ipsorum fieret: ② 4a², 3b, ubi vides opus esse virgula interiecta, et proinde vel alio signo turbante ne confundantur numeri dimensionum, cum numeris calculi. Et cum postea novenarii proba adhibenda est cavendum est ne hos quoque numeros dimensionum aliquando caeteris confundamus, et, cum caeteris interponantur, perpetuo mentem turbant, gerent, cum contra si semper superscribantur; nihil turbant, accedit una magna ratio, quod aliquando ipsi numeri dimensionum sint in caeteros reflexi ut 41²h², quod significat ipsum numerum 41. in se multiplicandum.

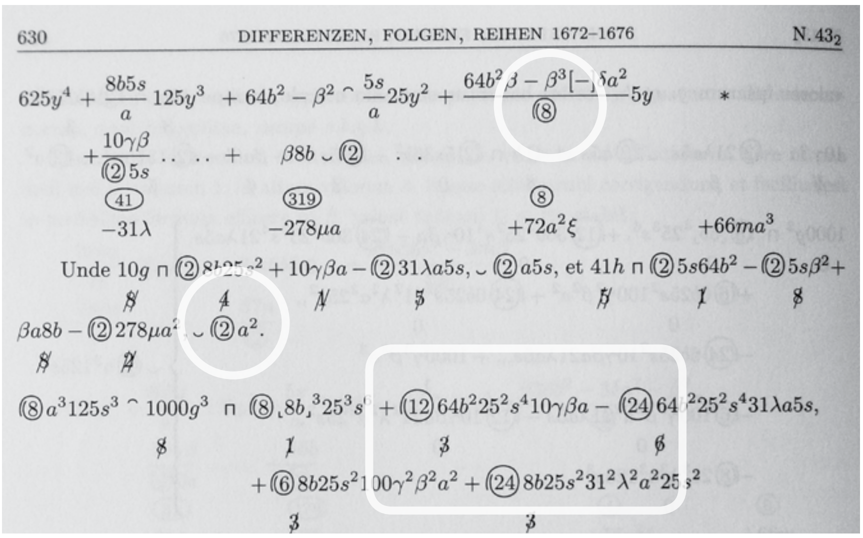
Dicet aliquis ad rationem praecedentem numeros calculo seu probae novenarii subiciendos, semper more communi, et ipsius Cartesii ac Wallisii initio termini ponendos; neque quenquam ante me, eos ipsi termino inseruisse nam in examplo praedicto pro ② 4a²3b scribendum esse more communi, (et si velis adhibito meo separare numeros essentiales a fictitiis,) ② 12a2b. Ita omnes numeri erunt ab initio, et ad secundam rationem. Dicetur: non esse ita scribendae 41², sed absolvendam operationem seu scribendum 1681. Verum hinc apparet perdi maximum methodi meae commodum quod est, numeros adhiberi,

◌ COMBINING ENCLOSING SPIRAL MARK
This non-spacing mark is to be combined with digits or letters.
LAA VII-1 p. 530



cuius aequationis ut tollatur terminus secundus, *(a)* fiet *(b)* ponemus $\frac{3}{2}z + \frac{2e}{④}$ ⊓ 2y, sive unde:
16y⁴ ⊓ $\frac{81}{16}$z + ④ $\frac{e}{2}$ $\frac{27}{8}$z + ⑥ $\frac{e^2}{4}$ $\frac{9}{4}$z² + ④ $\frac{e^3}{8}$ $\frac{3}{2}$z + $\frac{1}{16}$e⁴ ⊓ 0. *(2)* 2y — 12e *L* 4 2e *L ändert Hrs dreimal*

◌ COMBINING ENCLOSING SPIRAL MARK, ◌ COMBINING DOUBLE-WIDE ENCLOSING SPIRAL MARK – LAA VII-2 p. 180; VII-3 p. 630 (below)



630 DIFFERENZEN, FOLGEN, REIHEN 1672–1676 N. 43₂

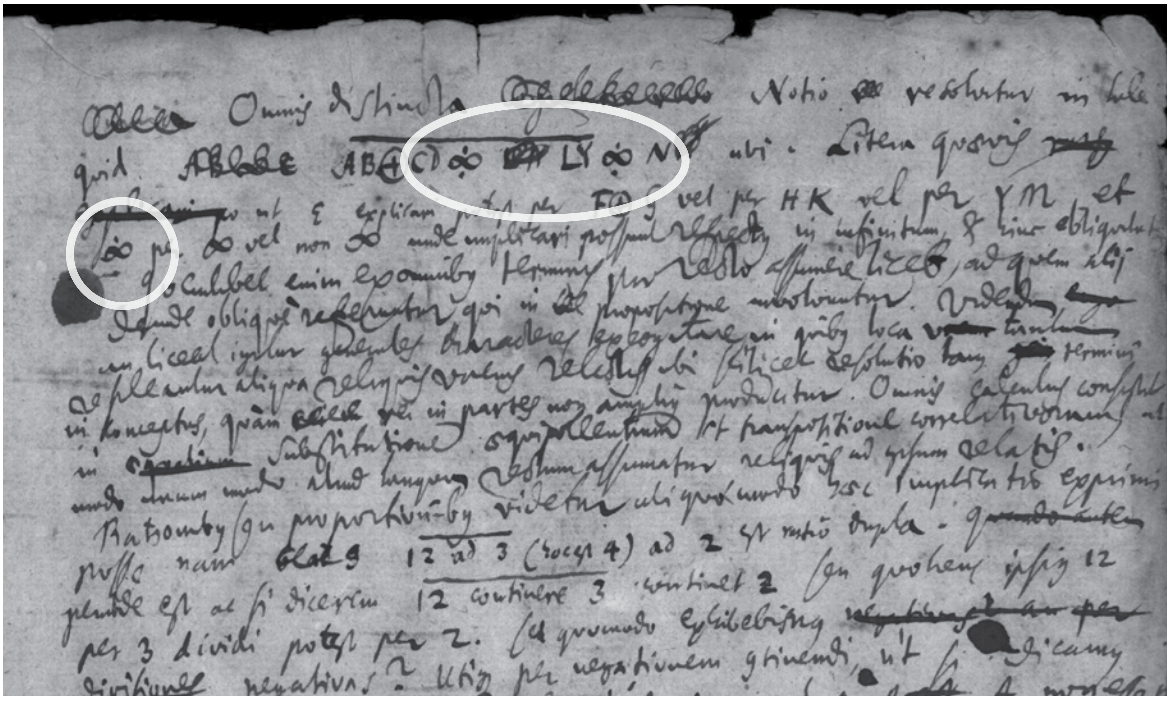
$625y^4 + \frac{8b5s}{a}125y^3 + 64b^2 - \beta^2 \frown \frac{5s}{a}25y^2 + \frac{64b^2\beta - \beta^3[-]5a^2}{⑧}5y$ *

$+\frac{10\gamma\beta}{②5s}$ . . . . + $\beta 8b \smile ②$ . .

㊶ −31λ ⑶⑴⑼ −278μa ⑧ +72a²ξ +66ma³

Unde 10g ⊓ ②8b25s² + 10γβa − ②31λa5s, ⌣ ②a5s, et 41h ⊓ ②5s64b² − ②5sβ² + βa8b − ②278μa² ⌣ ②a².

⑧a³125s³ ⌒ 1000g³ ⊓ ⑧8b³25³s³ + ⑫64b²25²s⁴10γβa − ㉔64b²25²s⁴31λa5s, + ⑥8b25s²100γ²β²a² + ㉔8b25s²31²λ²a²25s²



∞̣ INFINITY SIGN WITH DOTS
LH 4 VII B 2, fol. 73v

O r d o seu p r i u s e t p o s t e r i u s ex cogitationis plus minusve distinctae gradibus peti debent. Prius enim est quod altero simplicius concipitur. Quod si accedat relatio ad existentiam seu perceptionem fit prius tempore.

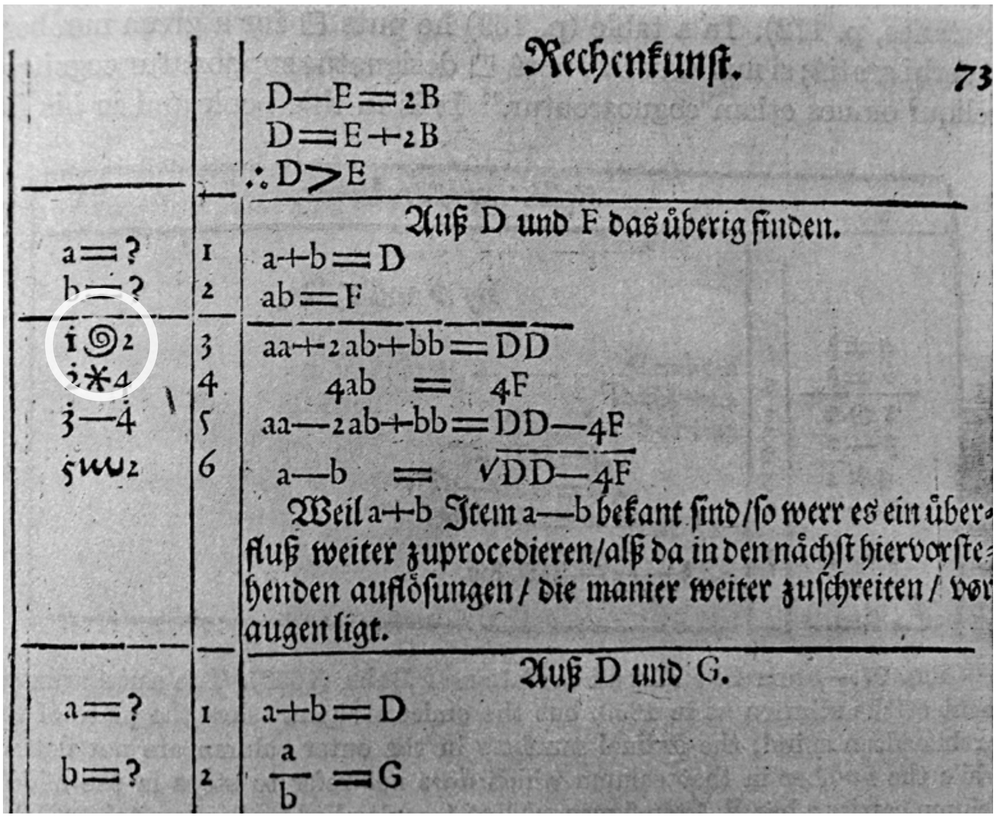
Omnia ad haec videntur revocari posse. Aliquidditas. Essentia. Existentia. Realitas. Perfectio. Uni[tas.] Convenientia. Veritas. Consequentia. Ordo. Causalitas. Mutatio. Magnitudo. Sensus. Appetitus. Cogitatio. Qualitates Sensibiles.

⟨ – ⟩ in characteristica omnia distincte cogitabilia revocari possunt ad $\overline{AB + CD \underset{\text{non } \infty}{\overset{\infty}{}} LM} \infty N$, hoc uno not⟨ato⟩ ⟨ – – – ⟩ et contra explicari ⟨ – – – ⟩ quod quaedam literae in ⟨ – ⟩ ut *Y* pro *S* pon⟨–⟩

Omnis distincta Notio resolvitur in tale quid $\overline{AB \oplus CD \,\dot{\infty}\, LY}\,\dot{\infty}\, N$ ubi Litera quaevis ut *E* explicari potest per *F* ⊕ *G* vel per *HK* vel per *YM* et $\dot{\infty}$ per ∞ vel *non* ∞ unde implicari possunt respectus in infinitum, et hinc obliquitat[es].

Quemlibet enim ex omnibus terminis pro recto assumere licet, ad quem alii deinde oblique referuntur, qui in propositione involvuntur. Videndum an liceat igitur generales characteres excogitare, in quibus loca tantum repleantur aliqua reliquis vacuis relictis, ubi scilicet resolutio tam termini in conceptus, quam rei in partes non amplius producitur.

∞̣ INFINITY SIGN WITH DOTS
LAA VI-4 p. 873



Rechenkunst. 73

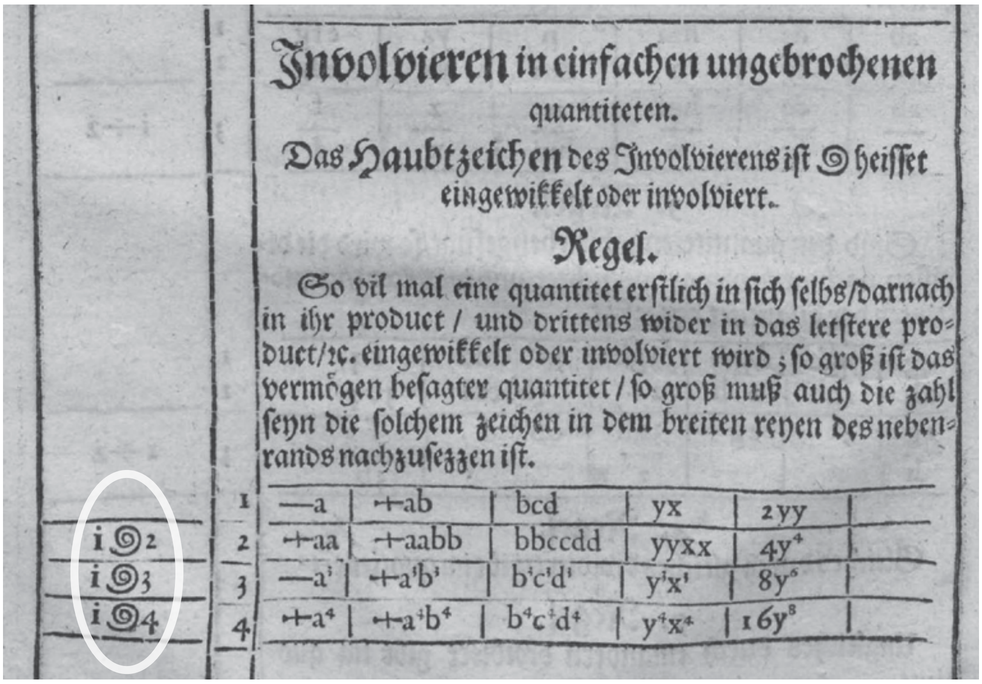
		D—E = 2B
		D = E + 2B
		∴ D > E
		Auß D und F das überig finden.
a = ?	1	a + b = D
b = ?	2	ab = F
i ◎ 2	3	aa + 2ab + bb = DD
2 * 4	4	4ab = 4F
3 — 4	5	aa — 2ab + bb = DD — 4F
5 ww 2	6	a — b = √DD — 4F

Weil a+b Item a—b bekant sind/so werr es ein überfluß weiter zuprocedieren/alß da in den nächst hiervorstehenden auflösungen/die manier weiter zuschreiten/vor augen ligt.

		Auß D und G.
a = ?	1	a + b = D
b = ?	2	$\frac{a}{b}$ = G

◎ INVOLVED SIGN – J. H. Rahn, Teutsche Algebra, 1659 (after Cajori).

In expressions of the form *a* ◎ *b*, the sign ◎ is used to denote the exponentiation of *a* by the power of *b*. In his “Teutsche Algebra” from 1659, the swiss mathematician Johann Heinrich Rahn refers to the operation as “involvieren” (= to involve).



Involvieren in einfachen ungebrochenen quantiteten.

Das Haubtzeichen des Involvierens ist ◎ heisset eingewikkelt oder involviert.

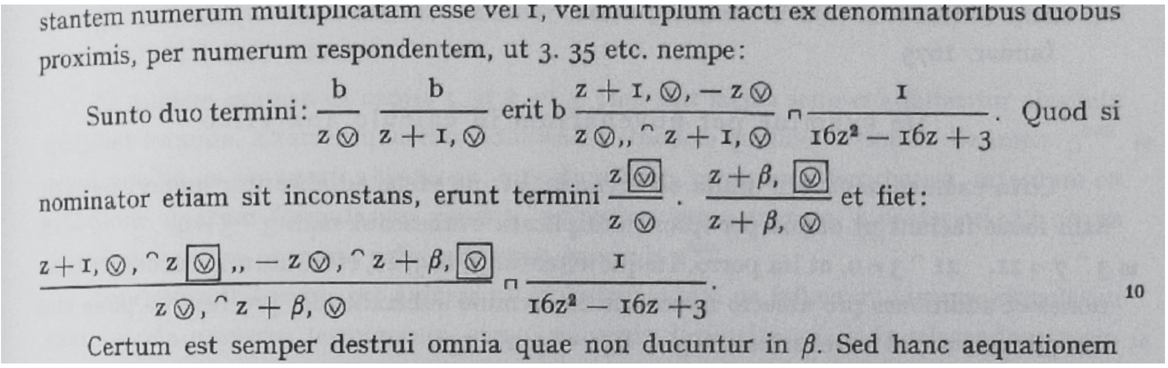
Regel.

So vil mal eine quantitet erstlich in sich selbs/darnach in ihr product / und drittens wider in das letstere product/2c. eingewikkelt oder involviert wird; so groß ist das vermögen besagter quantitet / so groß muß auch die zahl seyn die solchem zeichen in dem breiten reyen des nebenrands nachzusezzen ist.

	1	—a	+ab	bcd	yx	2yy
i ◎ 2	2	+aa	+aabb	bbccdd	yyxx	$4y^4$
i ◎ 3	3	$—a^3$	$+a^3b^3$	$b^3c^3d^3$	y^3x^3	$8y^6$
i ◎ 4	4	$+a^4$	$+a^4b^4$	$b^4c^4d^4$	y^4x^4	$16y^8$

◎ INVOLVED SIGN – J. H. Rahn, Teutsche Algebra, 1659.

In the time of Leibniz, the usual way of referring to curves or magnitudes is by giving equations that describe their specific relations. The concept of mapping as it is used in modern mathematics is not yet developed. Leibniz writes the signs $\textcircled{v}$ and $\boxed{\textcircled{v}}$ to the right of an expression (such as x $\textcircled{v}$ and $y+1$, $\boxed{\textcircled{v}}$) in order to denote two different arbitrary rules by which the expressions given in the left position are treated. The result is an expression. By this, the meaning is similar to writing $f(x)$ or $g(y+1)$ in modern mathematical notation with f and g denoting arbitrary functions.
In a similar way, Johann Bernoulli uses the sign φ (see p. 97) to denote a quantity depending on variables x and a (in modern terminology a function in x and a).



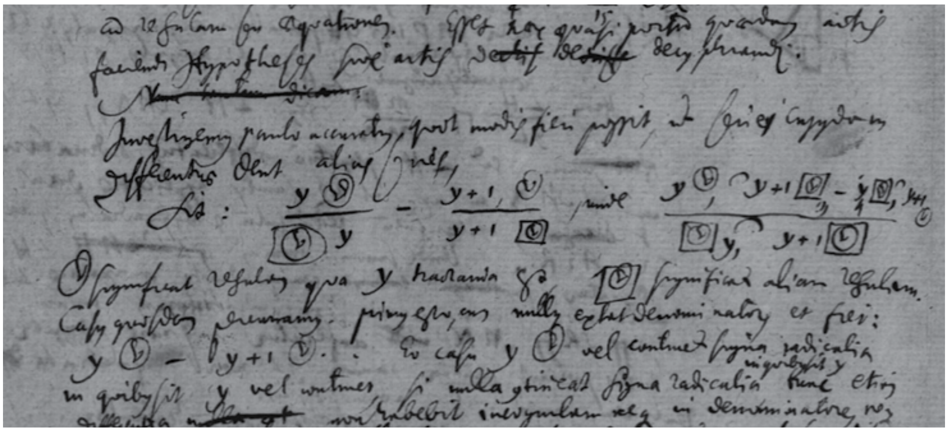
stantem numerum multiplicatam esse vel 1, vel multiplum facti ex denominatoribus duobus proximis, per numerum respondentem, ut 3. 35 etc. nempe:

Sunto duo termini: $\frac{b}{z\textcircled{v}}$ $\frac{b}{z+1,\textcircled{v}}$ erit b $\frac{z+1,\textcircled{v},-z\textcircled{v}}{z\textcircled{v},,\frown z+1,\textcircled{v}} \sqcap \frac{1}{16z^2-16z+3}$. Quod si nominator etiam sit inconstans, erunt termini $\frac{z\boxed{\textcircled{v}}}{z\textcircled{v}}$. $\frac{z+\beta,\boxed{\textcircled{v}}}{z+\beta,\textcircled{v}}$ et fiet:

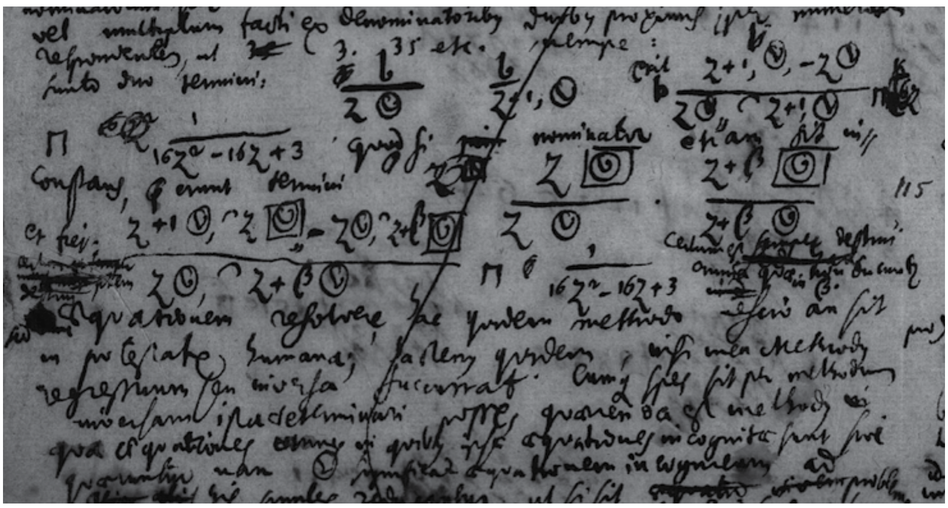
$$\frac{z+1,\textcircled{v},\frown z\boxed{\textcircled{v}},,-z\textcircled{v},\frown z+\beta,\boxed{\textcircled{v}}}{z\textcircled{v},\frown z+\beta,\textcircled{v}} \sqcap \frac{1}{16z^2-16z+3}.$$

Certum est semper destrui omnia quae non ducuntur in β. Sed hanc aequationem

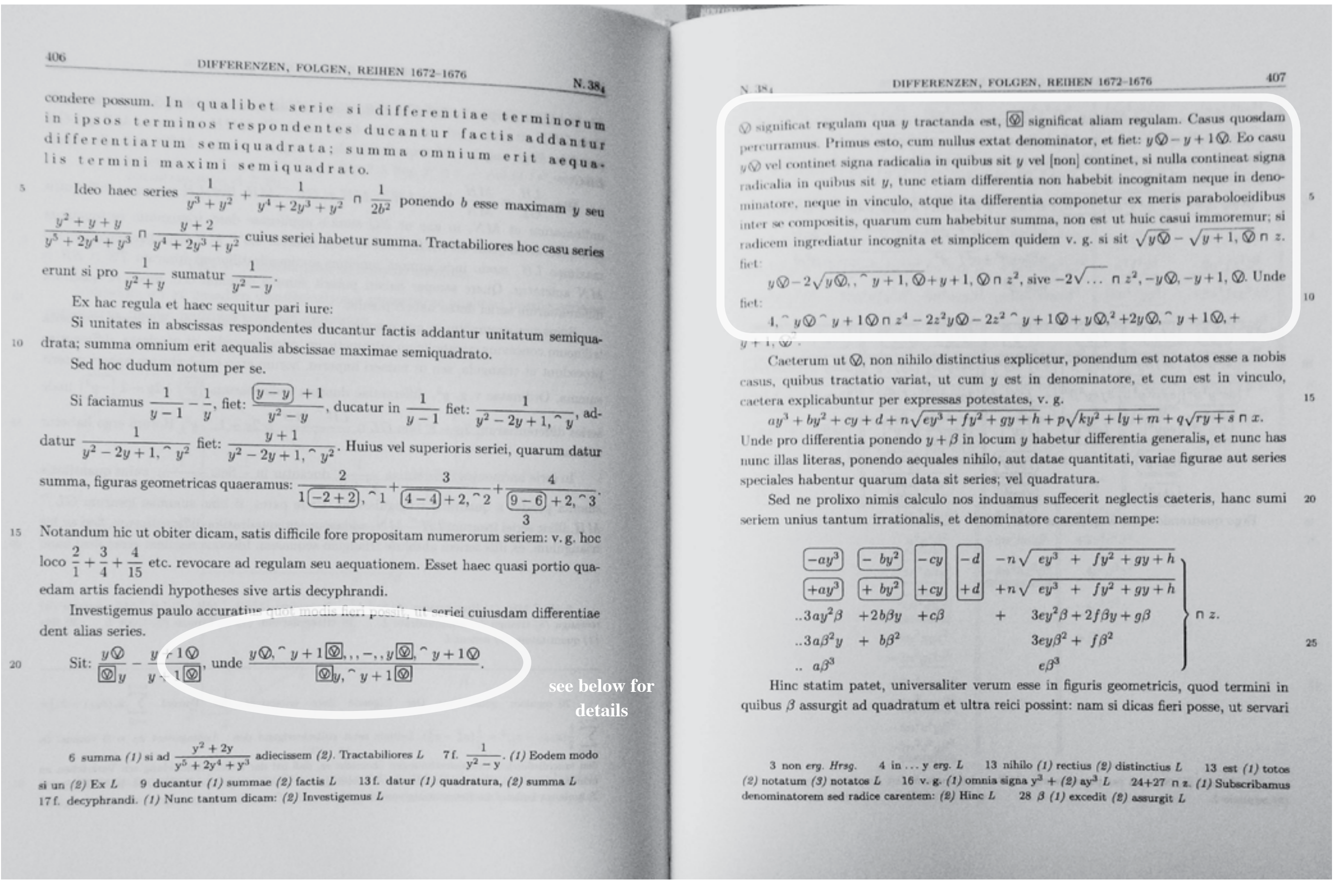
$\textcircled{v}$ LEIBNIZIAN ENCIRCLED V SIGN, $\boxed{\textcircled{v}}$ LEIBNIZIAN BOXED ENCIRCLED V SIGN
The following figures show manuscript specimen of the two characters.
LAA VII-1 p. 527



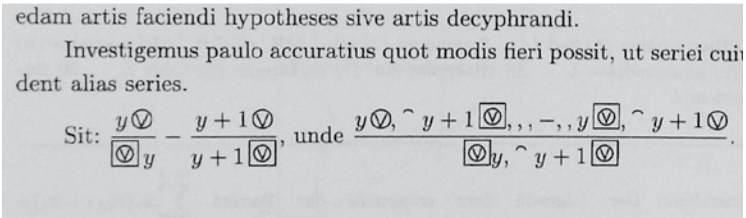
LH 35 V 4,
fol. 6



LH 35 VIII 30,
fol. 115r

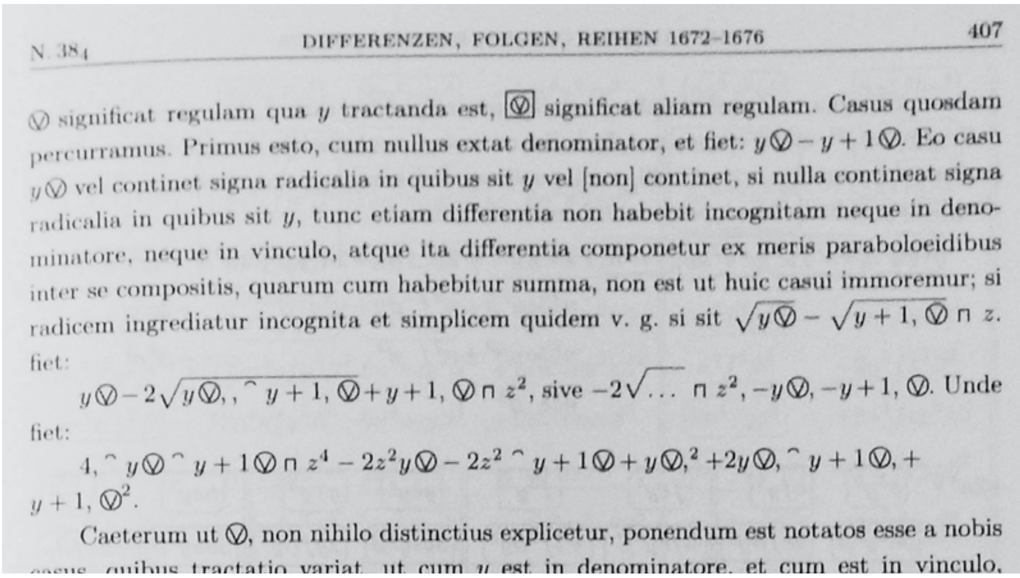


Ⓥ LEIBNIZIAN ENCIRCLED V SIGN, ⊡ LEIBNIZIAN BOXED ENCIRCLED V SIGN
Note that the representation of these characters in the edition is considered unfortunate. The round shape has to resemble a volute, similar as with @ (0040).
LAA VII-3 p. 406-407


edam artis faciendi hypotheses sive artis decyphrandi.

Investigemus paulo accuratius quot modis fieri possit, ut seriei cui
dent alias series.

Sit: $\frac{y\text{Ⓥ}}{\boxed{\text{Ⓥ}}y} - \frac{y+1\text{Ⓥ}}{y+1\boxed{\text{Ⓥ}}}$, unde $\frac{y\text{Ⓥ},\frown y+1\boxed{\text{Ⓥ}},,,-,,y\boxed{\text{Ⓥ}},\frown y+1\text{Ⓥ}}{\boxed{\text{Ⓥ}}y,\frown y+1\boxed{\text{Ⓥ}}}$.


N. 38₄ DIFFERENZEN, FOLGEN, REIHEN 1672–1676 407

Ⓥ significat regulam qua y tractanda est, $\boxed{\text{Ⓥ}}$ significat aliam regulam. Casus quosdam percurramus. Primus esto, cum nullus extat denominator, et fiet: $y\text{Ⓥ} - y + 1\text{Ⓥ}$. Eo casu $y\text{Ⓥ}$ vel continet signa radicalia in quibus sit y vel [non] continet, si nulla contineat signa radicalia in quibus sit y, tunc etiam differentia non habebit incognitam neque in denominatore, neque in vinculo, atque ita differentia componetur ex meris paraboloeidibus inter se compositis, quarum cum habebitur summa, non est ut huic casui immoremur; si radicem ingrediatur incognita et simplicem quidem v. g. si sit $\sqrt{y\text{Ⓥ}} - \sqrt{y+1,\text{Ⓥ}} \sqcap z$. fiet:

$$y\text{Ⓥ} - 2\sqrt{y\text{Ⓥ},,\frown y+1,\text{Ⓥ}} + y + 1,\text{Ⓥ} \sqcap z^2, \text{ sive } -2\sqrt{\ldots} \sqcap z^2, -y\text{Ⓥ}, -y+1,\text{Ⓥ}.$$ Unde fiet:

$$4,\frown y\text{Ⓥ} \frown y + 1\text{Ⓥ} \sqcap z^4 - 2z^2y\text{Ⓥ} - 2z^2 \frown y+1\text{Ⓥ} + y\text{Ⓥ},^2 + 2y\text{Ⓥ},\frown y+1\text{Ⓥ}, + y+1,\text{Ⓥ}^2.$$

Caeterum ut Ⓥ, non nihilo distinctius explicetur, ponendum est notatos esse a nobis casus, quibus tractatio variat, ut cum y est in denominatore, et cum est in vinculo, La Colonne C. du mesme feuïllet, contient l'aplication que j'ay faite de la premiere analogie de M. Leibnits, en ne se servant que de la ligne interrompue - - pour designer le Zero; et de la ligne entiere — pour marquer Un. La continuation de cette colonne est de l'autre coté du mesme feuillet.

Les colonnes D. E. F. G. H. font connoitre les diverses combinaisons qui se forment, lors que la ligne interrompue - - et la ligne entiere — se trouvent une à une; ou jointes deux à deux; ou trois à trois; ou 4. à 4. ou 5. à 5.

Et enfin la colonne I. donne les 64. caracteres ou figures de Fo-hi, arrangez dans l'ordre qu'ils doivent estre suivant la seconde analogie de M. Leibnits, pour marquer la suitte naturelle des nombres, depuis 0. jusques et compris 63. Mais par ce que depuis 31. les figures de cette Colonne I se raportent parfaitement au reste de ceux de la colonne C. je n'ay pas cru necessaire de les repeter.

On conclud donc de cette seconde analogie, qu'il faudroit sept lignes pour aller jusques à 127. Et 8 pour aller jusques à 255. etc. En sorte que si on vouloit exprimer 20000. de nostre arith. en la Binaire, il y faudroit employer suivant la premiere analogie, quinze rangs de lignes; ce qui seroit inevitable depuis 16384. qui s'exprimeroient ainsi, ||||||||||||||| . De maniere que pour connoitre la valeur de la ligne entiere, qui est à gauche, il faut bien sçavoir combien elle precede de lignes interrompues, ou de Zeros.

- - BROKEN EMDASH
LAA III-9, p. 606

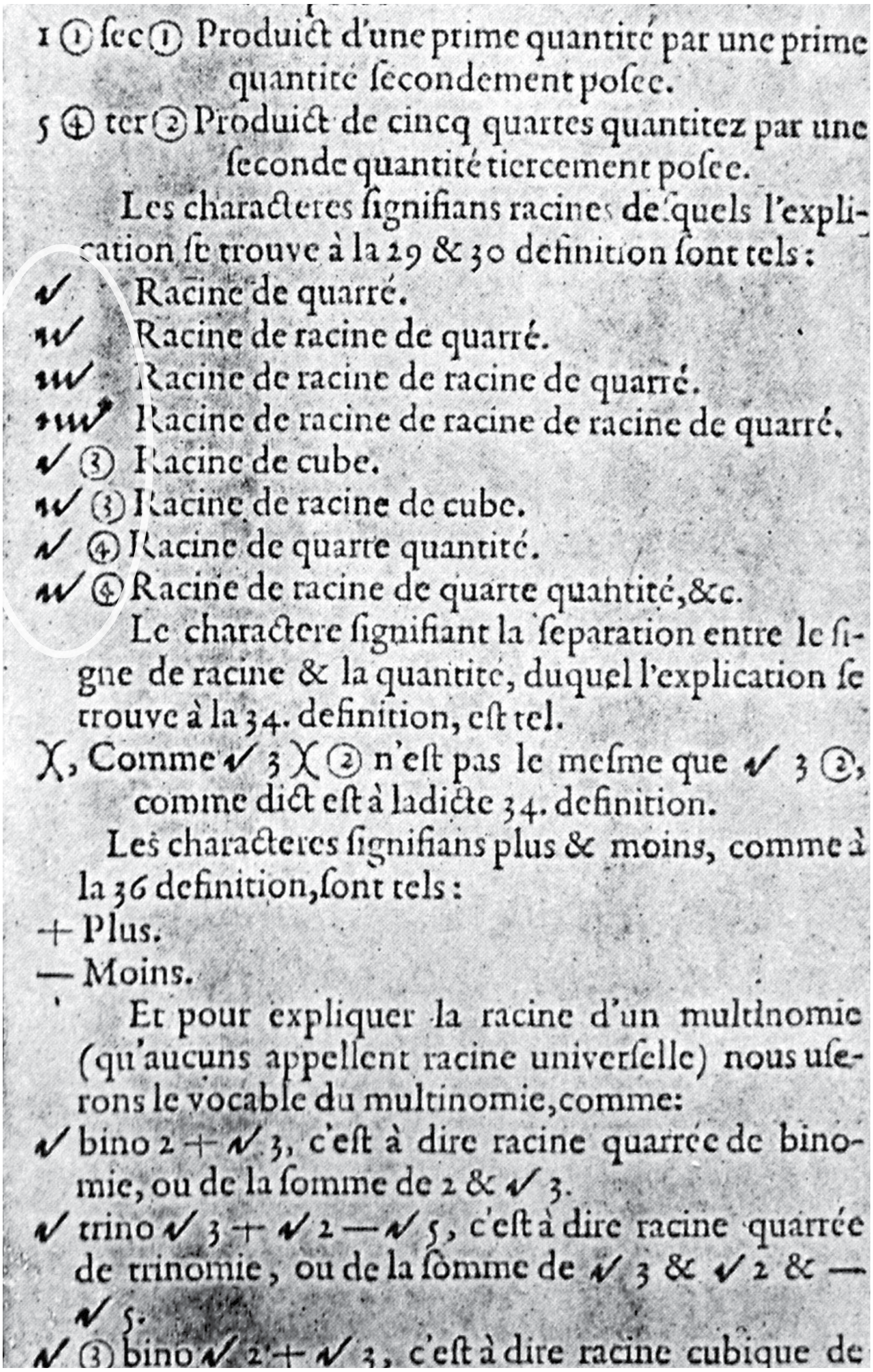
deux sortes de caracteres, qu'on assemble de six en six de toutes les manieres possibles, l'ordre naturel des combinaisons (de quelque façon qu'on s'y veuille prendre) les arrangera comme ils se trouvent dans le P. Martini; d'où l'on peut conclurre, que cet arrangem^{t} si bien suivi, ne procede d'autre chose, que de ce que n'y ayant que deux sortes de lignes à employer, il se faut necessairement servir, pour la combinaison de la progression double, qui estant la mesme qu'il faut employer dans l'arithmetique Binaire, il ne se faut pas étonner que le tout se raporte si parfaitement.

Cela est si vray que si les Figures de Fo-hi, estoient composées de trois sortes de Lignes, comme celles cy - - — -+- et qu'on en deut mettre pareillement six en chaque figure, dez que l'ordre de ces lignes aura esté fixé comme elles sont icy; si on cherche ensuitte toutes les Combinaisons selon la methode qu'il faut employer pour trois choses dissemblables, on trouvera 729 figures differentes de six lignes chacune; sans qu'il paroisse d'aucune necessité que cela se puisse raporter à aucune sorte d'arithmetique. Et cependant si on se propose d'apliquer l'arithm. ternaire à ces trois sortes de lignes, et que la premiere interrompue - - designe le Zero. La suivante entiere — Un. Et la trois[i]esme croisée par le milieu -+- deux, on trouvera en descendant de haut en bas, ou en remontant de bas en haut, que toute la suite des figures, donnera exactem^{t} toute la suitte des nombres depuis 0. Jusques et compris 728.

Il n'est donc pas facile à mon sens de determiner certainement, si les 64 caracteres de Fo-hi, doivent estre regardez comme une simple Combinaison, ou comme une arith. binaire complette, puis qu'il y a un si parfait raport entre ces deux choses; sur tout si

- - BROKEN EMDASH, -+- CROSSED EMDASH
LAA III-9, p. 610
These two characters group with the existent EMDASH (2014) in lines 18 and 24 of this sample.

see also next page



1 ① ſec ① Produict d'une prime quantité par une prime quantité ſecondement poſee.

5 ④ ter ② Produict de cincq quartes quantitez par une ſeconde quantité tiercement poſee.

Les characteres ſignifians racines deſquels l'explication ſe trouve à la 29 & 30 definition ſont tels:

√ Racine de quarré.

√√ Racine de racine de quarré.

√√√ Racine de racine de racine de quarré.

√√√√ Racine de racine de racine de racine de quarré.

√ ③ Racine de cube.

√√ ③ Racine de racine de cube.

√ ④ Racine de quarte quantité.

√√ ④ Racine de racine de quarte quantité, &c.

Le charactere ſignifiant la ſeparation entre le ſigne de racine & la quantité, duquel l'explication ſe trouve à la 34. definition, eſt tel.

Ⅹ, Comme √ 3 Ⅹ ② n'eſt pas le meſme que √ 3 ②, comme dict eſt à ladicte 34. definition.

Les characteres ſignifians plus & moins, comme à la 36 definition, ſont tels:

\+ Plus.

— Moins.

Et pour expliquer la racine d'un multinomie (qu'aucuns appellent racine univerſelle) nous uſerons le vocable du multinomie, comme:

√ bino 2 + √ 3, c'eſt à dire racine quarrée de binomie, ou de la ſomme de 2 & √ 3.

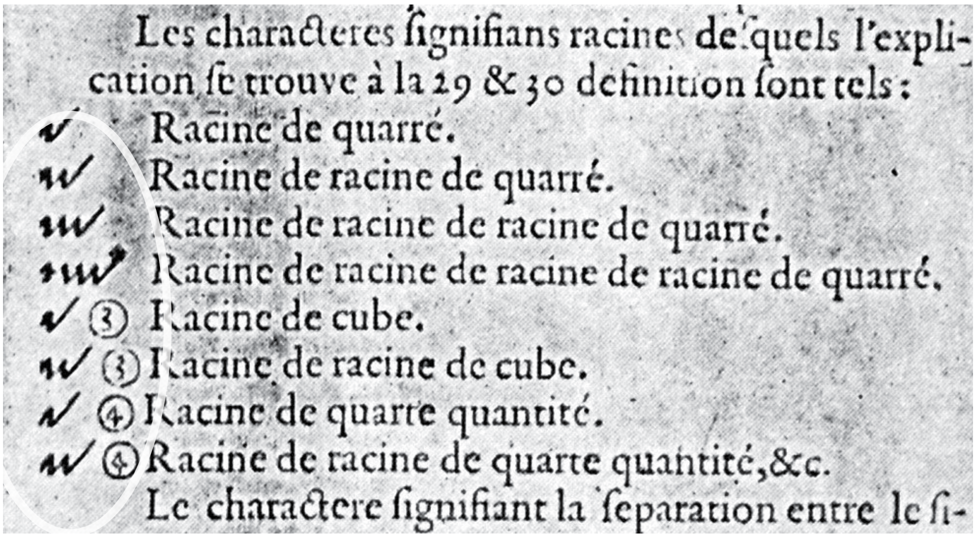
√ trino √ 3 + √ 2 — √ 5, c'eſt à dire racine quarrée de trinomie, ou de la ſomme de √ 3 & √ 2 & — √ 5.

√ ③ bino √ 2 + √ 3, c'eſt à dire racine cubique de

RADIX SIGN 1, RADIX SIGN 2, RADIX SIGN 3

These characters can be seen related to the established radix symbol √ (221A).

Simon Stevin, L'arithmétique in Œuvres mathématiques, 1634 (after Cajori)

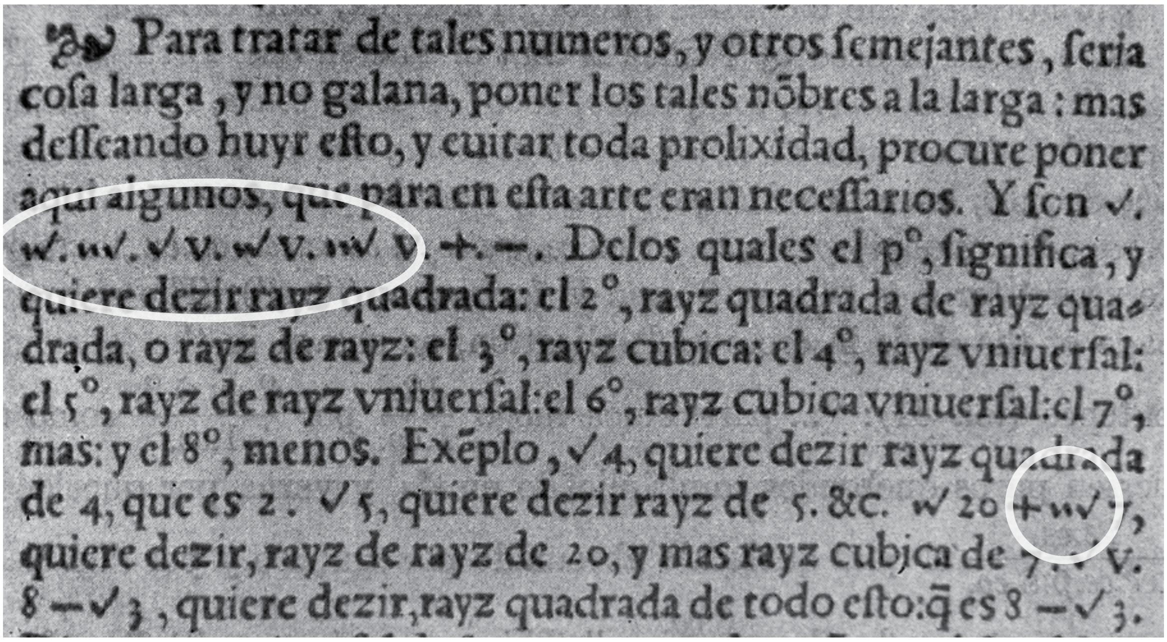


Les characteres signifians racines desquels l'explication se trouve à la 29 & 30 definition sont tels:
√ Racine de quarré.
ʌ√ Racine de racine de quarré.
ʌʌ√ Racine de racine de racine de quarré.
ʌʌʌ√ Racine de racine de racine de racine de quarré.
√ ③ Racine de cube.
ʌ√ ③ Racine de racine de cube.
√ ④ Racine de quarte quantité.
ʌ√ ④ Racine de racine de quarte quantité, &c.
Le charactere signifiant la separation entre le si-

Simon Stevin, L'arithmétique in Œvres mathématiques, 1634 (after Cajori)

The number of ascending lines indicates how often an operation of root determination is performed on an expression. In the Stevin example the combination with an encircled number indicates, which type of root is meant. If there is no such number, the square root is to be considered. For example, the combinations denote the following:

ʌ√ square root of square root, which corresponds to the forth root;
ʌʌ√ square root of square root of square root, which corresponds to the eighth root;
ʌʌʌ√ square root of square root of square root of square root, which corresponds to the sixteenth root;
ʌ√ ③ cubic root of cubic root, which corresponds to the ninth root;
ʌ√ ④ forth root of forth root, which corresponds to the sixteenth root.



Para tratar de tales numeros, y otros semejantes, seria cosa larga, y no galana, poner los tales nõbres a la larga: mas desseando huyr esto, y euitar toda prolixidad, procure poner aqui algunos, que para en esta arte eran necessarios. Y son √. ʌ√. ʌʌ√. √v. ʌ√v. ʌʌ√v. +. —. Delos quales el p°, significa, y quiere dezir rayz quadrada: el 2°, rayz quadrada de rayz quadrada, o rayz de rayz: el 3°, rayz cubica: el 4°, rayz vniuersal: el 5°, rayz de rayz vniuersal: el 6°, rayz cubica vniuersal: el 7°, mas: y el 8°, menos. Exẽplo, √4, quiere dezir rayz quadrada de 4, que es 2. √5, quiere dezir rayz de 5. &c. ʌ√20 + ʌʌ√7, quiere dezir, rayz de rayz de 20, y mas rayz cubica de 7. √v. 8 — √3, quiere dezir, rayz quadrada de todo esto: q̃ es 8 — √3.

ʌ√ RADIX SIGN 1, ʌʌ√ RADIX SIGN 2, ʌʌʌ√ RADIX SIGN 3
Marco Aurel, Arithmetica algebratica, 1552 (after Cajori)

tantum ponendo a^2t minorem quam q^3.
Quae aequatio ponatur esse eadem cum aequatione 4. erit in aequatione 4[:]

$$P \overset{(43)}{\sqcap} 0. \quad \text{et} \quad Q \overset{(44)}{\sqcap} -\!\textbf{.}\, Q \sqcap -1\textbf{.}\, \frac{3q^2}{a}.$$

Nam per hoc signum **.**Q. semper designo ipsam molem affirmativam, ideoque per -**.** designo signum affectionis cuiusque quantitatis, quod ex signis calculi non semper apparet. (Quando autem quantitas ipsa est quadratica aut alia parium dimensionum, ut

. BOLD PERIOD
The glyphic representation being optically close to BULLET (2022) and BULLET OPERATOR (2219), this character is defined by its position on the base line, like PERIOD (002E).
LAA VII-2 P. 609; VII-8 p. 18, 19 (below)

et … il *(a)* faut *(b)* vaut *L* 7 j'avois *(1)* $\frac{1}{2}$ *(2)* $\frac{3}{4}$ *L* 10 joue *(1)* trois jeux *(2)* à trois partis *L*
11–19,1 suite; *(1)* item qv'il en gagne *(2)* | | | item qv'il gagne *(a)* **.** | | | vel | **.** | | vel | | **.** | vel *(b)* | | **.** | *(aa)* vel | **.** | | vel **.** | | | *(bb)* ou *L*

qu'il gagne | | **.** | ou | **.** | | ou **.** | | |. Et s'il gagne en 5 jeux il peut gagner ainsi: **. .** | | | | **. .** | | | | **. .** | **.** | | **.** | **.** | **.** | | | **.** | **.** |. *Ecce modos omnes quibus vinci potest. Et notandum hoc combinationis plane singularis genus. Nunc ut ex*

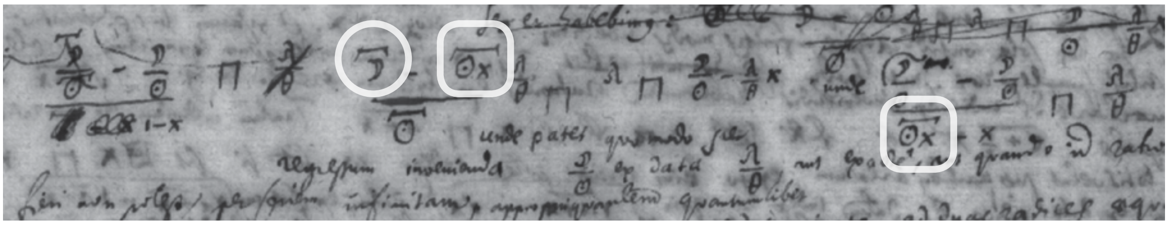
$$1\,\widehat{}\, + b\ 2\,\widehat{}\, + c\,x\ 3\,\widehat{}\, + d\,x^2 - 4\frac{l\lambda}{\theta}x^3 \qquad \frac{☽}{⊙}$$

$$\text{unde } \lambda \sqcap \frac{-\frac{f\lambda}{\theta} \quad -\frac{g\lambda}{\theta}. \quad -\frac{h\lambda}{\theta}..}{1g + 2hx + 3lx^2} \sqcap y - \frac{\lambda}{\theta}x.$$ Fiet ☽ determinata ad 2

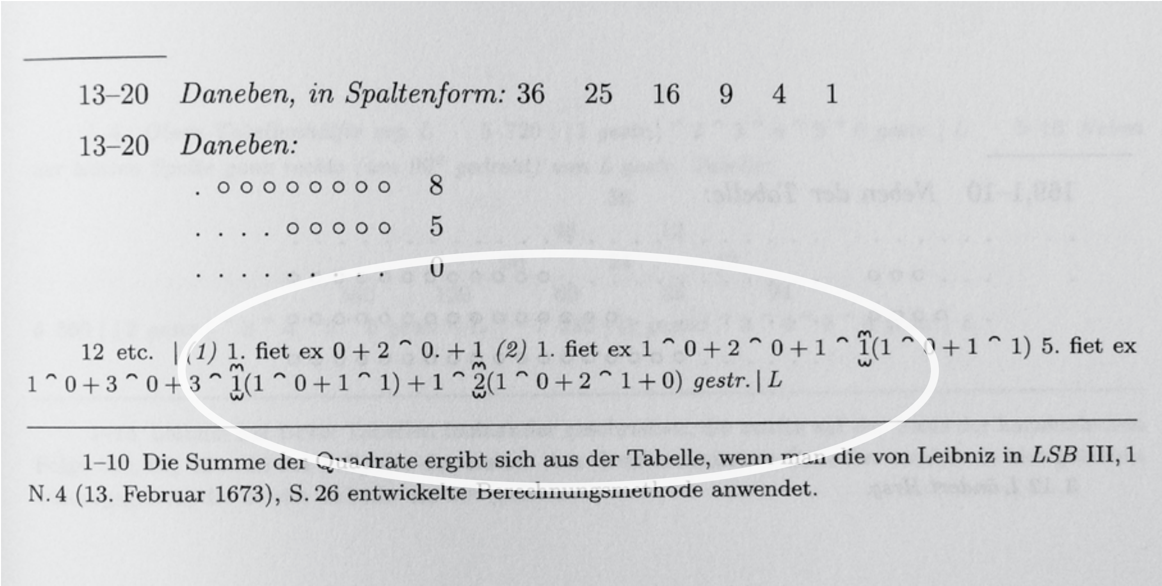
radices sit signum $\overline{☽}$ et ⊙ determ. sit sign. $\overline{⊙}$. $\frac{\overline{☽} - \overline{⊙x}\frac{\lambda}{\theta}}{⊙} \sqcap \lambda \sqcap \frac{☽}{⊙} - \frac{\lambda}{\theta}x.$ unde $\frac{\frac{\overline{☽}}{\overline{⊙}} - \frac{☽}{⊙}}{\overline{⊙x} - x} \sqcap$
$\frac{\lambda}{\theta}$ unde patet quomodo per regressum invenienda $\frac{☽}{⊙}$ ex data $\frac{\lambda}{\theta}$, aut exacte, aut quando id rationaliter fieri non potest, per seriem infinitam, appropinquantem quantumlibet.

Aequationes notabiles: $a + bx + cx^2 + dx^3$ etc. ⊓ ☽ determinata ad duas radices aequales dat: $b + 2cx + 3dx^2$ etc. ⊓ $\overline{☽}$. Ducatur hoc in illud fiet $☽\overline{☽}$. Ajo $\int \overline{☽\overline{☽}}$ id est $\int \overline{☽ d ☽} \sqcap \frac{☽^2}{2}$. Itaque

◌̅ COMBINING OVERLINE WITH TERMINALS
◌͞ COMBINING DOUBLE-WIDE OVERLINE WITH TERMINALS
LAA VII-5 p. 587



◌̅ COMBINING OVERLINE WITH TERMINALS, ◌͞ COMBINING DOUBLE-WIDE OVERLINE WITH TERMINALS – LAA VII-5 p. 587.
The manuscript of this text (below) shows the different widths of the two characters.



13–20 *Daneben, in Spaltenform:* 36 25 16 9 4 1

13–20 *Daneben:*

. o o o o o o o o 8

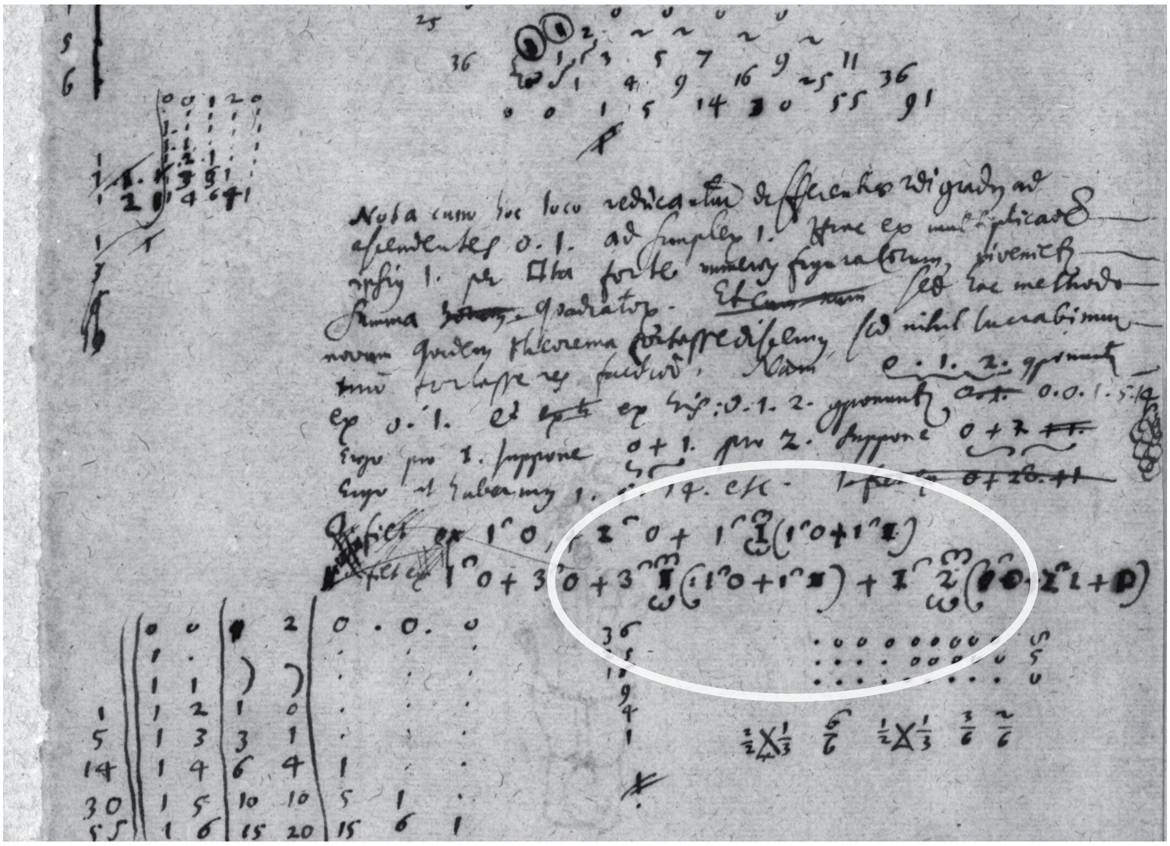
. . . . o o o o o 5

. . . . 0

12 etc. | *(1)* 1. fiet ex 0 + 2 ⌢ 0. + 1 *(2)* 1. fiet ex 1 ⌢ 0 + 2 ⌢ 0 + 1 ⌢ 1(1 ⌢ 0 + 1 ⌢ 1) 5. fiet ex 1 ⌢ 0 + 3 ⌢ 0 + 3 ⌢ 1(1 ⌢ 0 + 1 ⌢ 1) + 1 ⌢ 2(1 ⌢ 0 + 2 ⌢ 1 + 0) *gestr.* | *L*

1–10 Die Summe der Quadrate ergibt sich aus der Tabelle, wenn man die von Leibniz in *LSB* III, 1 N. 4 (13. Februar 1673), S. 26 entwickelte Berechnungsmethode anwendet.

◌ COMBINING FACTOR MARK
LAA VII-3 p. 167



◌ COMBINING FACTOR MARK
LH 35 XII 1 fol. 138r

Esto jam aequatio ad sectionem Conicam $y \sqcap \sqrt{2ax \mp \frac{r}{t}x^2}\, B$. Primum ponendo $x \sqcap v$ videamus quod eventurum sit, nam si res non succedit, valorem mutabimus: At y esto $\sqcap w + \varphi + k - \psi$. fietque $B + \psi \sqcap A$. et quadrando $B^2 + 2\psi B + \psi^2 \sqcap A^2$, unde ordinando:

$$\begin{array}{llllll} +1 & v^2 & -e & v & -\lambda^2 & \sqcap \ -2\psi B. \\ \mp\frac{r}{t} & & -2h & & -eh & \\ \overset{\frown}{\odot} & & +2a & & +h^2 & \\ & & \overset{\frown}{☽} & & +2\varphi k & \\ & & & & +k^2 & \\ & & & & +\psi^2 & \\ & & & & \overset{\frown}{☿} & \end{array}$$

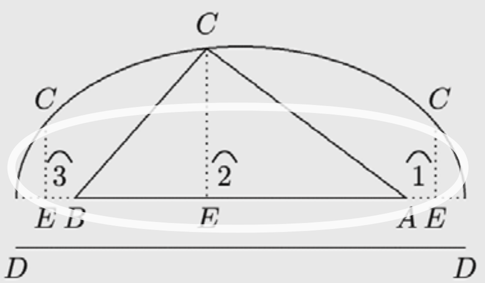
et rursus quadrando atque ordinando

$$\begin{array}{lllll} \odot^2 v^4 & +2\odot ☽ v^3 & +2\odot ☿ \ v^2 & [+] 2 ☽ ☿ \ v & +☿^2 \sqcap 0. \\ & & + ☽^2 & \ldots \ - 8\psi^2 a \ldots & \\ & & \mp 4\psi^2 \frac{r}{t} \ldots & & \end{array}$$

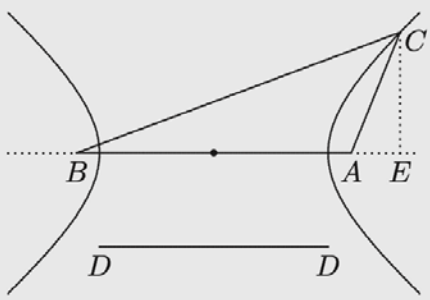
◌⏜ COMBINING HORIZONTAL PARANTHESIS
LAA VII-7 p. 329

S c h e d i a s m a d e F o c i s C o n i c a r u m , Octob. 1674

Invenire locum, unde ductae ad data duo puncta rectae datam faciant summam, aut dato differant intervallo. Quod ita reperiemus:



[Fig. 1]



[Fig. 2]

Datorum punctorum distantia AB appelletur, a, data summa vel datum intervallum, b. Ex puncto loci quaesiti assumto, C, demittatur perpendicularis in AB productam si opus est. CE, appellanda y, et AE vocetur x. Erit $AC^2 \sqcap y^2 + x^2$, porro $\overset{\frown}{1}$ $EB \sqcap AB + AE$, vel $\overset{\frown}{2}$ $AB - AE$, vel $\overset{\frown}{3}$, $AE - AB$, ergo $EB \sqcap (\pm)a(\pm)x$, ejusque quadratum, $EB^2 \sqcap$

◌⏜ COMBINING HORIZONTAL PARANTHESIS
LAA VII-7 p. 357 – The glyphs applied in those two samples are not ideal.

122 DIFFERENZEN, FOLGEN, REIHEN 1672–1676 N. 8

z terminus primus. $z-a$ terminus 2^{dus}. $z-a-b$ terminus tertius. $z=\frac{c}{d}$. $z-a=\frac{c}{d+e}$. $z-a-b=\frac{c}{d+e+f}$. Ergo $\frac{c}{d}-\frac{c}{d+e}=a$. $\frac{c}{d}\times\frac{c}{d+e}$ [=] $\frac{\cancel{cd}+ce-\cancel{cd}}{dd+de}$. Ergo $a=\frac{ce}{dd+de}$. $b=\frac{c}{\llcorner d+e}\overset{-}{\times}\frac{c}{d+e+f\lrcorner}$ $\frac{\cancel{cd}+\cancel{ce}+cf-\cancel{cd}-\cancel{ce}}{(\llcorner d+e\lrcorner Q.)dd+ee+2de+df+ef}=b$. $\frac{\cancel{c}}{dd+de}\times\frac{\cancel{c}f}{\llcorner d+e\lrcorner Q.+df+ef}=\frac{\cancel{c}}{d}\times\frac{\cancel{c}}{d+e+f}$. $\frac{d+e+f}{\cancel{d}}=\frac{\llcorner d+e\lrcorner Q.+df+ef}{\cancel{d}df}$.

⸗ COMBINING DOUBLE-WIDE SLASH

This character is similar to COMBINING LONG SOLIDUS OVERLAY (0338). Its function is to create a strike-through mark for *two* neighbouring base characters, so it is supposed to work in the same way as e.g. the characters 035C to 0362.

LAA VII-3 p.122

[*Teil 1*]

Fig. 3.

$\cancel{AN}=AE=AK$

$AI=ID=IG=A\beta=\gamma M=\frac{AG}{2}$ modo β sit in linea DE.

$IB=AZ=B\gamma=\gamma\delta=\frac{CG}{2}=ZG$

$BI=AZ\quad B\alpha=BE\quad \beta\alpha=\beta E\quad A\beta=D\beta$

NB. recta $A\alpha$ non incidit in rectam AZ.

⸗ COMBINING DOUBLE-WIDE SLASH

LAA VII-4 p. 409

1 puncta *(1)* D(D) *(2)* N(N) *L* 5 f. et A(A). *(1)* | Sane *erg.* | Esto NB ⊓ b. N(N) ⊓ n. fiet: *(a)* NB vel N(B) ⊓ b + $\frac{\text{yn}}{\text{b}}$ sive b + ☘n *(aa)* $\frac{\text{e}}{\text{n}}$ ⊓ $\frac{\text{C(E)}}{}$ *(bb)* $\frac{\text{E(E)}}{\text{N(N)}}$ ⊓ $\frac{\text{C(E)}}{\text{CN}}$ qvae CN velut data appelletur c, et fiet C(E) ⊓ $\frac{\text{E(E)} \frown \text{c}}{\text{N(N)}}$ datur porro et CB ⊓ ∓b ± c. Datur et NE ⊓ ν unde scilicet calculum incipimus. datur ergo et AB. Nam est $\frac{\text{AB}}{\text{NE} ⊓ \nu}$ ⊓ $\frac{\text{CB ⊓ ∓b ± c}}{\text{c ⊓ NC}}$. Ergo AB ⊓ ∓b±c, ⌢ *(aaa)* $\frac{\text{n}}{\text{c}}$ *(bbb)* $\frac{\text{N(N)}}{\text{C}}$ sit B(B) ⊓ β. *(aaaa)* | NL ⊓ *streicht Hrsg.* | λ *(bbbb)* | E(E) ⊓ λ. *streicht Hrsg.* | Rectius ita | Porro cum Triangula NL(N) et CEN sunt similia erit $\frac{\text{N(N)}}{\text{N(L)}}$ ⊓ $\frac{\text{NC} ⊓ \cancel{\text{c}}}{\frac{\text{E(E)} \frown \cancel{\text{c}}}{\text{N(N)}}}$ *erg. u. gestr.* | *(2)* omnes *L* 6 f. cognitas, *(1)* | erit CE ad EN *streicht Hrsg.* | (hic negliguntur signa includentia ob infinitas parvitates) ut NL ⊓ E(E) ⊓ λ ad (N)L, seu λ ad $\sqrt{\text{n}^2-\lambda^2}$ sive EC ⊓ $\frac{\lambda\text{n}}{\sqrt{\text{n}^2-\lambda^2}}$ Jam alias EC ⊓ $\sqrt{\text{c}^2-\text{e}^2}$ *(2)* (hic … parvitates) | : *erg. Hrsg.* | EC ⊓ $\sqrt{\text{c}^2-\text{e}^2}$ *L* 8 NL ⊓ λ *L ändert Hrsg.* 9 $\frac{\text{b}-\text{c}}{\text{b}}$ *(1)* demonstravit jam Hugenius:

☘ CLOVERLEAF SIGN

LAA VII-5 P. 136

4.g) Superscript characters

auf der ein Punkt D mit der Ordinate y liegt. Wird an diese Kurve eine Tangente angelegt, die jene eben in D berührt, so bezeichnet Leibniz den Tangentenabschnitt zu y (also den Abstand zwischen D und dem Schnittpunkt der Tangente mit der x-Achse) als $y^{\boxed{t}}$. Die Abszisse des Punktes D bezeichnet er auf analoge Weise mit $y^{\boxed{x}}$. Um Platz für einen etwaigen Exponenten zu schaffen, verwendet er hierfür auch die Schreibweise $\overset{\boxed{x}}{y}$. Ob Leibniz diese Notation — die man modern gesprochen als eine Schreibweise für Funktionen bezeichnen könnte und die er selbst als besonders leistungsfähig einschätzt — in seinem weiteren Werk verwendet, ist noch nicht geklärt. Beispiele:

$EG \sqcap FG^{\boxed{t}}$

$DN \sqcap e^{\boxed{x}} - y^{\boxed{x}}$

$DG \sqcap \sqrt{e^2 - 2ey + y^2 + \overset{\boxed{x}}{e}{}^2 - 2\,\overset{\boxed{x}}{e}, \overset{\boxed{x}}{y} + \overset{\boxed{x}}{y}{}^2}$ (alle aus N. 66)

Zu den im Band auftretenden mathematischen Symbolen siehe auch S. 674 f.

$^{\boxed{t}}$ SUPERSCRIPT ENCLOSED SMALL T SIGN
$^{\boxed{x}}$ SUPERSCRIPT ENCLOSED SMALL X SIGN
LAA VII-7 p. LIII

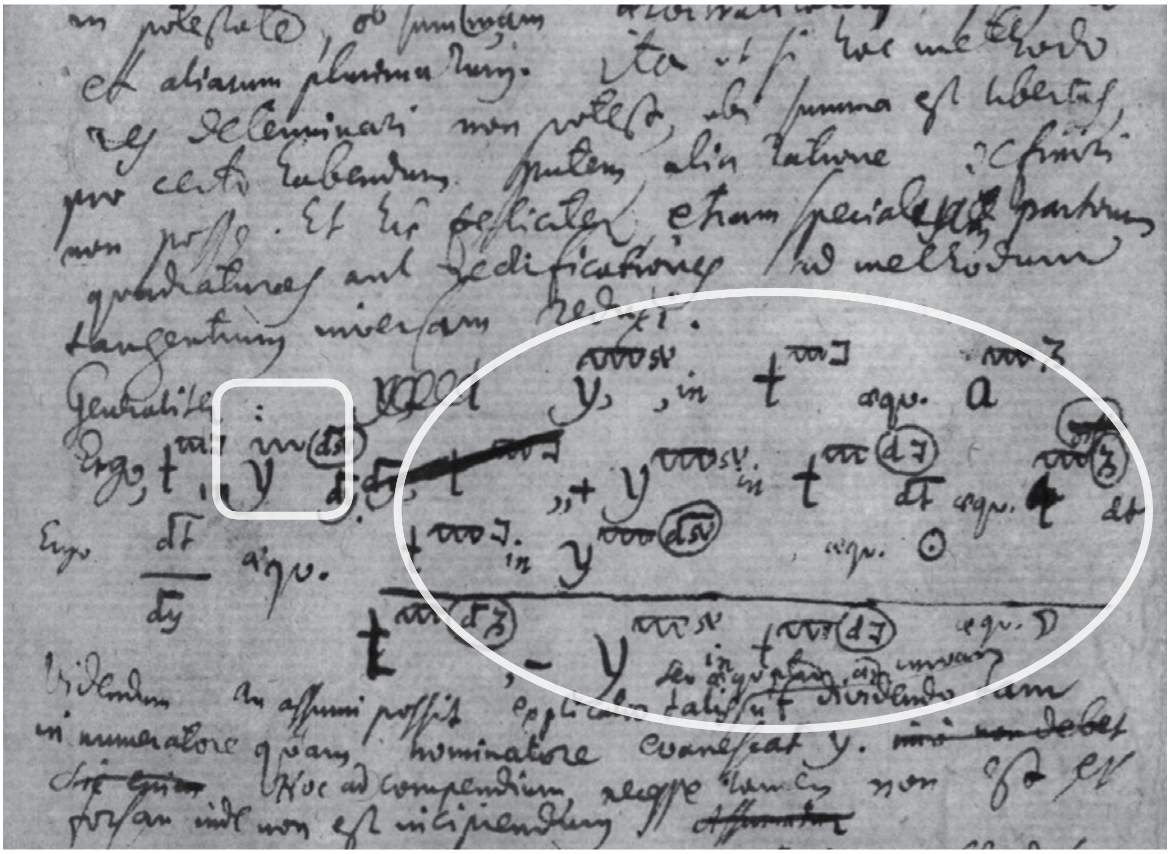
3 $FG^{\boxed{t}}$: Gemeint ist der Tangentenabschnitt zu FG. Die Notation steht also für den Abstand des Berührpunktes einer Tangente von ihrem Schnittpunkt mit der Achse. Zunächst hatte Leibniz diese Größe als $FG^{\boxed{z}}$ bezeichnet, das z dann aber durch t ersetzt. Vorstufen dieser Notation finden sich in den gelöschten Varianten zu S. 595 Z. 4 f. 3 f. $FG \sqcap \frac{\omega - y}{v}, \frac{1^{\boxed{t}}}{1}$: Dies impliziert $FG^{\boxed{t}} = 1^{\boxed{t}}$, was im Allgemeinen nicht zutrifft. 6 $e^{\boxed{x}}$: Die Notation steht für die zur Ordinate e (also FG) gehörende Abszisse. Diese Ausdrucksweise hält Leibniz für sehr leistungsfähig. 7 $\overset{\boxed{x}}{e}{}^2$: Leibniz verändert ein weiteres Mal die Notation. Es liegt also eine Entwicklungslinie vor — von den verworfenen Formen $x^{\textcircled{z}}$ und $\frac{z}{x}$ über $x^{\boxed{z}}$ bzw. $x^{\boxed{t}}$ bis zu einer Schreibweise, die dasselbe als $\overset{\boxed{t}}{x}$ ausdrücken würde.

$^{\boxed{t}}$ SUPERSCRIPT ENCLOSED SMALL T SIGN
$^{\boxed{x}}$ SUPERSCRIPT ENCLOSED SMALL X SIGN
$^{\boxed{z}}$ SUPERSCRIPT ENCLOSED SMALL Z SIGN
$^{\textcircled{z}}$ SUPERSCRIPT ENCIRCLED SMALL Z SIGN
LAA VII-7 p. 596

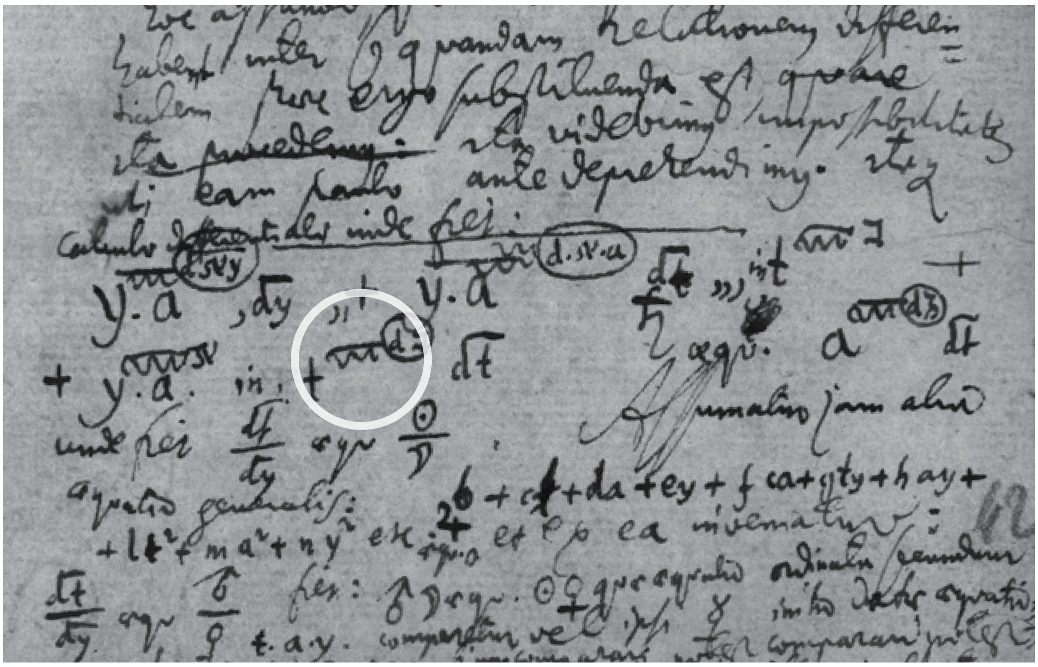
Haec maximi momenti. Si $h = 1$ fit g integer posito m integro. Sed hinc nihil lucramur. Si faciamus $v = \boxed{n} \overline{e + fz^h}$ fiet $v^{1:n} - e = fz^h$ et $v^{\overline{1:n}-1} dv : nf = hz^{h-1} dz$ et $z = \overline{v^{1:n} - e, : f}^{\boxed{1:h}}$ et $dz = \dfrac{dv}{hnf} v^{\overline{1:n}-1}$ in $\overline{v^{1:n} - e, : f}^{\boxed{\frac{1-h}{h}}}$ et fit $\int dz\, z^m \overline{e + fz^h}^n\ \overline{b + dz^c}^r$ et $dz \cdot z^m e + fz^h$ etc. $= \dfrac{dv}{hnf} v^{1:n}\ (-1)$ in $\overline{v^{1:n} - e, : f}^{\boxed{\frac{1-h+m}{h}}}$ in (v) in $\overline{b + d \dfrac{v^{1:n} - e}{f}^{\boxed{c}}}^r$, ita revera, posito $\dfrac{1 - h + m}{h}$ esse integrum, obtenta est depressio. Si h sit 1, quantitate sub irrationali contenta resoluta in plures divisores, et unum ex his irrationalem ponendo v, habetur depressio.

12 integer |posito m integro *erg.*| (*1*) et hac methodo licebit nodere et pro pluribus invicem ducere unus. Si qvaeratur $\odot = \int dzz^m\, \overline{\overline{e + fz} : \overline{b + dz} : \overline{k + iz}}^{\boxed{n}}$ non eos reducitur ad $\odot = \int, \dfrac{w - e^{\boxed{g}}}{f} \omega^n$ (*2*) Sed hinc *LiA*[1] 16 depressio, (*1*) imo generaliter (*2*) si h fiet, (*3*) si h sit *LiA*[1] 16 1, (*1*) adhibitis qvotcunqve (*2*) qvantitate *LiA*[1]

🄶 SUPERSCRIPT ENCLOSED SMALL G SIGN
🄽 SUPERSCRIPT ENCLOSED SMALL N SIGN
LAA III-2 p. 94

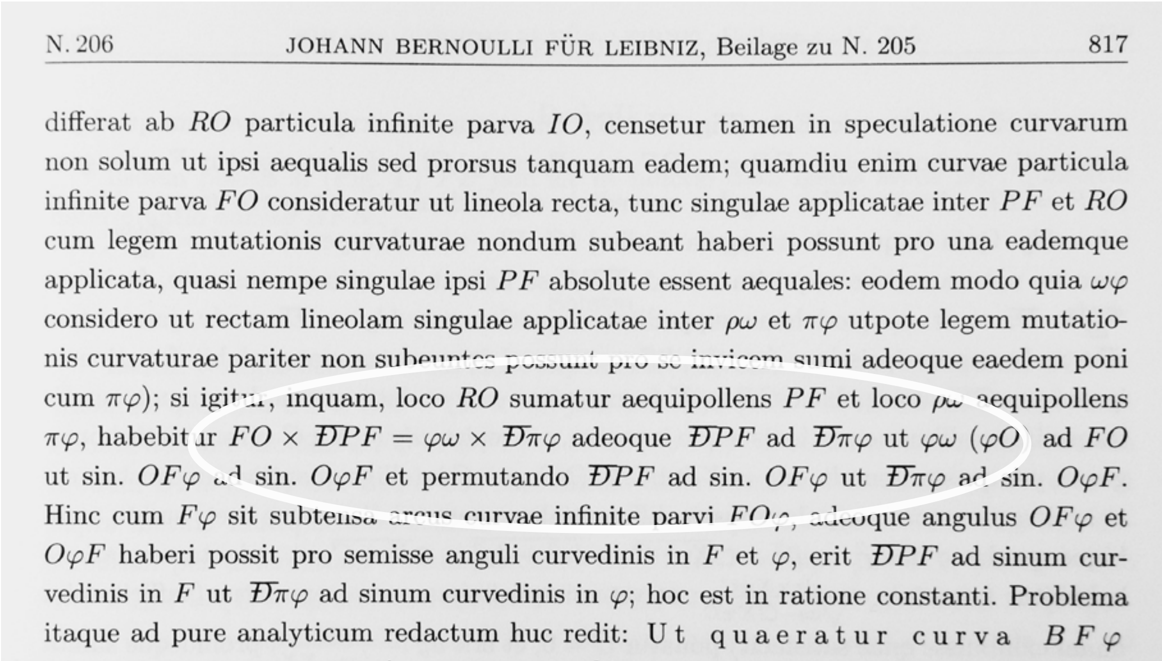


ᵚ SUPERSCRIPT WAVE , ᵚ̄ SUPERSCRIPT WAVE WITH TOP LINE
LH 35 7 I, fol. 39r. *The edition of this manuscript is currently in progress*.



ᵚ̄ SUPERSCRIPT WAVE WITH TOP LINE
LH 35 7 I, fol. 41v. *The edition of this manuscript is currently in progress*.

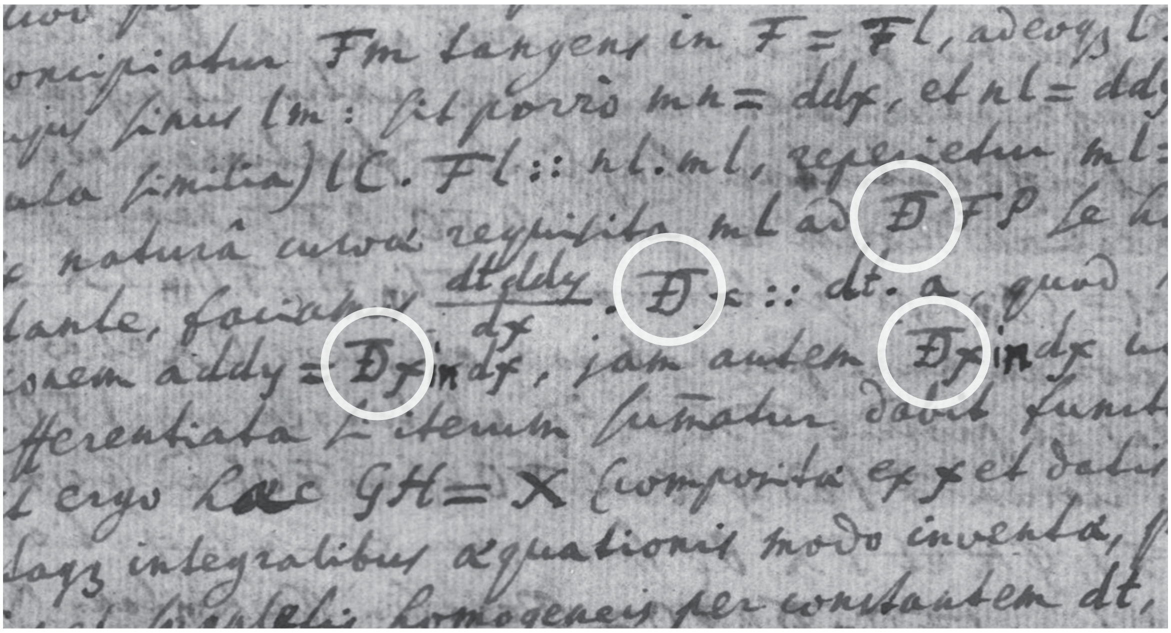
4.h) Letterlike symbols



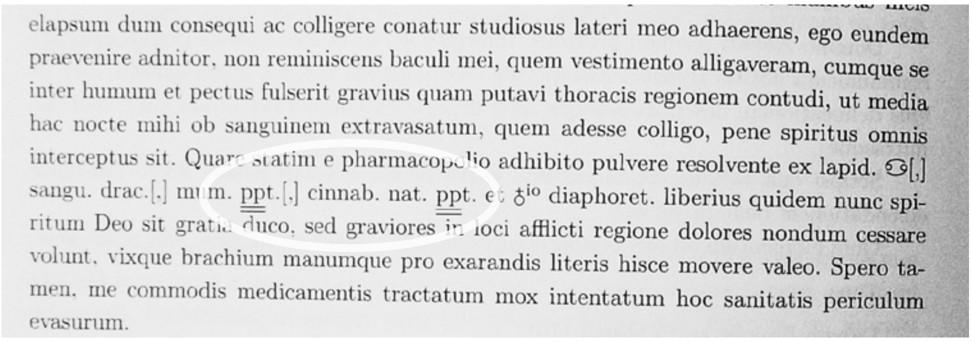
N. 206 JOHANN BERNOULLI FÜR LEIBNIZ, Beilage zu N. 205 817

differat ab RO particula infinite parva IO, censetur tamen in speculatione curvarum non solum ut ipsi aequalis sed prorsus tanquam eadem; quamdiu enim curvae particula infinite parva FO consideratur ut lineola recta, tunc singulae applicatae inter PF et RO cum legem mutationis curvaturae nondum subeant haberi possunt pro una eademque applicata, quasi nempe singulae ipsi PF absolute essent aequales: eodem modo quia $\omega\varphi$ considero ut rectam lineolam singulae applicatae inter $\rho\omega$ et $\pi\varphi$ utpote legem mutationis curvaturae pariter non subeuntes possunt pro se invicem sumi adeoque eaedem poni cum $\pi\varphi$); si igitur, inquam, loco RO sumatur aequipollens PF et loco $\rho\omega$ aequipollens $\pi\varphi$, habebitur $FO \times$ Đ̄$PF = \varphi\omega \times$ Đ̄$\pi\varphi$ adeoque Đ̄PF ad Đ̄$\pi\varphi$ ut $\varphi\omega$ (φO) ad FO ut sin. $OF\varphi$ ad sin. $O\varphi F$ et permutando Đ̄PF ad sin. $OF\varphi$ ut Đ̄$\pi\varphi$ ad sin. $O\varphi F$. Hinc cum $F\varphi$ sit subtensa arcus curvae infinite parvi $FO\varphi$, adeoque angulus $OF\varphi$ et $O\varphi F$ haberi possit pro semisse anguli curvedinis in F et φ, erit Đ̄PF ad sinum curvedinis in F ut Đ̄$\pi\varphi$ ad sinum curvedinis in φ; hoc est in ratione constanti. Problema itaque ad pure analyticum redactum huc redit: U t q u a e r a t u r c u r v a $BF\varphi$

Đ̄ LATIN CAPITAL D WITH TOP BAR AND CROSSBAR
The D with top bar and crossbar is used here to denote the differential quotient.
LAA III-7 p. 817

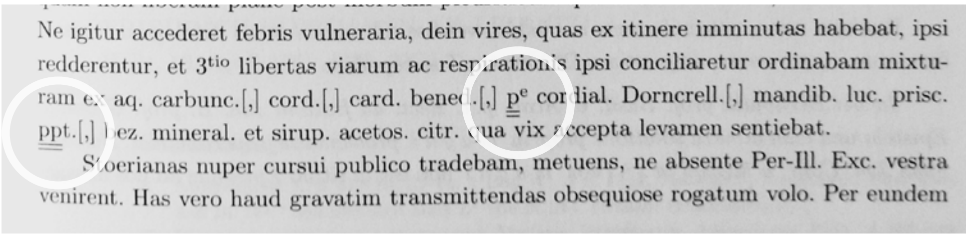


Đ̄ LATIN CAPITAL D WITH TOP BAR AND CROSSBAR
The D with top bar and crossbar is used here to denote the differential quotient. The D shape and the top stroke are written in one single movement, which reveals that the stroke is intended as a part of the letterform itself, not as a virgula.
Leibniz manuscript, GWLB, LBr. 57,1 239v°



elapsum dum consequi ac colligere conatur studiosus lateri meo adhaerens, ego eundem praevenire adnitor, non reminiscens baculi mei, quem vestimento alligaveram, cumque se inter humum et pectus fulserit gravius quam putavi thoracis regionem contudi, ut media hac nocte mihi ob sanguinem extravasatum, quem adesse colligo, pene spiritus omnis interceptus sit. Quare statim e pharmacopolio adhibito pulvere resolvente ex lapid. ♋[,] sangu. drac.[.] mum. p̳p̳t.[.] cinnab. nat. p̳p̳t. et ♁^{io} diaphoret. liberius quidem nunc spiritum Deo sit gratias duco, sed graviores in loci afflicti regione dolores nondum cessare volunt, vixque brachium manumque pro exarandis literis hisce movere valeo. Spero tamen, me commodis medicamentis tractatum mox intentatum hoc sanitatis periculum evasurum.

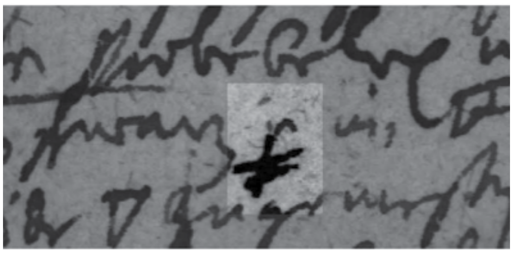
p̳ LOWERCASE P WITH DOUBLE CROSSBAR
In this specimen p̳p̳t means *präpariert* (prepared). The connecting double stroke must however cross the descender, the glyph used in the edition is not quite appropriate. LAA III-8 p. 572



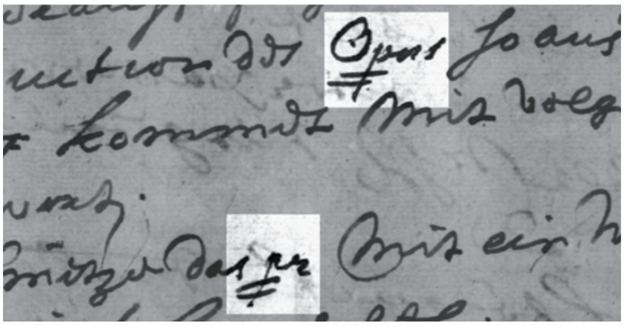
Ne igitur accederet febris vulneraria, dein vires, quas ex itinere imminutas habebat, ipsi redderentur, et 3^{tio} libertas viarum ac respirationis ipsi conciliaretur ordinabam mixturam ex aq. carbunc.[,] cord.[,] card. bened.[,] p̳^{e} cordial. Dorncrell.[,] mandib. luc. prisc. p̳p̳t.[,] bez. mineral. et sirup. acetos. citr. qua vix accepta levamen sentiebat.
Stoerianas nuper cursui publico tradebam, metuens, ne absente Per-Ill. Exc. vestra venirent. Has vero haud gravatim transmittendas obsequiose rogatum volo. Per eundem

p̳ LOWERCASE P WITH DOUBLE CROSSBAR
The character is used in at least two alchemical expressions: a single p̳ denotes *pulvis* (powder) whereas the double p̳p̳ in combination with t stands for *präpariert* (prepared). In this specimen both single and double usage are represented. LAA III-8 p. 605

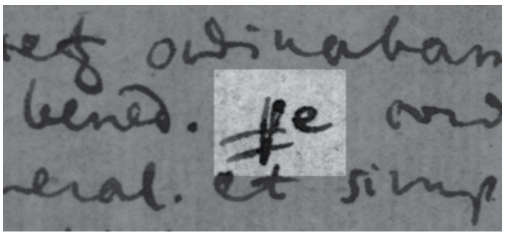
Some examples of the usage of p̳ LOWERCASE P WITH DOUBLE CROSSBAR by Leibniz and some of his correspondents (below). In all these cases the abbreviation is used for Pulvis. (top left: Leibniz; top right: Martin Elers; bottom left and right: Rudolf Christian Wagner).



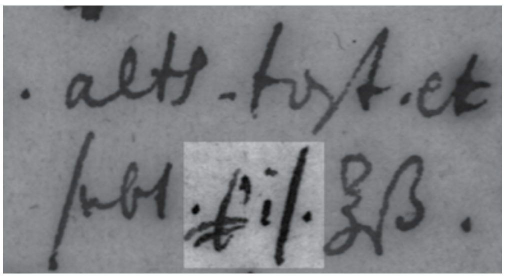
LH XLI 2 Bl. 3r



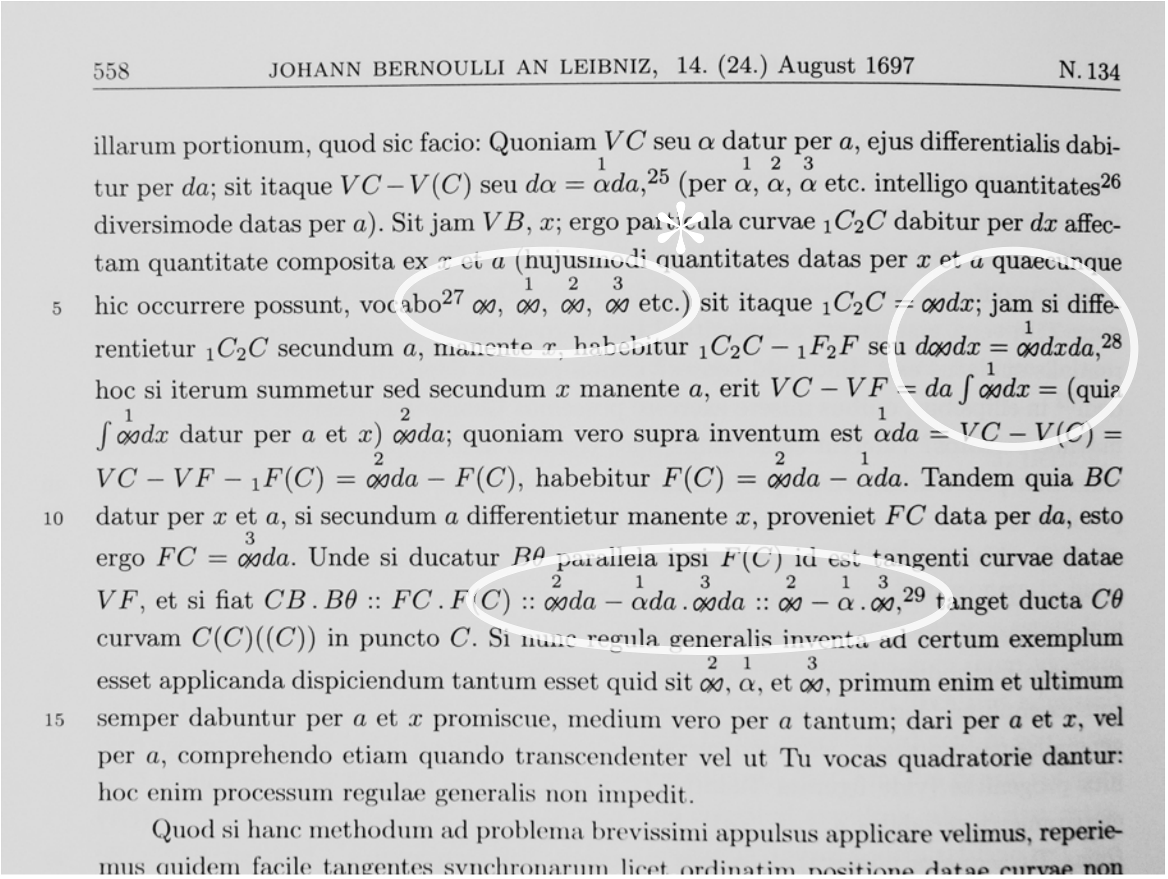
LBr. 237 Bl. 88v



LBr. 973 Bl. 95r



LBr. 973 Bl. 95r



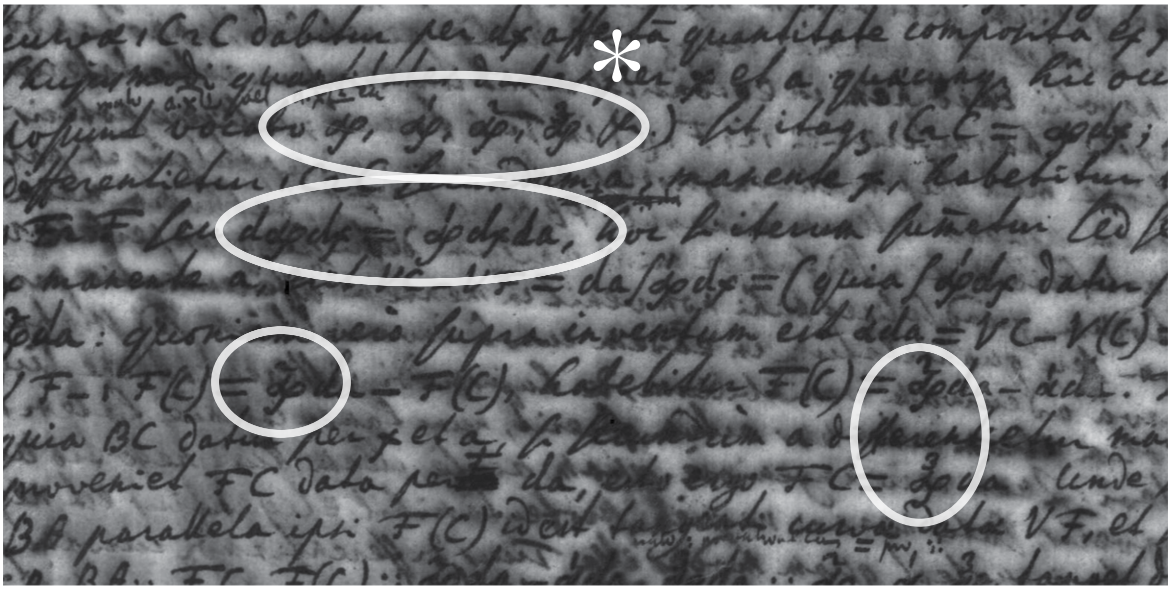
558 JOHANN BERNOULLI AN LEIBNIZ, 14. (24.) August 1697 N. 134

illarum portionum, quod sic facio: Quoniam VC seu α datur per a, ejus differentialis dabitur per da; sit itaque $VC - V(C)$ seu $d\alpha = \overset{1}{\alpha} da$,[25] (per $\overset{1}{\alpha}$, $\overset{2}{\alpha}$, $\overset{3}{\alpha}$ etc. intelligo quantitates[26] diversimode datas per a). Sit jam VB, x; ergo particula curvae ${}_1C_2C$ dabitur per dx affectam quantitate composita ex x et a (hujusmodi quantitates datas per x et a quaecunque hic occurrere possunt, vocabo[27] $\alpha\!x$, $\overset{1}{\alpha\!x}$, $\overset{2}{\alpha\!x}$, $\overset{3}{\alpha\!x}$ etc.) sit itaque ${}_1C_2C = \alpha\!x\,dx$; jam si differentietur ${}_1C_2C$ secundum a, manente x, habebitur ${}_1C_2C - {}_1F_2F$ seu $d\alpha\!x\,dx = \overset{1}{\alpha\!x}\,dx\,da$,[28] hoc si iterum summetur sed secundum x manente a, erit $VC - VF = da \int \overset{1}{\alpha\!x}\,dx =$ (quia $\int \overset{1}{\alpha\!x}\,dx$ datur per a et x) $\overset{2}{\alpha\!x}\,da$; quoniam vero supra inventum est $\overset{1}{\alpha} da = VC - V(C) = VC - VF - {}_1F(C) = \overset{2}{\alpha\!x}\,da - F(C)$, habebitur $F(C) = \overset{2}{\alpha\!x}\,da - \overset{1}{\alpha} da$. Tandem quia BC datur per x et a, si secundum a differentietur manente x, proveniet FC data per da, esto ergo $FC = \overset{3}{\alpha\!x}\,da$. Unde si ducatur $B\theta$ parallela ipsi $F(C)$ id est tangenti curvae datae VF, et si fiat $CB\,.\,B\theta :: FC\,.\,F(C) :: \overset{2}{\alpha\!x}\,da - \overset{1}{\alpha} da\,.\,\overset{3}{\alpha\!x}\,da :: \overset{2}{\alpha\!x} - \overset{1}{\alpha}\,.\,\overset{3}{\alpha\!x}$,[29] tanget ducta $C\theta$ curvam $C(C)((C))$ in puncto C. Si hanc regula generalis inventa ad certum exemplum esset applicanda dispiciendum tantum esset quid sit $\overset{2}{\alpha\!x}$, $\overset{1}{\alpha}$, et $\overset{3}{\alpha\!x}$, primum enim et ultimum semper dabuntur per a et x promiscue, medium vero per a tantum; dari per a et x, vel per a, comprehendo etiam quando transcendenter vel ut Tu vocas quadratorie dantur: hoc enim processum regulae generalis non impedit.

Quod si hanc methodum ad problema brevissimi appulsus applicare velimus, reperiemus quidem facile tangentes synchronarum licet ordinatim positione datae curvae non

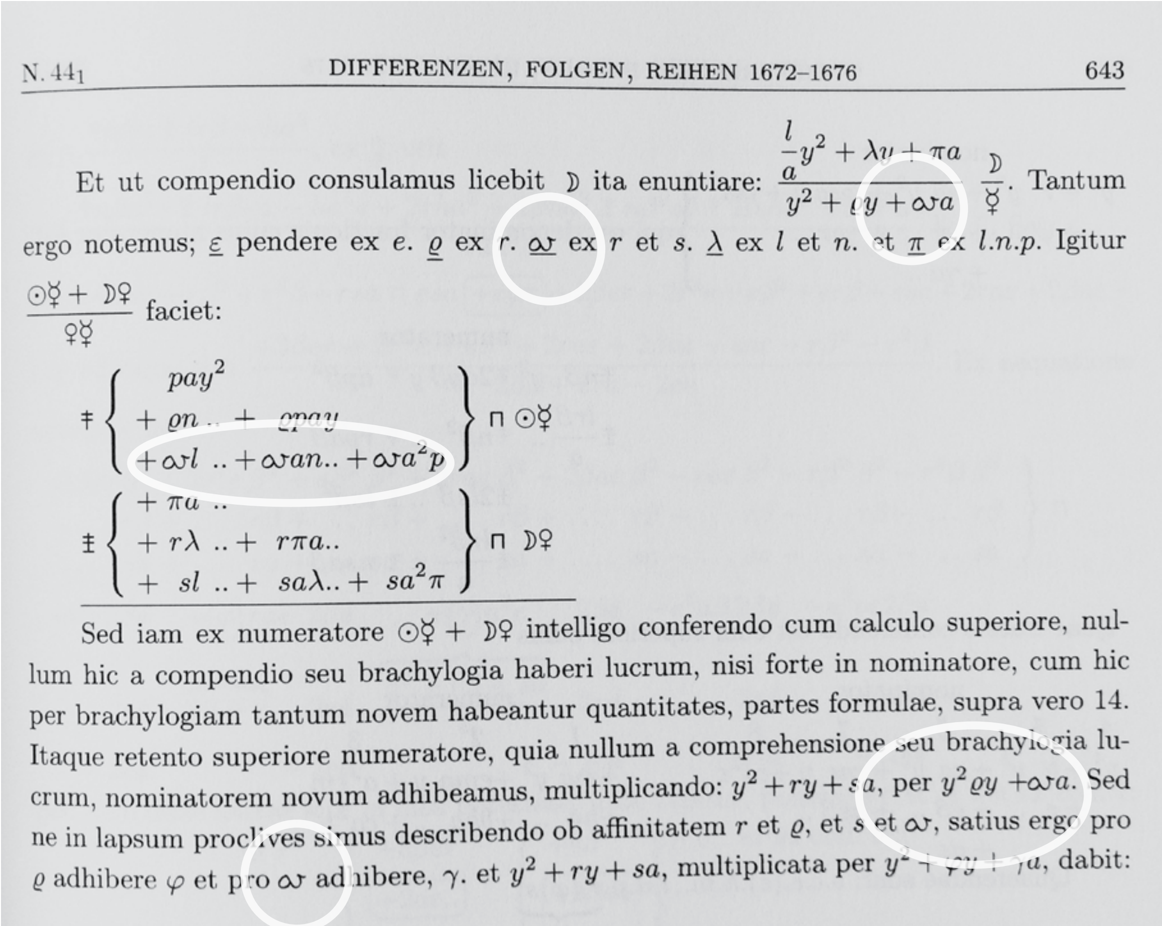
αx BERNOULLIAN ALPHA-X SIGN

The author Johann Bernoulli uses α as a quantity depending on *a*. In analogy, he combines an α and a cursive *x* to denote a quantity depending on the variables *a* and *x* (in modern terminology a function in a and x). LAA III-7 p. 558



αx BERNOULLIAN ALPHA-X SIGN

Handwriting of Johann Bernoulli, 1697. Approximately the same part of text as in the image above. GWLB, LBr. 57,1 211v°


N. 44₁ DIFFERENZEN, FOLGEN, REIHEN 1672–1676 643

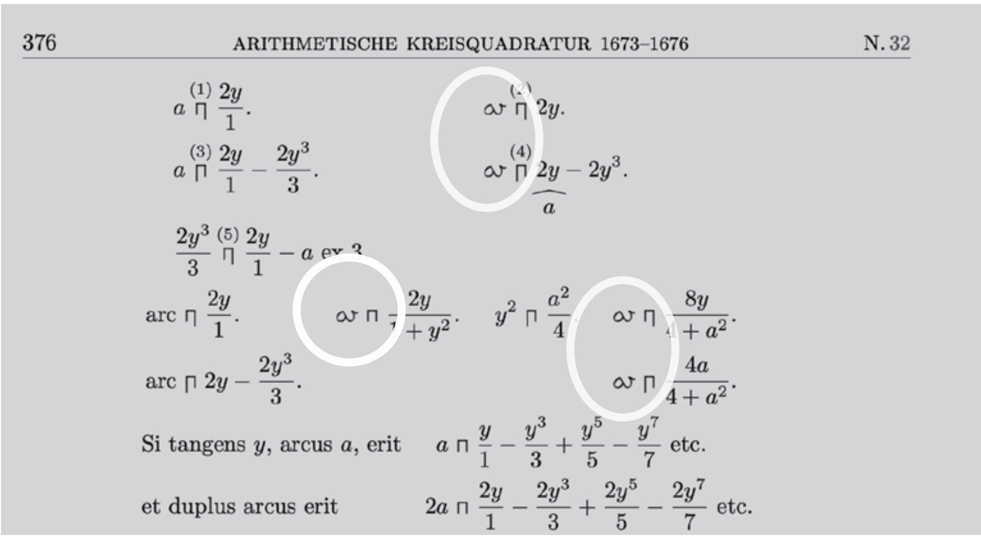
Et ut compendio consulamus licebit ☽ ita enuntiare: $\dfrac{\frac{l}{a}y^2 + \lambda y + \pi a}{y^2 + \varrho y + ϭa}\ \dfrac{☽}{☿}$. Tantum ergo notemus; $\underline{\varepsilon}$ pendere ex e. $\underline{\varrho}$ ex r. $\underline{ϭ}$ ex r et s. $\underline{\lambda}$ ex l et n. et $\underline{\pi}$ ex $l.n.p.$ Igitur $\dfrac{☉☿ + ☽♀}{♀☿}$ faciet:

$$\mp \left\{ \begin{array}{l} pay^2 \\ + \varrho n .. + \varrho pay \\ + ϭl .. + ϭan .. + ϭa^2p \end{array} \right\} ⊓ ☉☿$$

$$\pm \left\{ \begin{array}{l} + \pi a .. \\ + r\lambda .. + r\pi a .. \\ + sl .. + sa\lambda .. + sa^2\pi \end{array} \right\} ⊓ ☽♀$$

Sed iam ex numeratore ☉☿ + ☽♀ intelligo conferendo cum calculo superiore, nullum hic a compendio seu brachylogia haberi lucrum, nisi forte in nominatore, cum hic per brachylogiam tantum novem habeantur quantitates, partes formulae, supra vero 14. Itaque retento superiore numeratore, quia nullum a comprehensione seu brachylogia lucrum, nominatorem novum adhibeamus, multiplicando: $y^2 + ry + sa$, per $y^2 \varrho y + ϭa$. Sed ne in lapsum proclives simus describendo ob affinitatem r et ϱ, et s et $ϭ$, satius ergo pro ϱ adhibere φ et pro $ϭ$ adhibere, γ. et $y^2 + ry + sa$, multiplicata per $y^2 + \varphi y + \gamma a$, dabit:

ϭ SIGMA-SIGMA SIGN
Leibniz uses this symbol for a quantity in the same way as he uses roman letters or other Greek letters, such as gamma, epsilon, lambda, pi, phi or omega; as shown in this example.
LAA VII-3 p. 643


376 ARITHMETISCHE KREISQUADRATUR 1673–1676 N. 32

$a \overset{(1)}{⊓} \dfrac{2y}{1}$. $\quad ϭ \overset{(2)}{⊓} 2y$.

$a \overset{(3)}{⊓} \dfrac{2y}{1} - \dfrac{2y^3}{3}$. $\quad ϭ \overset{(4)}{⊓} \dfrac{2y - 2y^3}{a}$.

$\dfrac{2y^3}{3} \overset{(5)}{⊓} \dfrac{2y}{1} - a$ ex 3

arc $⊓ \dfrac{2y}{1}$. $\quad ϭ ⊓ \dfrac{2y}{1 + y^2}$. $\quad y^2 ⊓ \dfrac{a^2}{4}$. $\quad ϭ ⊓ \dfrac{8y}{4 + a^2}$.

arc $⊓ 2y - \dfrac{2y^3}{3}$. $\quad ϭ ⊓ \dfrac{4a}{4 + a^2}$.

Si tangens y, arcus a, erit $\quad a ⊓ \dfrac{y}{1} - \dfrac{y^3}{3} + \dfrac{y^5}{5} - \dfrac{y^7}{7}$ etc.

et duplus arcus erit $\quad 2a ⊓ \dfrac{2y}{1} - \dfrac{2y^3}{3} + \dfrac{2y^5}{5} - \dfrac{2y^7}{7}$ etc.

ϭ SIGMA-SIGMA SIGN
LAA VII-6 p. 376
This sample also shows the two signs ⊓ LEIBNIZIAN GREATER and ⊓ LEIBNIZIAN LESS.

$-\text{♂}^2 \sqcap \frac{-d^2+2d\text{☿}-\text{☿}^2}{4}$. Ergo $ca \sqcap \boxed{\mp\frac{a}{q}\text{♀}^2} + 2a\text{♀} \pm \frac{\boxed{2}a}{q}\text{♀}^2 \frac{-d^2\boxed{+2d\text{☿}}-\text{☿}^2}{4}\ \frac{\boxed{-d\text{☿}}+\text{☿}^2}{2}$.

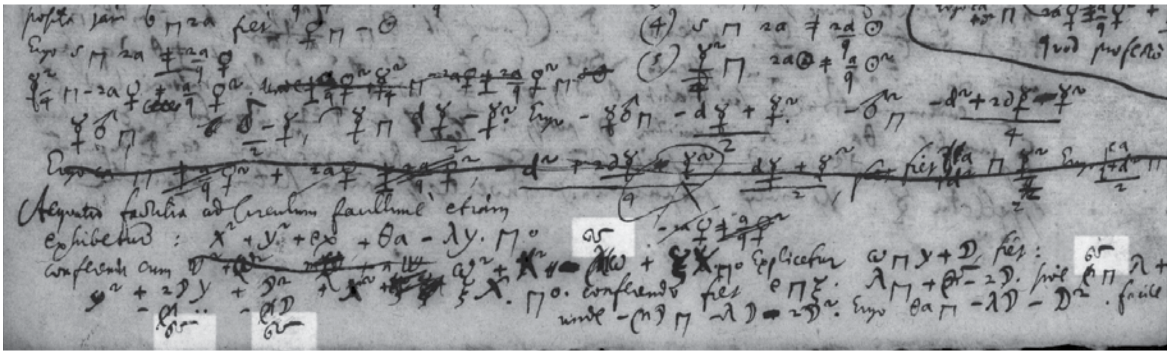
Ergo $\frac{ca}{+d^2} \sqcap 2a\text{♀} \pm \frac{a}{q}\text{♀}^2 + \frac{\text{☿}^2}{4}$. Ergo $\frac{\left\{\begin{matrix} ca \\ +d^2 \end{matrix}\right.}{2} \sqcap 2a\text{♀} \mp \frac{a}{q}\text{♀}^2$ vel $\sqcap \frac{\text{☿}^2}{4}$. Quod profecto elegans est theorema. $s \sqcap \frac{\text{☿}^2}{4\text{♀}}$.

Aequatio factitia ad Circulum facillime etiam exhibetur: $x^2+y^2+ex+\theta a-\lambda y \sqcap 0$. conferenda cum $\omega^2+x^2-\text{ϭ}\omega+\xi x \sqcap 0$. Explicetur $\omega \sqcap y+\text{☽}$, fiet:

$$y^2+2\,\text{☽}y+\ \text{☽}^2+x^2+\xi x \sqcap 0.$$
$$-\text{ϭ}\ ..\ -\text{ϭ}\,\text{☽}$$

Conferendo fiet $e \sqcap \xi$. $\lambda \sqcap \text{ϭ}-2\,\text{☽}$. sive $\text{ϭ} \sqcap \lambda+2\,\text{☽}$. Unde $-\text{ϭ}\,\text{☽} \sqcap -\lambda\,\text{☽}-2\,\text{☽}^2$. Ergo $\theta a \sqcap -\lambda\,\text{☽}-\ \text{☽}^2$. Facile ergo habetur ☽ ergo et ϭ.

ϭ SIGMA-SIGMA SIGN – LAA VII-7 p. 414
The following figure shows the manuscript source of that text (LH 35 XIII 3, fol. 161r).

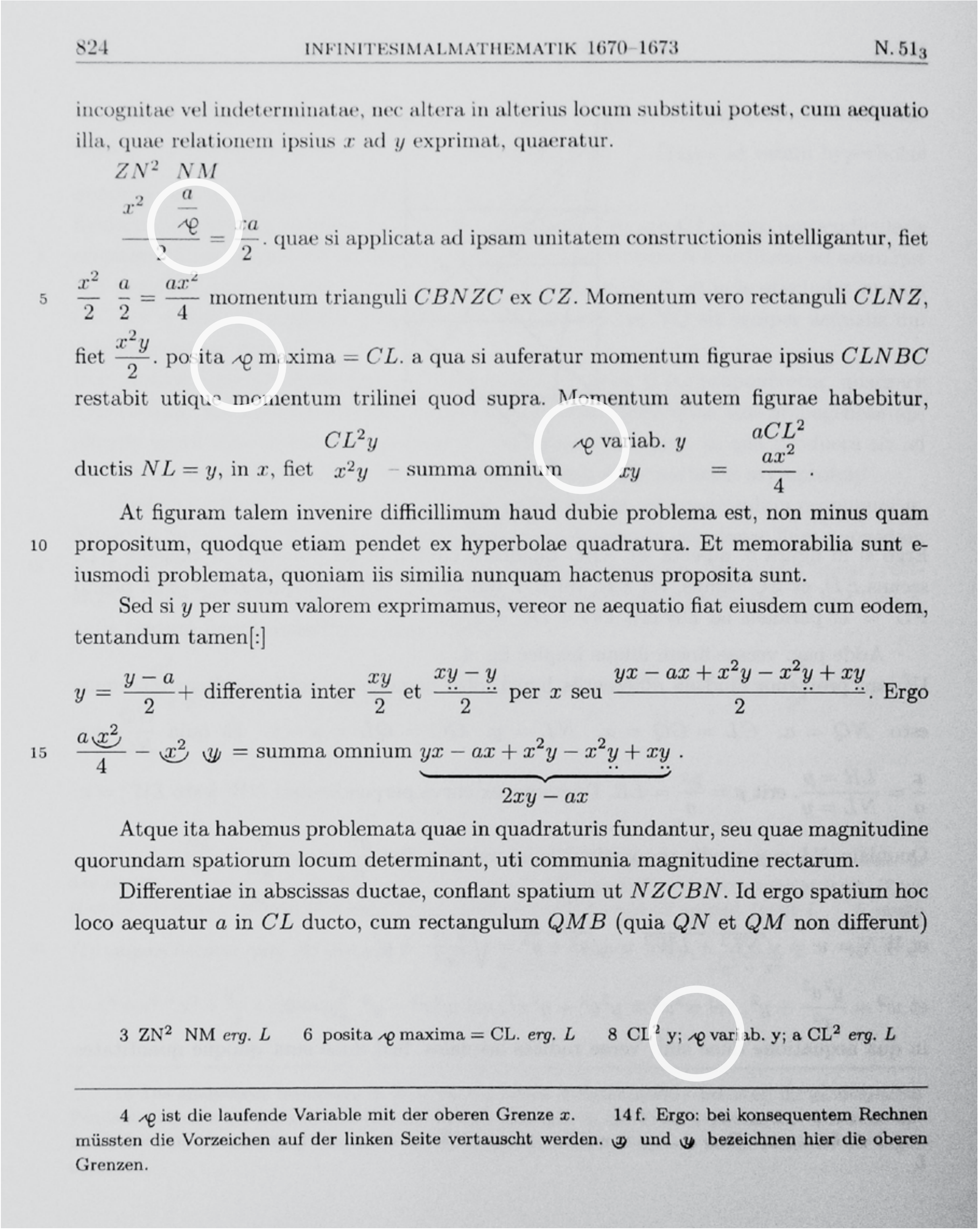


7–14 *Am Rand:*

$$1\ -\frac{a^2}{1,2}\ +\frac{a^4}{1,2,3,4}\ -\frac{a^6}{1,2,3,4,5,6}$$
$$\frac{a}{1}\ -\frac{a^3}{1,2,3}\ +\frac{a^5}{1,2,3,4,5}\ -\frac{a^7}{1,2,3,4,5,6,7}$$
$$\frac{a^2}{1,2}\ -\frac{a^4}{1,2,3,4}\ +\frac{a^6}{1,2,3,4,5,6}\ -\frac{a^8}{1,2,3,4,5,6,7,8}$$

14 *Darunter:* $\int \overline{d\bar{a}v} \sqcap$ segm. $\sqcap$ ϭ. $\int \overline{ad\text{ϭ}} \sqcap \int \overline{ad\bar{a}v}$. $d\bar{a}v \sqcap d$ϭ. Ergo vel $v \sqcap \frac{d\text{ϭ}}{d\bar{a}}$ vel $a \sqcap \int \overline{\frac{d\text{ϭ}}{v}}$. $\int \overline{d\bar{a}v} \sqcap$ ϭ.

ϭ SIGMA-SIGMA SIGN
LAA VII-6 p. 401



incognitae vel indeterminatae, nec altera in alterius locum substitui potest, cum aequatio illa, quae relationem ipsius x ad y exprimat, quaeratur.

$$\frac{\overset{ZN^2}{x^2}\ \overset{NM}{\frac{a}{\mathfrak{x}}}}{2} = \frac{\mathfrak{x}a}{2}.$$ quae si applicata ad ipsam unitatem constructionis intelligantur, fiet $\frac{x^2}{2}\,\frac{a}{2} = \frac{ax^2}{4}$ momentum trianguli $CBNZC$ ex CZ. Momentum vero rectanguli $CLNZ$, fiet $\frac{x^2y}{2}$. posita $\mathfrak{x}$ maxima $= CL$. a qua si auferatur momentum figurae ipsius $CLNBC$ restabit utique momentum trilinei quod supra. Momentum autem figurae habebitur, ductis $NL = y$, in x, fiet $\overset{CL^2y}{x^2y}$ – summa omnium $\overset{\mathfrak{x} \text{ variab. } y}{xy} = \overset{aCL^2}{\frac{ax^2}{4}}$.

At figuram talem invenire difficillimum haud dubie problema est, non minus quam propositum, quodque etiam pendet ex hyperbolae quadratura. Et memorabilia sunt eiusmodi problemata, quoniam iis similia nunquam hactenus proposita sunt.

Sed si y per suum valorem exprimamus, vereor ne aequatio fiat eiusdem cum eodem, tentandum tamen[:]

$$y = \frac{y-a}{2} + \text{ differentia inter } \frac{xy}{2} \text{ et } \frac{\ddot{x}y - y}{2} \text{ per } x \text{ seu } \frac{yx - ax + x^2y - \ddot{x}^2y + \ddot{x}y}{2}. \text{ Ergo}$$

$$\frac{a\mathfrak{x}^2}{4} - \mathfrak{x}^2\ \mathfrak{y} = \text{summa omnium } \underbrace{yx - ax + x^2y - \ddot{x}^2y + \ddot{x}y}_{2xy - ax}.$$

Atque ita habemus problemata quae in quadraturis fundantur, seu quae magnitudine quorundam spatiorum locum determinant, uti communia magnitudine rectarum.

Differentiae in abscissas ductae, conflant spatium ut $NZCBN$. Id ergo spatium hoc loco aequatur a in CL ducto, cum rectangulum QMB (quia QN et QM non differunt)

3 ZN^2 NM *erg. L* 6 posita $\mathfrak{x}$ maxima = CL. *erg. L* 8 CL^2 y; $\mathfrak{x}$ variab. y; a CL^2 *erg. L*

4 $\mathfrak{x}$ ist die laufende Variable mit der oberen Grenze x. 14 f. Ergo: bei konsequentem Rechnen müssten die Vorzeichen auf der linken Seite vertauscht werden. $\mathfrak{x}$ und $\mathfrak{y}$ bezeichnen hier die oberen Grenzen.

$\mathfrak{x}$ LOWERCASE KURRENT X SIGN

This page shows a deliberate distinction between the normal Latin x and a kurrent x, which has been ‘borrowed’ from the German cursive “Kurrentschrift” style. In this case, the LOWERCASE KURRENT X SIGN is used in the context of analyzing properties of curves. In a modern correspondence, it could be described as a variable on which the curve depends and which is limited by a given x. Therefore, to choose the LOWERCASE KURRENT X SIGN is motivated by the need to apply a *different* kind of x.

LAA VII-4 p. 824

superficiem cylindricam sub arcu et abscissa ab extremo radii IB. ad habendum arcus momentum. Iam ut arcus decrescunt, ita abscissae crescunt, in ratione altitudinum, seu numerorum naturalium. Ergo cylindro quem dixi segmenti, addenda est summa talium productorum.

$$\begin{matrix} x & 2x & 3x & 4x & 5x & \text{etc.} \\ a-1 & a-2 & a-3 & a-4 & a-5 & \end{matrix}$$

Posito radio a, infinitis arcus partibus x. fiet: $ax + 2ax + 3ax$ seu pro omnibus x. seu arcu sumto $\mathbb{X}$ fiet $\dfrac{\mathbb{X}^2 a}{2}$.

$$\begin{matrix} x & 2x & 3x & 4x. \\ 1 & 2 & 3 & 4 \end{matrix}$$

Seu posito $a =$ infinitis b seu $= \beta b$ fiet $1xb.$ $4xb.$ $9xb.$ $16xb.$ Erit tertia pars cubi sub media proportionali inter arcum et radium a. Idemque sic probatur: manifestum est ista

$\mathbb{X}$ LATIN CAPITAL DOUBLE X
With this symbol Leibniz denotes "all *x*". As in the case of the LOWERCASE KURRENT X SIGN, Leibniz needed a different kind of *X*.
LAA VII-4 p. 273, 274 (below)

274 INFINITESIMALMATHEMATIK 1670–1673 N. 16_2

$1xb.$ $2 \frown 2xb.$ $3 \frown 3xb.$ esse nihil aliud quam summam $\bigtriangledown^{\text{lorum}}$ quorum altitudo omnia b. vel ipsa a. basis omnia x. vel ipsa $\mathbb{X}$ eaque continue diminuta. Inde a basi, sibi superposita horum elementa crescunt ut parallelepipeda, quorum latera crescunt in eadem ratione numerorum naturalium seu ut quadrata, quorum radices sunt numeri naturales: nam v.g. parallelepipedum $4xb$. ergo radix $\square^{\text{ta}}$ aequalis: $2Rqxb$. et pro $Rq_{\llcorner\llcorner}9_{\llcorner}xb_{\lrcorner}$ fiet $3\,Rq\,xb$. et ita porro.

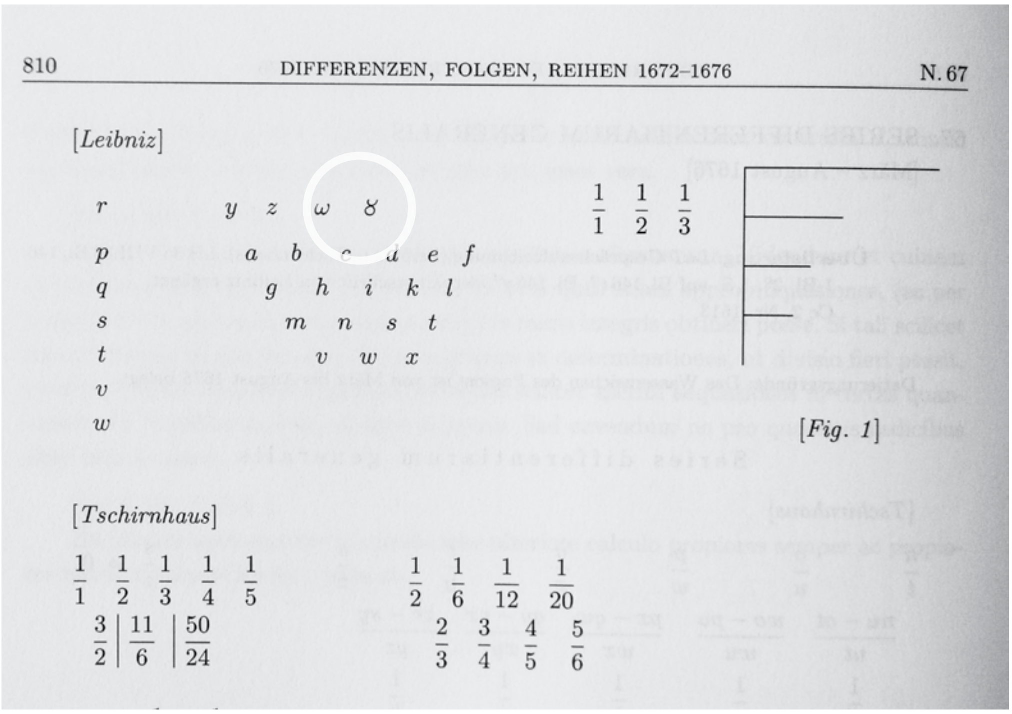
SECONDO 251

Agguaglisi 4.p.R.q.L 24.m.20 $\overset{1}{\smile}$ ⅃ à 1 $\overset{1}{\smile}$ in simili agguagliamenti bisogna sempre cercare, che la R.q.legata resti sola, però si leuarà il 4. ad ambedue le parti, e si hauerà R.q.L 24.m.20 $\overset{1}{\smile}$ ⅃. eguale à 2 $\overset{1}{\smile}$ m.4. Quadrisi ciascuna delle parti, si hauerà 24.m. 20. $\overset{1}{\smile}$ eguale à 4 $\overset{2}{\smile}$ m.16. $\overset{1}{\smile}$ p.16. lieuinsi li meni da ciascuna delle parti, e ponganssi dall'altra parte si hauerà 4 $\overset{2}{\smile}$ p.20 $\overset{1}{\smile}$ p.16. eguale à 24.p.16 $\overset{1}{\smile}$. lieuinsi li 16 $\overset{1}{\smile}$ à ciascuna delle parti, e si hauerà 4 $\overset{2}{\smile}$ p.4 $\overset{1}{\smile}$ p.16. eguale à 24. lieuisi il 16. da ogni parte si haueranno 4 $\overset{2}{\smile}$ p. 4 $\overset{1}{\smile}$ eguale à 8. riduchisi à 1 $\overset{2}{\smile}$ si hauerà 1 $\overset{2}{\smile}$ p.1 $\overset{1}{\smile}$ eguale à 2 (seguitisi il Capitolo) che il Tanto ualerà 1.

4.p.R.q.L 24.m. 20.⅃ $\overset{1}{\smile}$ Eguale à 2. $\overset{1}{\smile}$

R.q.L 24. m. 20.⅃ $\overset{1}{\smile}$ Eguale à 2. m.4. $\overset{1}{\smile}$

⅃ REVERSED CAPITAL L
Bombelli, L'algebra, 1572 (after Cajori I p. 125)



810 DIFFERENZEN, FOLGEN, REIHEN 1672–1676 N. 67

[*Leibniz*]

r		y	z	ω	ȣ					
p			a	b	c	d	e	f		
q				g	h	i	k	l		
s					m	n	s	t		
t						v	w	x		
v										
w										

$\frac{1}{1}\ \frac{1}{2}\ \frac{1}{3}$

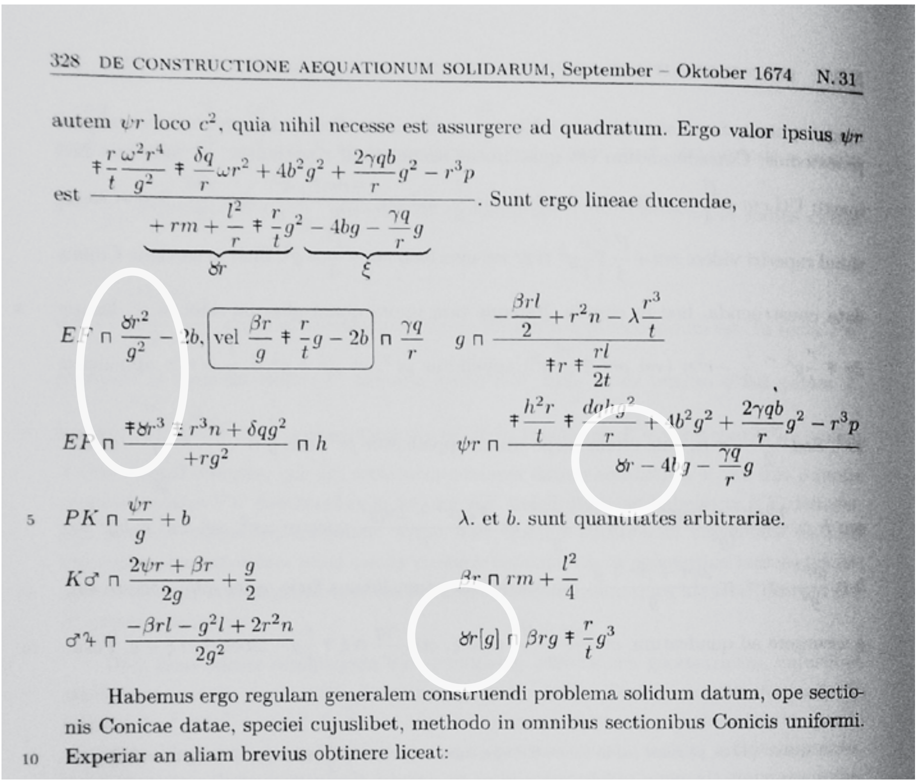
[*Fig. 1*]

[*Tschirnhaus*]

$\frac{1}{1}\ \frac{1}{2}\ \frac{1}{3}\ \frac{1}{4}\ \frac{1}{5}$ $\qquad$ $\frac{1}{2}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{20}$

$\frac{3}{2}\ \Big|\ \frac{11}{6}\ \Big|\ \frac{50}{24}$ $\qquad$ $\frac{2}{3}\ \frac{3}{4}\ \frac{4}{5}\ \frac{5}{6}$

ȣ OMICRON-UPSILON SIGN, LAA VII-3 p. 810



328 DE CONSTRUCTIONE AEQUATIONUM SOLIDARUM, September – Oktober 1674 N. 31

autem ψr loco c^2, quia nihil necesse est assurgere ad quadratum. Ergo valor ipsius ψr est $\dfrac{\mp\frac{r}{t}\frac{\omega^2 r^4}{g^2} \mp \frac{\delta q}{r}\omega r^2 + 4b^2g^2 + \frac{2\gamma qb}{r}g^2 - r^3p}{\underbrace{+rm + \frac{l^2}{r} \mp \frac{r}{t}g^2}_{ȣr} \underbrace{- 4bg - \frac{\gamma q}{r}g}_{\xi}}$. Sunt ergo lineae ducendae,

$EF \sqcap \dfrac{ȣr^2}{g^2} - 2b$, vel $\boxed{\dfrac{\beta r}{g} \mp \dfrac{r}{t}g - 2b} \sqcap \dfrac{\gamma q}{r}$ $\qquad$ $g \sqcap \dfrac{-\frac{\beta rl}{2} + r^2n - \lambda\frac{r^3}{t}}{\mp r \mp \frac{rl}{2t}}$

$EF \sqcap \dfrac{\mp ȣr^3 \pm r^3n + \delta qg^2}{+rg^2} \sqcap h$ $\qquad$ $\psi r \sqcap \dfrac{\mp\frac{h^2r}{t} \mp \frac{d\omega hg^2}{r} + 4b^2g^2 + \frac{2\gamma qb}{r}g^2 - r^3p}{ȣr - 4bg - \frac{\gamma q}{r}g}$

$PK \sqcap \dfrac{\psi r}{g} + b$ $\qquad$ λ. et b. sunt quantitates arbitrariae.

$K♂ \sqcap \dfrac{2\psi r + \beta r}{2g} + \dfrac{g}{2}$ $\qquad$ $\beta r \sqcap rm + \dfrac{l^2}{4}$

$♂♃ \sqcap \dfrac{-\beta rl - g^2l + 2r^2n}{2g^2}$ $\qquad$ $ȣr[g] \sqcap \beta rg \mp \dfrac{r}{t}g^3$

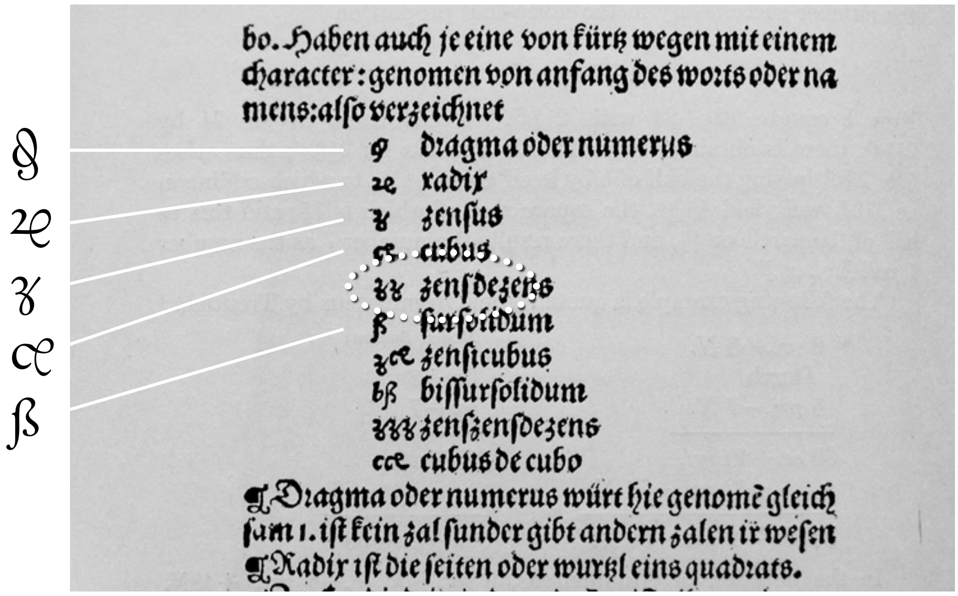
Habemus ergo regulam generalem construendi problema solidum datum, ope sectionis Conicae datae, speciei cujuslibet, methodo in omnibus sectionibus Conicis uniformi. Experiar an aliam brevius obtinere liceat:

ȣ GREEK LOWERCASE OMICRON-UPSILON – LAA VII-7 p. 328

Leibniz uses that symbol, which is derived from a Greek minuscule ligature ου, for denoting a variable, alongside with e.g. β or ω and latin lowercase letters. Because of that specific context and function the character ought to be distinguished from ȣ PLUSMINUS SIGN which has a similar basic grapheme but is used as a mathematical operator symbol instead. – Optional is an encoding of a Greek letter pair (upper and lowercase), the capital Ȣ has been proposed by M. Everson 1998 (N1743).

4.i) Coss symbols

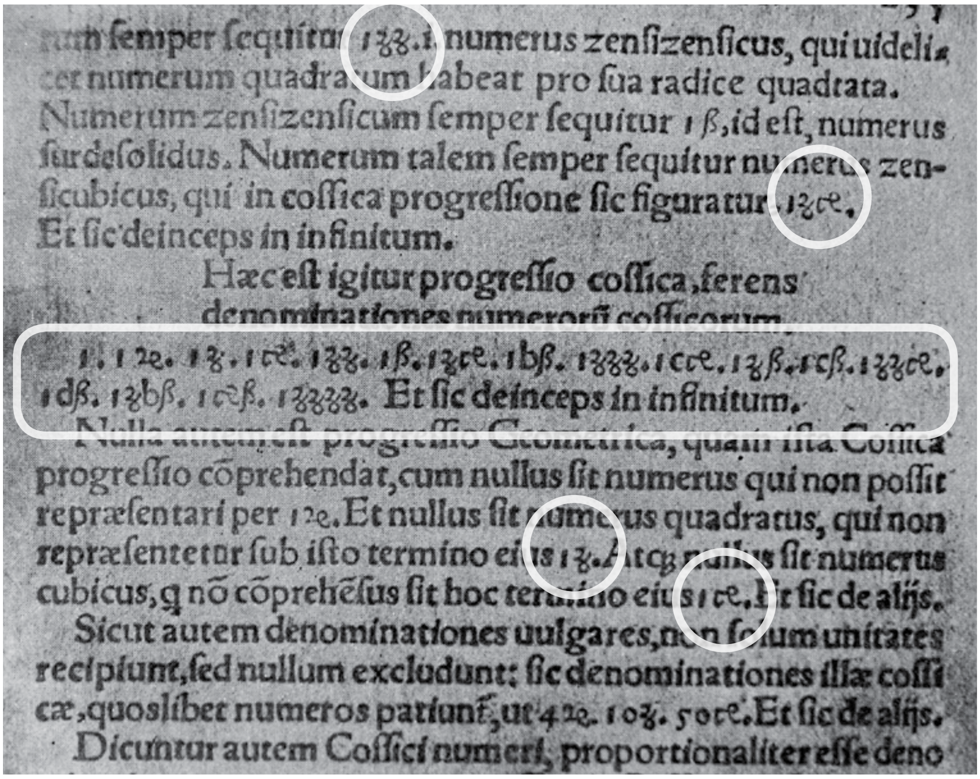
The so-called *Coss* or *cossic* symbols where a widely adopted set of characters for denoting powers, in the 16th and 17th century. They are derivates from Latin letters c, d, r, ſ and z which were modified into special shaped unique symbols. Although they bear some optical similarities to existing characters, we see a case for encoding this whole set in its particular form and determined by its special meaning and function. For editions of sources such as shown here in facsimile mode it is neccessary to reproduce these symbols accurately and unambiguously.
We show a couple of instances from printed sources, and also a piece of manuscript evidence by Leibniz. See page 111 for a synopsis of these 9 (10) characters.


bo. Haben auch ie eine von kürtz wegen mit einem
character: genomen von anfang des worts oder na
mens: also verzeichnet
dragma oder numerus
radix
zensus
cubus
zensdezens
sursolidum
zensicubus
bissursolidum
zenszensdezens
cubus de cubo
Dragma oder numerus würt hie genomē gleich
sam 1. ist kein zal sunder gibt andern zalen ir wesen
Radix ist die seiten oder wurtzl eins quadrats.

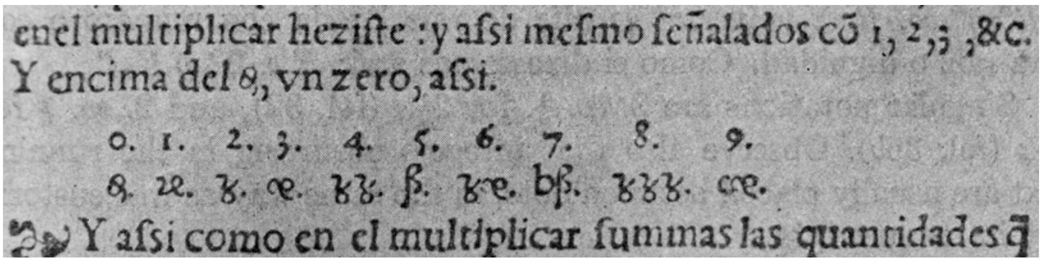
Rudolff 1525 (after Cajori). This sample shows ʒ LOWERCASE KURRENT Z SIGN, ꝺ LOWERCASE D ROTUNDA WITH CROSSING LOOP, ꝝ LOWERCASE R ROTUNDA WITH LOOP, ꞓ LOWERCASE C WITH RIGHT LOOP and ß DOUBLE S ABBREVIATION SIGN.

This print demonstrates the deliberate distinction between the cossic character ʒ and the normal fraktur 𝔷 (see at ⁘).
The character ʒ LOWERCASE KURRENT Z SIGN is denoting *zensus*. It is semantically determined by this one unambiguous meaning, and graphically characterized by a) a round-shaped upper part (mostly), and b) a prominent loop descender which crosses upwards. The origin of its shape is neither *Fraktur* type nor Latin script style but the German *Kurrent* writing style. Therefore the character is neither to be unified with *ezh* (0292) nor with any of the mathematical alphanumeric characters 1D4B5 etc[a], in order to secure its specific semantic content.

The character ß should not be unified with LATIN SMALL LETTER SHARP S (00DF, its somewhat obscure origin going back to medieval long-s abbreviation characters and later becoming typographically a ligature of long s and z in blackletter styles). Wheras the ß character is clearly and unambiguously based on the ligation of ſ and s in order to facilitate a crisp abbreviation sign for frequently occuring words like *ſemis*, *ſolidus* or *ſurſolidus*. Since the German **ß** appears in various shapes nowadays in typefaces (e.g. in Times – like seen here – it is definitely not a ſ-s ligature), it would be unappropriate to assign 00DF to the special usage shown in these examples.

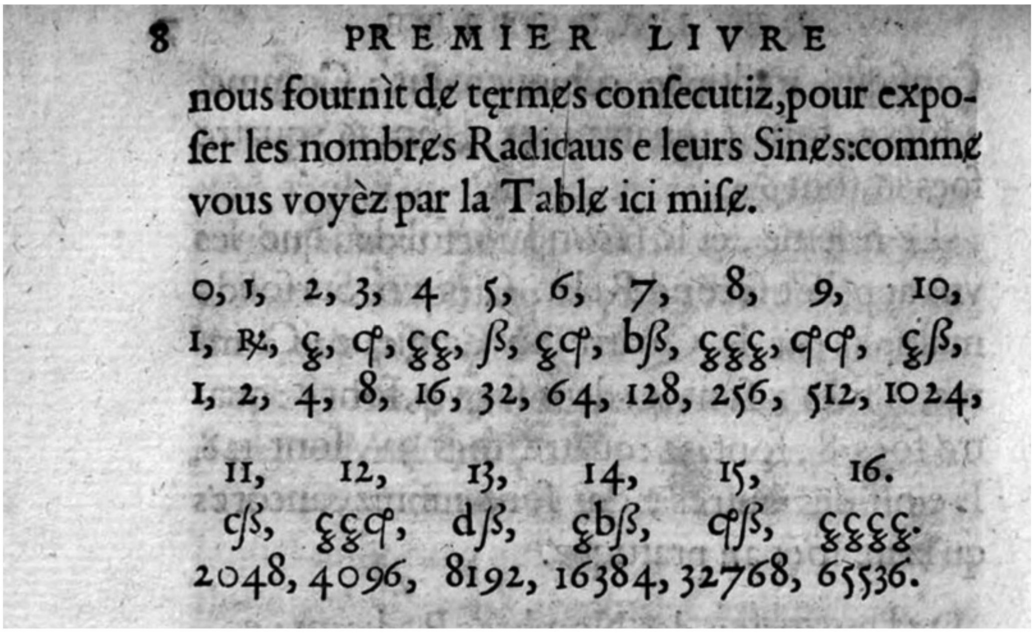


Stifel 1544 (after Cajori). This sample shows 𝔷 LOWERCASE KURRENT Z SIGN, ꝛ LOWERCASE R ROTUNDA WITH LOOP, c LOWERCASE C WITH RIGHT LOOP and ß DOUBLE S ABBREVIATION SIGN.



Aurel 1552, fol. 73B (after Cajori). This sample shows 𝔷 LOWERCASE KURRENT Z SIGN (2., 4., 6., 8.), ꝺ LOWERCASE D ROTUNDA WITH CROSSING LOOP (0.), ꝛ LOWERCASE R ROTUNDA WITH LOOP (1.), c LOWERCASE C WITH RIGHT LOOP 3., 6., 9.), and ß DOUBLE S ABBREVIATION SIGN (7.).

These samples also show how those characters were used in combination to express the powers 4th and so on.

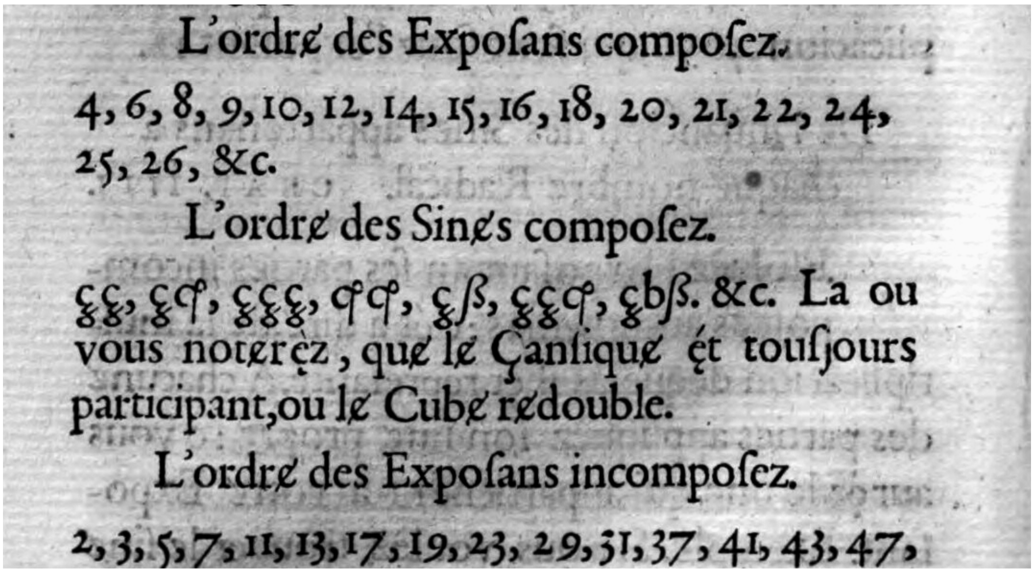


8 PREMIER LIVRE

nous fournìt de termes consecutiz,pour expo-
ser les nombres Radicaus e leurs Sines:comme
vous voyèz par la Table ici mise.

0,	1,	2,	3,	4	5,	6,	7,	8,	9,	10,
1,	℞,	ꞔ,	ꝯ,	ꞔꞔ,	ß,	ꞔꝯ,	bß,	ꞔꞔꞔ,	ꝯꝯ,	ꞔß,
1,	2,	4,	8,	16,	32,	64,	128,	256,	512,	1024,

11,	12,	13,	14,	15,	16.
cß,	ꞔꞔꝯ,	dß,	ꞔbß,	ꝯß,	ꞔꞔꞔꞔ.
2048,	4096,	8192,	16384,	32768,	65536.



L'ordre des Exposans composez.

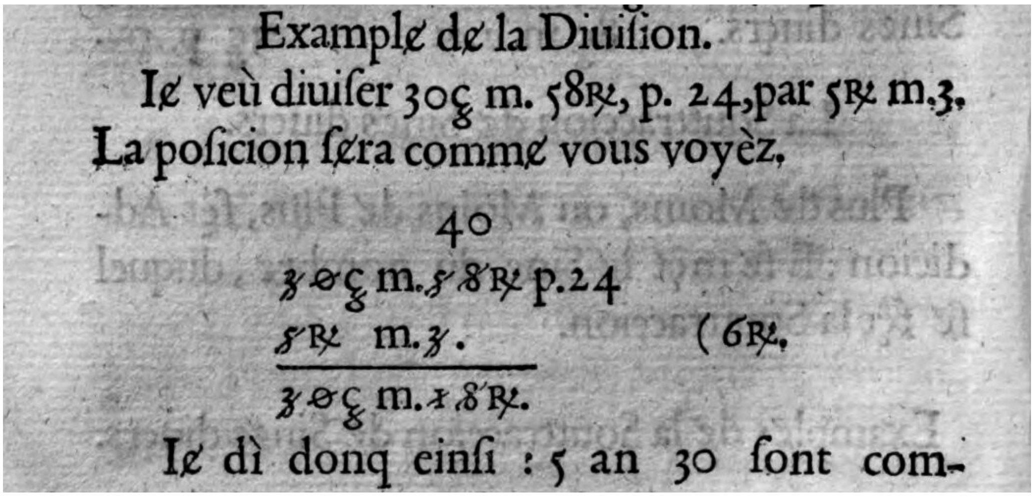
4,6,8,9,10,12,14,15,16,18,20,21,22,24,
25,26,&c.

L'ordre des Sines composez.

ꞔꞔ, ꞔꝯ, ꞔꞔꞔ, ꝯꝯ, ꞔß, ꞔꞔꝯ, ꞔbß. &c. La ou
vous noterèz, que le Çansique ét tousjours
participant,ou le Cube redouble.

L'ordre des Exposans incomposez.

2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,



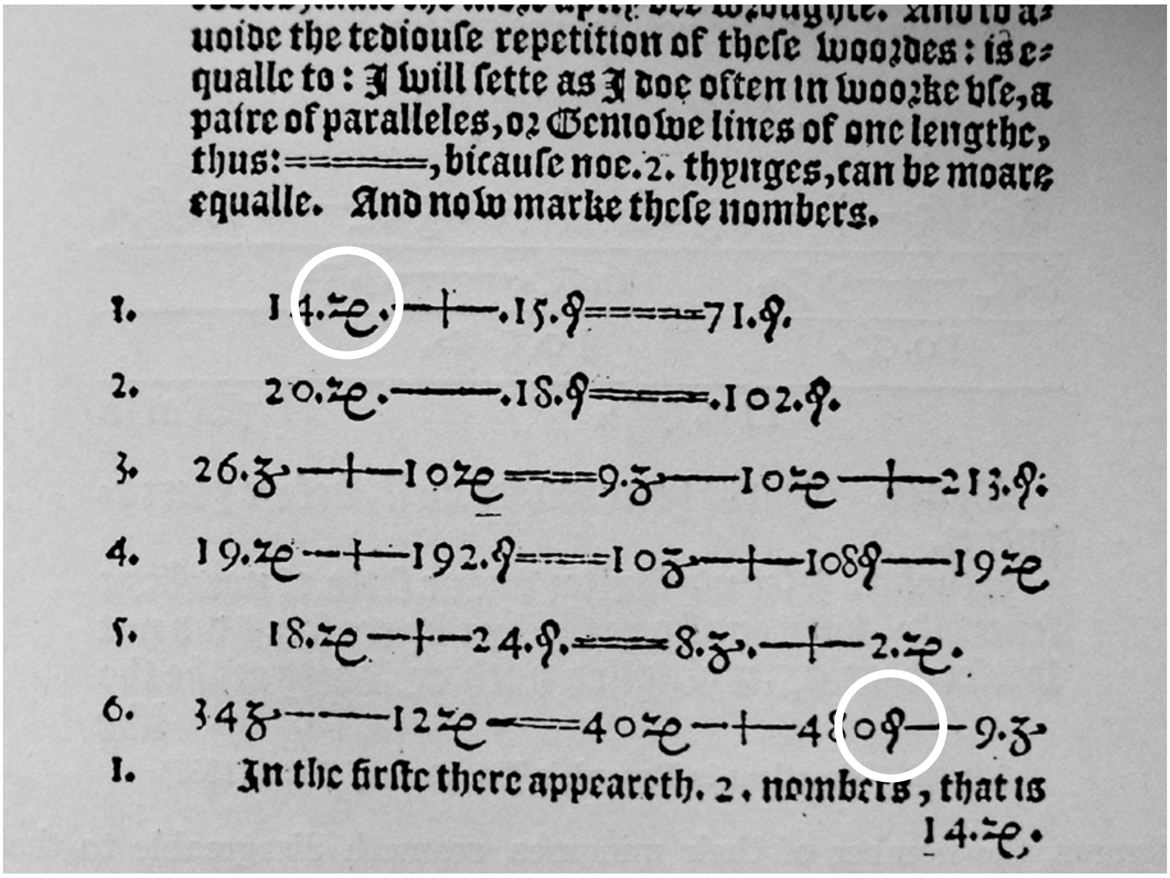
Example de la Diuision.

Ie veù diuiser 30ꞔ m. 58℞, p. 24,par 5℞ m.3.
La posicion sera comme vous voyèz.

40
30ꞔ m.58℞ p.24
5℞ m.3. (6℞,
30ꞔ m.18℞.

Ie dì donq einsi : 5 an 30 sont com-

Three extracts from Peletier 1554: ꞔ LOWERCASE C WITH DESCENDER, ꝯ LOWERCASE C WITH RIGHT LOOP, ℞ SMALL CAPITAL R WITH SLASH and ß DOUBLE S ABBREVIATION SIGN.

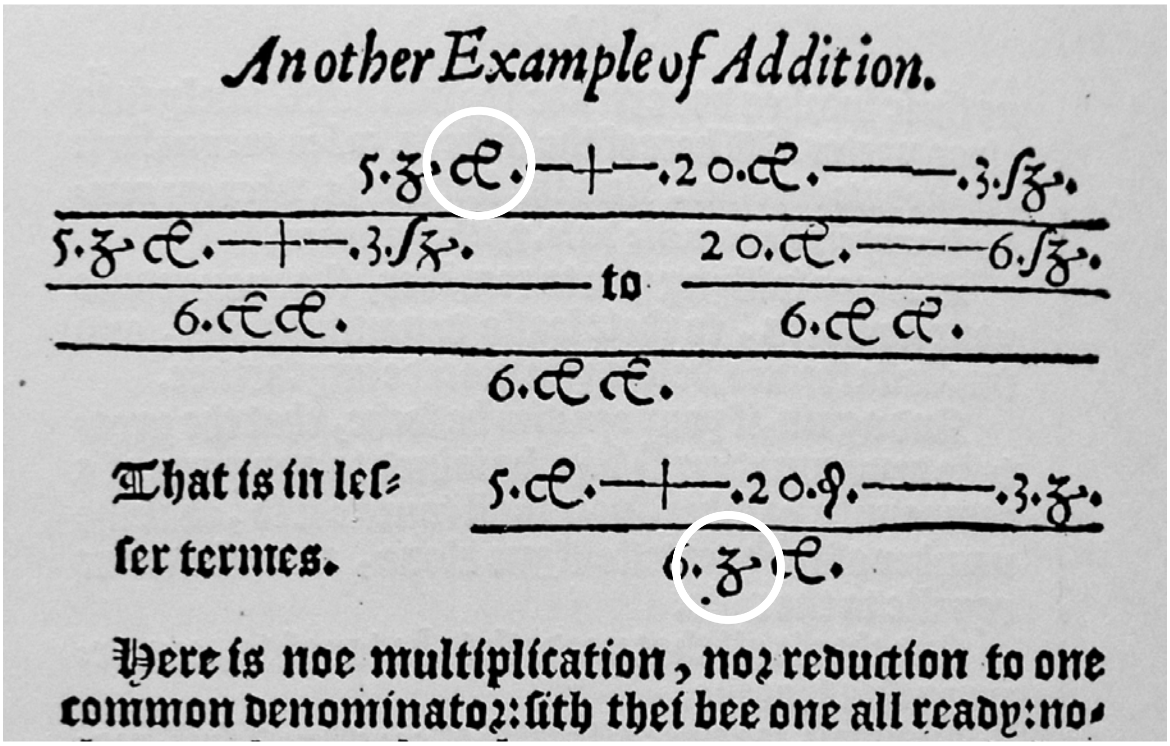


uoide the tediouse repetition of these woordes: is equalle to: I will sette as I doe often in woorke use, a paire of paralleles, or Gemowe lines of one lengthe, thus: =====, bicause noe .2. thynges, can be moare equalle. And now marke these nombers.

1. 14.ꝛ. —+— .15.ꝺ ===== 71.ꝺ.
2. 20.ꝛ. ——— .18.ꝺ ===== .102.ꝺ.
3. 26.ʒ —+— 10ꝛ ===== 9.ʒ ——— 10ꝛ —+— 213.ꝺ.
4. 19.ꝛ —+— 192.ꝺ ===== 10ʒ —+— 108ꝺ ——— 19ꝛ
5. 18.ꝛ —+— 24.ꝺ. ===== 8.ʒ. —+— 2.ꝛ.
6. 34ʒ ——— 12ꝛ ===== 40ꝛ —+— 480ꝺ ——— 9.ʒ.

1. In the firste there appeareth .2. nombers, that is 14.ꝛ.

P



Another Example of Addition.

5.ʒ.ꞔ. —+— .20.ꞔ. ——— .3.∫ʒ.

5.ʒ.ꞔ. —+— .3.∫ʒ. / 6.ꞔꞔ. to 20.ꞔ. ——— 6.∫ʒ. / 6.ꞔꞔ.

6.ꞔꞔ.

That is in lesser termes. 5.ꞔ. —+— .20.ꝺ. ——— .3.ʒ. / 6.ʒ.ꞔ.

Here is noe multiplication, nor reduction to one common denominator: sith thei bee one all readie: no-

Two extracts from Recorde 1557 (after Cajori): ꞔ LOWERCASE C WITH RIGHT LOOP, ꝺ LOWERCASE D ROTUNDA WITH CROSSING LOOP, ꝛ LOWERCASE R ROTUNDA WITH LOOP and ʒ LOWERCASE KURRENT Z SIGN.

this, a diuerse *Arithmetike* from the other. Practise bryngeth in, here, diuerse compoundyng of Numbers: as some tyme, two, three, foure (or more) *Radicall* nūbers, diuersly knit, by signes, of More & Lesse: as thus √ʒ 12 + √ꝯ 15. Or thus √ʒʒ 19 + √ꝯ 12 — √ʒ 2. &c. And some tyme with whole numbers, or fractions of whole Number, amōg them: as 20 + √ʒ 24. √ꝯ 16 + 33 — √ʒ 10. √ʒʒ 44 + 12 ½ + √ꝯ 9. And so infinitely, may hap the varietie. After this: Both the one and the other.

Example from Dee 1570 (after Cajori): ꝯ LOWERCASE C WITH RIGHT LOOP and ʒ LOWERCASE KURRENT Z SIGN.

0,	1,	2,	3,	4,	5,	6,	7,	8,	9,	10,
1,	R,	ʒ,	ꝯ,	ʒʒ,	ß,	ʒꝯ,	bß,	ʒʒʒ,	ꝯꝯ,	ʒß,
1,	2,	4,	8,	16,	32,	64,	128,	256,	512,	1024,

11,	12,	13,	14,	15,	16,
cß,	ʒʒꝯ,	dß,	ʒbß,	ꝯß,	ʒʒʒʒ,
2048,	4096,	8192,	16384,	32768,	65536.

From Peletier 1620.

CAP. XXVIII. 159

Sit rursus Binomium primum 72 + √ʒ 2880. Maius nomen 72. secabitur in duas partes producentes 720. quartam partem quadrati 2880. maioris nominis, hac ratione. Semissis maioris nominis 72. est 36. a cuius quadrato 1296. detracta quarta pars prædicta 720. relinquit 576. cuius radix 24. addita ad semissem nominatam 36. & detracta ab eadem, facit partes quæsitas 60. & 12. Ergo radix Binomij est √ʒ 60 + √ʒ 12. quod hic probatum est per multiplicationem radicis in se quadratè.

√ʒ 60 + √ʒ 12
√ʒ 60 + √ʒ 12
60 + √ʒ 720 + 12
60 + √ʒ 720
72 + √ʒ 2880

Sit quoque elicienda radix ex hoc residuo sexto √ʒ 60 — √ʒ 12. Maius nomen √ʒ 60. distribuetur in duas partes producētes 3. quartam partem quadrati 12. minoris nominis, hoc pacto. Semissis maioris nominis √ʒ 60. est √ʒ 15. a cuius quadrato 15. detracta nominata pars quarta 3. relinquit 12. cuius radix √ʒ 12. addita ad semissem √ʒ 15. prædictam, & ab eadem sublata facit partes √ʒ 15 + √ʒ 12. & √ʒ 15 — √ʒ 12. Ergo radix dicti Residui sexti est √ʒ (√ʒ 15 + √ʒ 12) — √ʒ (√ʒ 15 — √ʒ 12) quod hic probatum est.

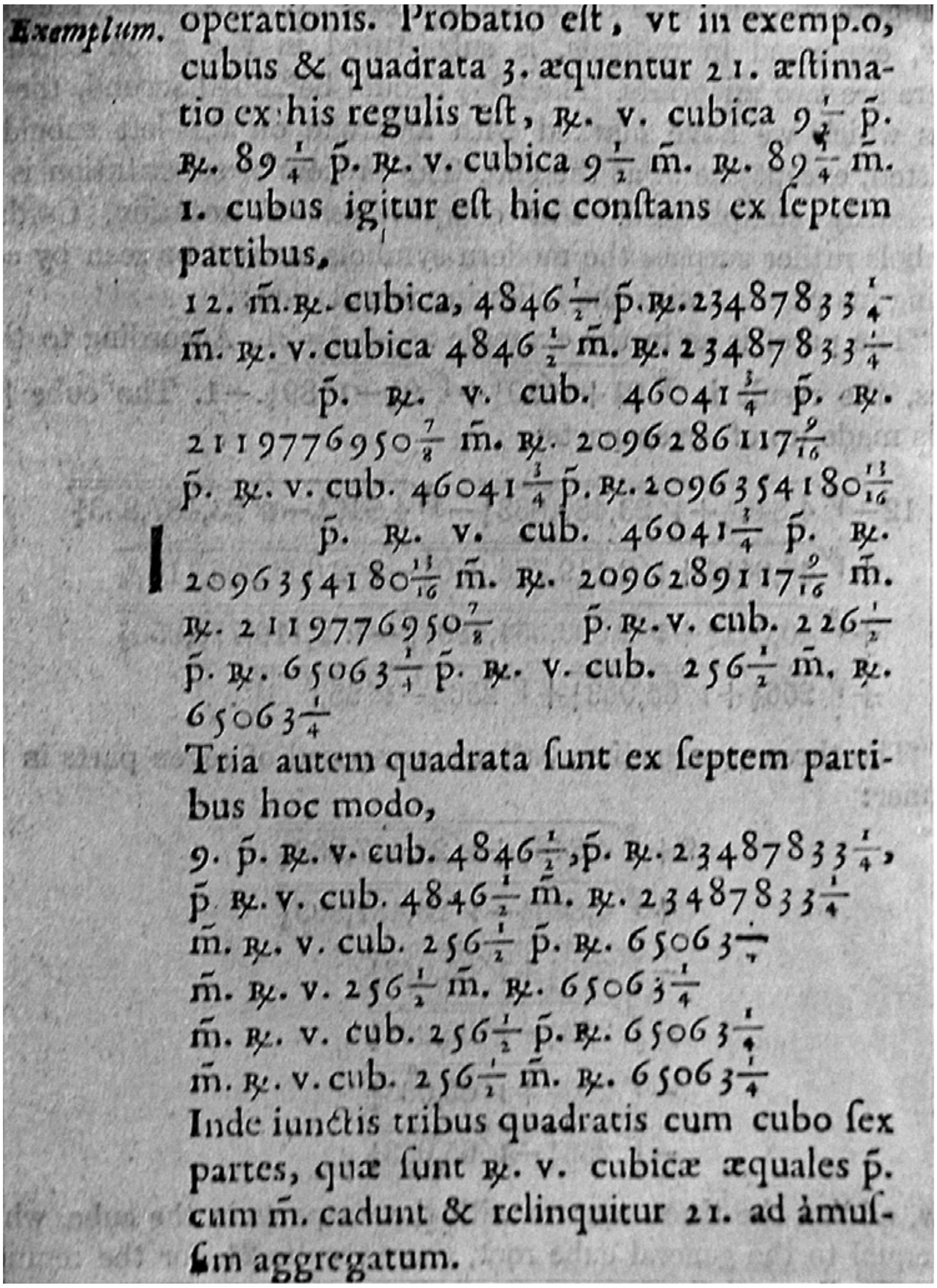
√ʒ (√ʒ 15 + √ʒ 12) — √ʒ (√ʒ 15 — √ʒ 12)
√ʒ (√ʒ 15 + √ʒ 12) — √ʒ (√ʒ 15 — √ʒ 12)

Quadrata partium. √ʒ 15 + √ʒ 12 & √ʒ 15 — √ʒ 12
— √ʒ 3
— √ʒ 3

Summa. √ʒ 60 — √ʒ 12

Clavius 1608 (after Cajori): ʒ LOWERCASE KURRENT Z SIGN.

From Wallis, Operum mathematicorum, 1657 (after Cajori) shows the use of ꟓ LOWERCASE LONG S WITH TOP LOOP, an abbreviation sign based on the letter long s.



Exemplum. operationis. Probatio eſt, vt in exemplo, cubus & quadrata 3. æquentur 21. æſtimatio ex his regulis eſt, ℞. v. cubica $9\frac{1}{2}$ p̄. ℞. $89\frac{1}{4}$ p̄. ℞. v. cubica $9\frac{1}{2}$ m̄. ℞. $89\frac{1}{4}$ m̄. 1. cubus igitur eſt hic conſtans ex ſeptem partibus,

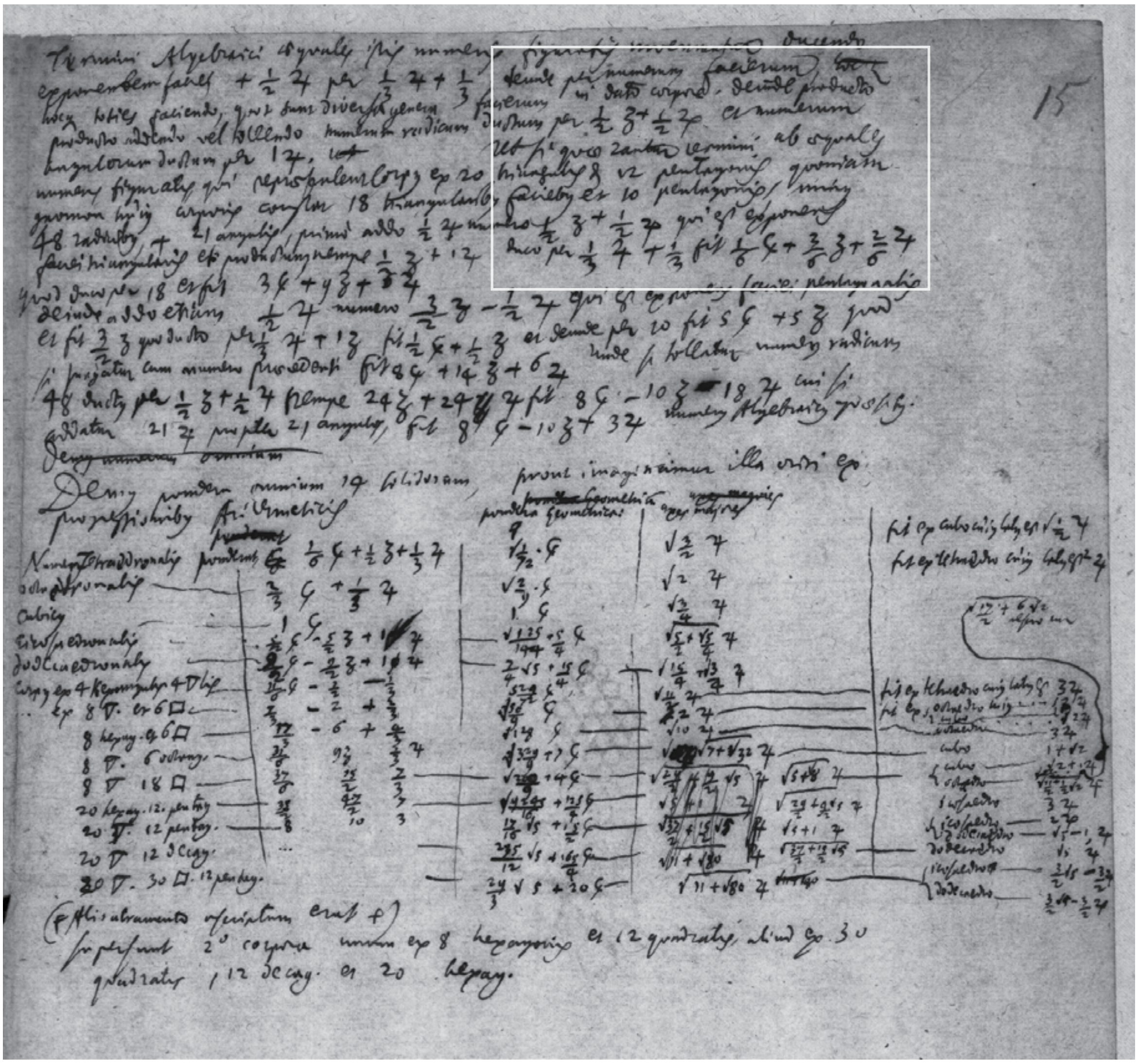
12. m̄. ℞. cubica, $4846\frac{1}{2}$ p̄. ℞. $23487833\frac{1}{4}$
m̄. ℞. v. cubica $4846\frac{1}{2}$ m̄. ℞. $23487833\frac{1}{4}$
p̄. ℞. v. cub. $46041\frac{3}{4}$ p̄. ℞. $2119776950\frac{7}{8}$ m̄. ℞. $2096286117\frac{9}{16}$
p̄. ℞. v. cub. $46041\frac{3}{4}$ p̄. ℞. $2096354180\frac{13}{16}$
p̄. ℞. v. cub. $46041\frac{3}{4}$ p̄. ℞. $2096354180\frac{13}{16}$ m̄. ℞. $2096289117\frac{9}{16}$ m̄. ℞. $2119776950\frac{7}{8}$
p̄. ℞. v. cub. $226\frac{1}{2}$ p̄. ℞. $65063\frac{1}{4}$ p̄. ℞. v. cub. $256\frac{1}{2}$ m̄. ℞. $65063\frac{1}{4}$

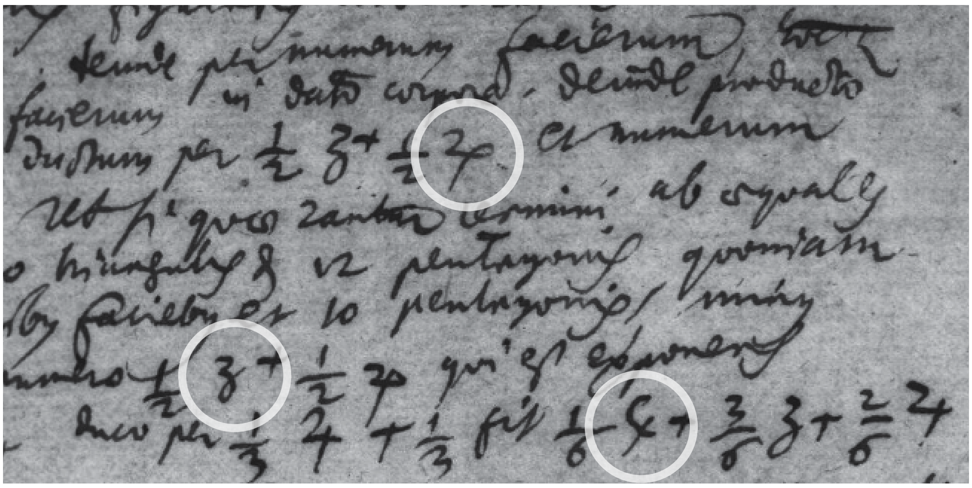
Tria autem quadrata ſunt ex ſeptem partibus hoc modo,

9. p̄. ℞. v. cub. $4846\frac{1}{2}$, p̄. ℞. $23487833\frac{1}{4}$,
p̄ ℞. v. cub. $4846\frac{1}{2}$ m̄. ℞. $23487833\frac{1}{4}$
m̄. ℞. v. cub. $256\frac{1}{2}$ p̄. ℞. $65063\frac{1}{4}$
m̄. ℞. v. $256\frac{1}{2}$ m̄. ℞. $65063\frac{1}{4}$
m̄. ℞. v. cub. $256\frac{1}{2}$ p̄. ℞. $65063\frac{1}{4}$
m̄. ℞. v. cub. $256\frac{1}{2}$ m̄. ℞. $65063\frac{1}{4}$

Inde iunctis tribus quadratis cum cubo ſex partes, quæ ſunt ℞. v. cubicæ æquales p̄. cum m̄. cadunt & relinquitur 21. ad amuſſim aggregatum.

A page from Cardano 1663 (after Cajori) shows a frequent use of ℞ SMALL CAPITAL R WITH SLASH.



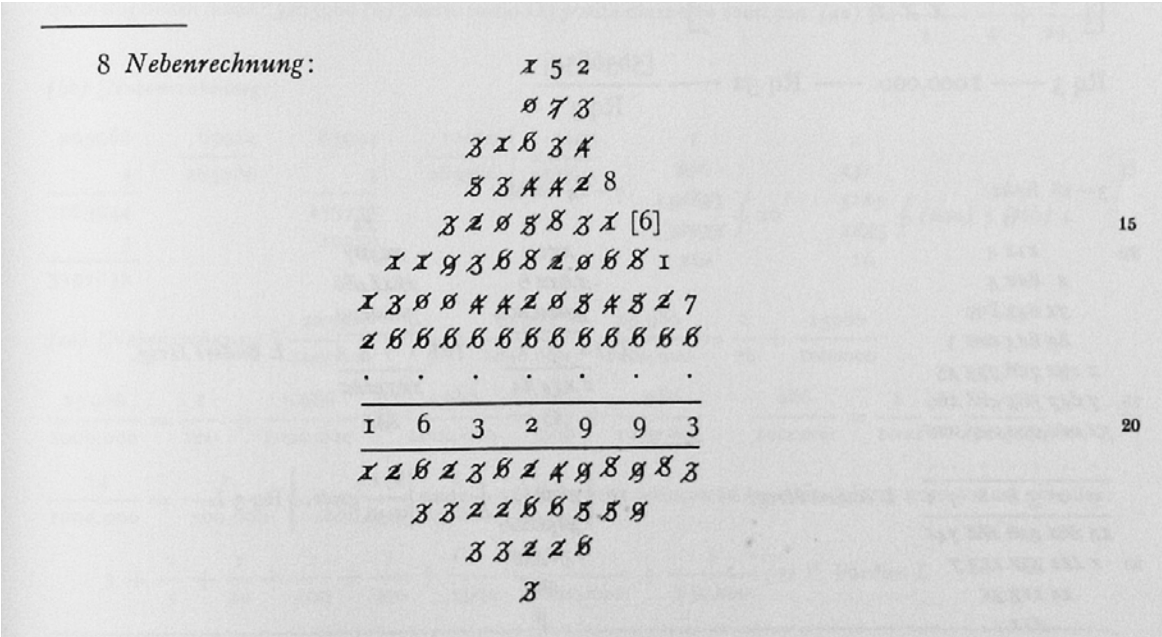


Ms. LH 4 I 4b 1v., Leibniz 1676, shows a frequent use of cossic signs: ç LOWERCASE C WITH SMALL SLASH for *cubus*, ꝝ LOWERCASE R ROTUNDA WITH LOOP for *radix* (which, in this case, gets often a simplified shape without the loop) and ʒ LOWERCASE KURRENT Z SIGN for *zensus*.
The use of the simplier ç instead of cꝝ for *cubus* is believed to originate from writings of Descartes, from who Leibniz (and other authors) made text copies.

	Glyph									
	Character	LOWERCASE D ROTUNDA WITH CROSSING LOOP	SMALL CAPITAL R WITH SLASH	LOWERCASE R ROTUNDA WITH LOOP	LOWERCASE KURRENT Z SIGN	LOWERCASE C WITH DESCENDER	LOWERCASE C WITH SMALL SLASH	LOWERCASE C WITH RIGHT LOOP	DOUBLE S ABBREVIATION SIGN	LOWERCASE LONG S WITH TOP LOOP
	Meaning	dragma	radix	radix	zensus	census	cubus	cubus	solidus sursolidum semis	sursolidum
1	Rudolf 1525									
2	Stifel 1544									
3	Aurel 1552									
4	Peletier 1554									
5	Recorde 1557									
6	Dee 1570									
7	Peletier 1620									
8	Clavius 1608/12									
9	Beeckmann 1628									
10	Wallis 1657									
11	Cardano 1663									
12	Leibniz MS 1676									
13	MS Leiden 17. c.									
14	MS Hamburg 17. c.									

Comparative survey of Coss symbols in various sources, 1525 to 1676.

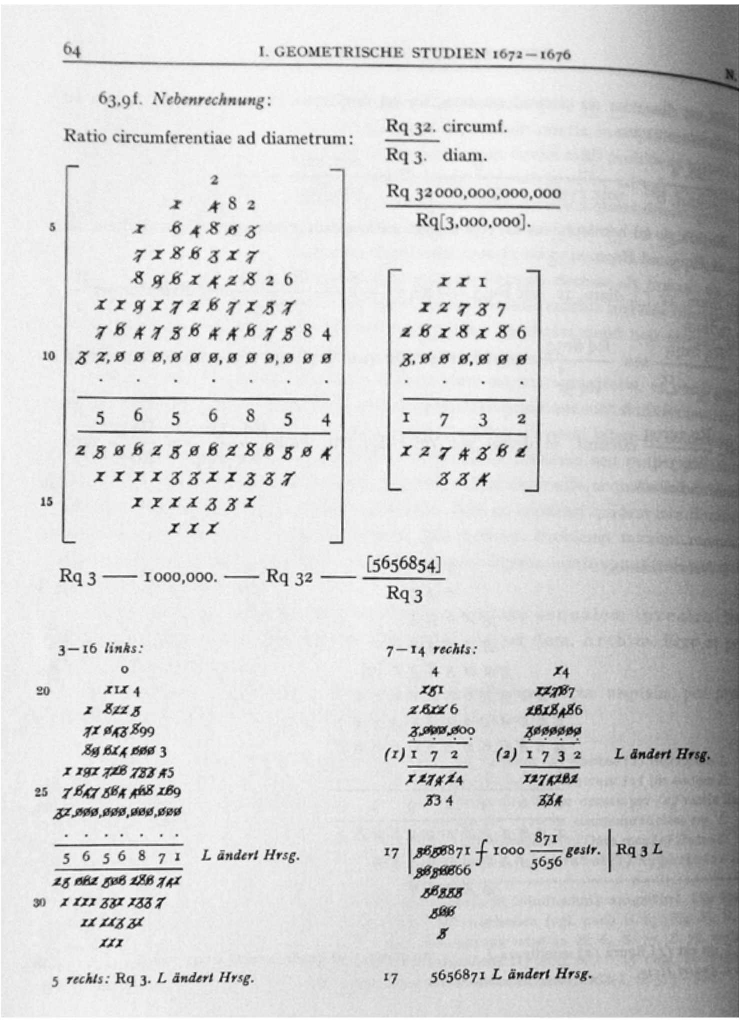
4.k) Digit characters



8 *Nebenrechnung*:

1 6 3 2 9 9 3

Ø 1̸ 2̸ 3̸ 4̸ 5̸ 6̸ 7̸ 8̸ 9̸ SLASHED DIGITS ZERO to NINE.
LAA VII-1 p. 63 (top), 64 (below)



64 I. GEOMETRISCHE STUDIEN 1672–1676

63,9f. *Nebenrechnung*:

Ratio circumferentiae ad diametrum: Rq 32. circumf. / Rq 3. diam.

Rq 32000,000,000,000 / Rq[3,000,000].

5 6 5 6 8 5 4

1 7 3 2

Rq 3 —— 1000,000. —— Rq 32 —— [5656854] / Rq 3

3–16 *links:*

5 6 5 6 8 7 1 *L ändert Hrsg.*

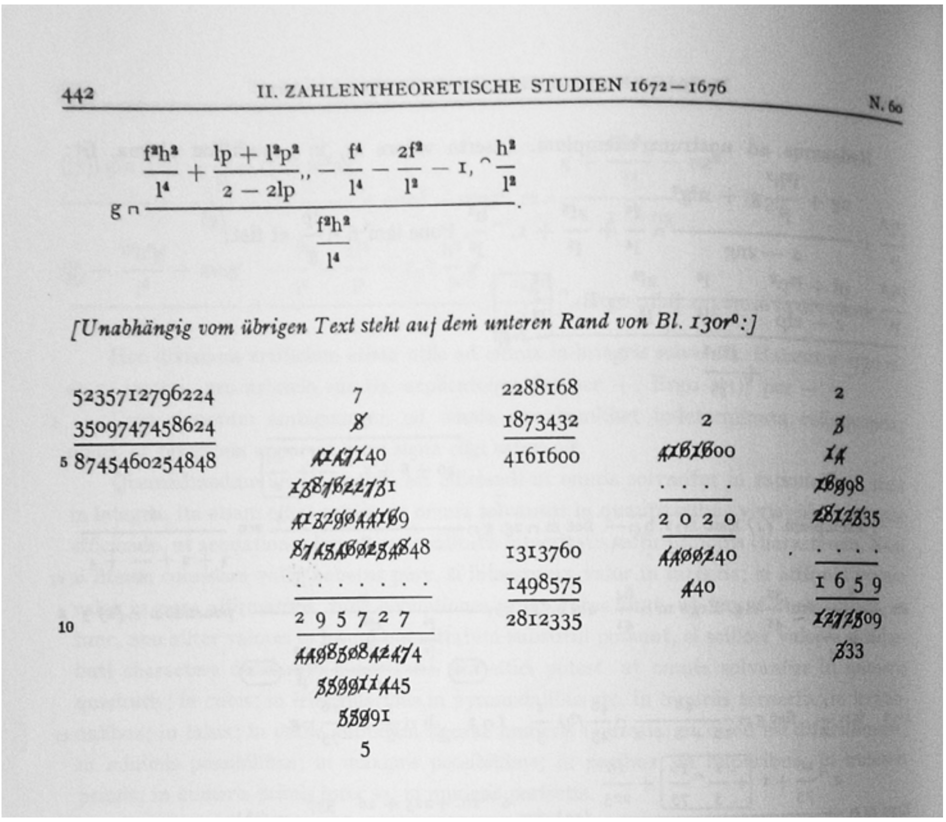
7–14 *rechts:*

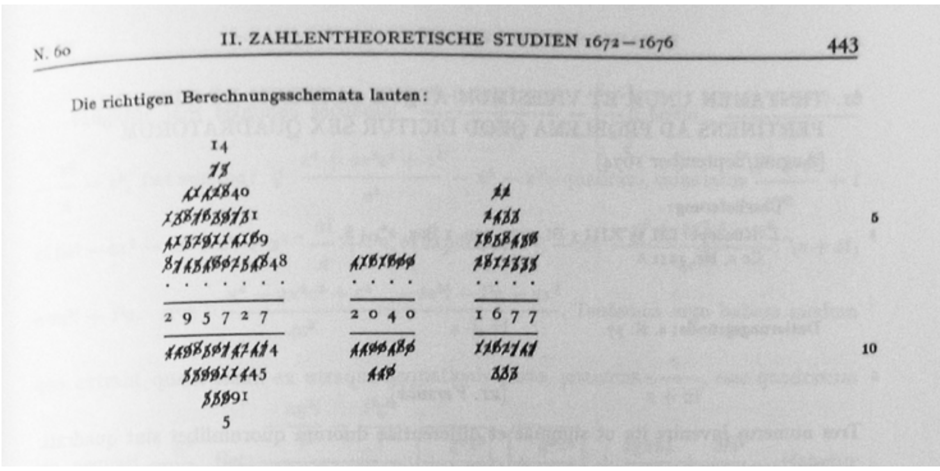
(1) 1 7 2 (2) 1 7 3 2 *L ändert Hrsg.*

17 ... ⨍ 1000 871/5656 *gestr.* | Rq 3 *L*

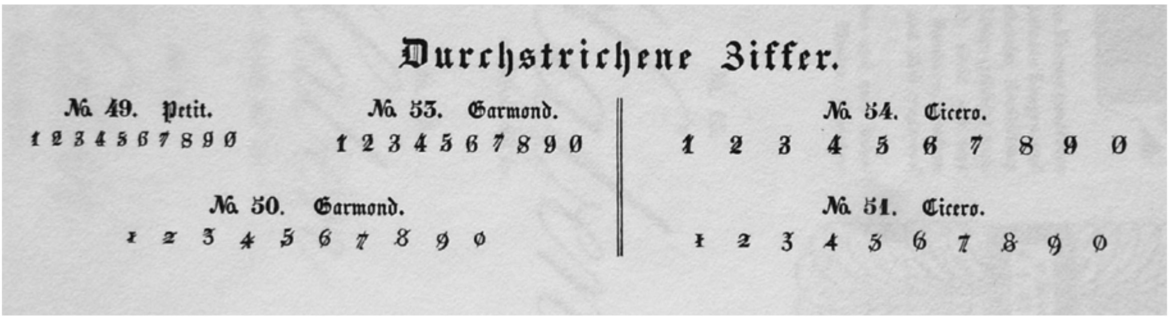
5 *rechts:* Rq 3. *L ändert Hrsg.*

17 5656871 *L ändert Hrsg.*

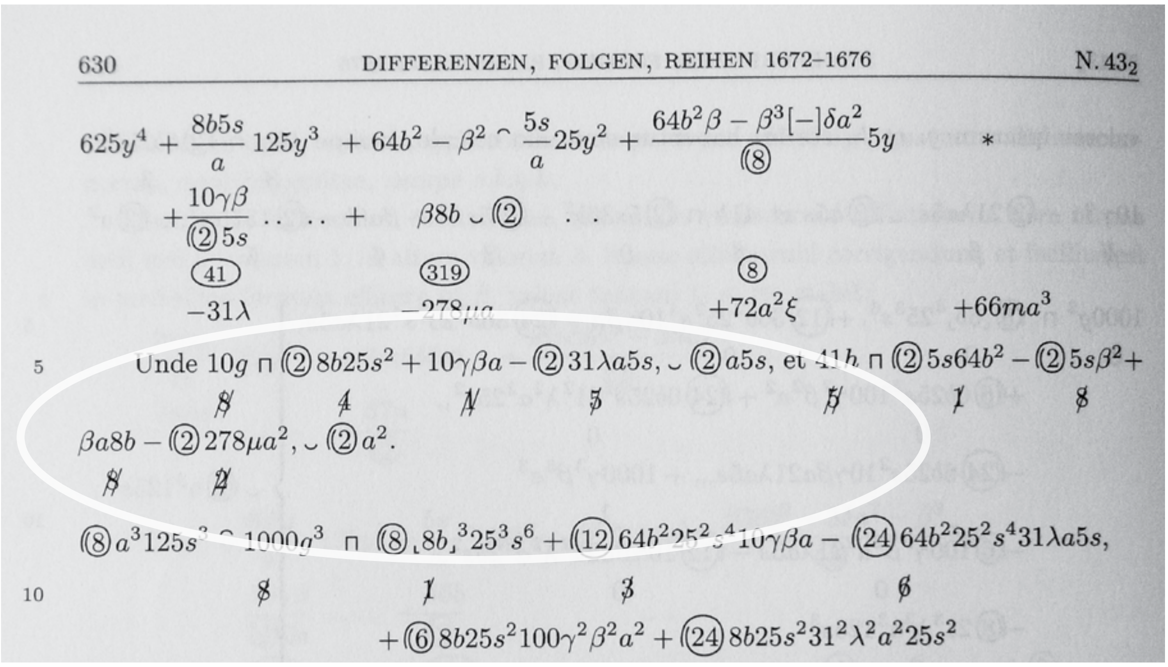




0̸ 1̸ 2̸ 3̸ 4̸ 5̸ 6̸ 7̸ 8̸ 9̸ SLASHED DIGITS ZERO to NINE.
LAA VII-1 p. 442, 443



0̸ 1̸ 2̸ 3̸ 4̸ 5̸ 6̸ 7̸ 8̸ 9̸ SLASHED DIGITS ZERO to NINE.
From Andreä's Type specimen book, 1834


630 DIFFERENZEN, FOLGEN, REIHEN 1672–1676 N. 43₂

$625y^4 + \frac{8b5s}{a}125y^3 + 64b^2 - \beta^2 \frown \frac{5s}{a}25y^2 + \frac{64b^2\beta - \beta^3[-]\delta a^2}{⑧}5y$ *

$+\frac{10\gamma\beta}{②5s} \ldots \ldots + \beta 8b \smile ② \ldots$

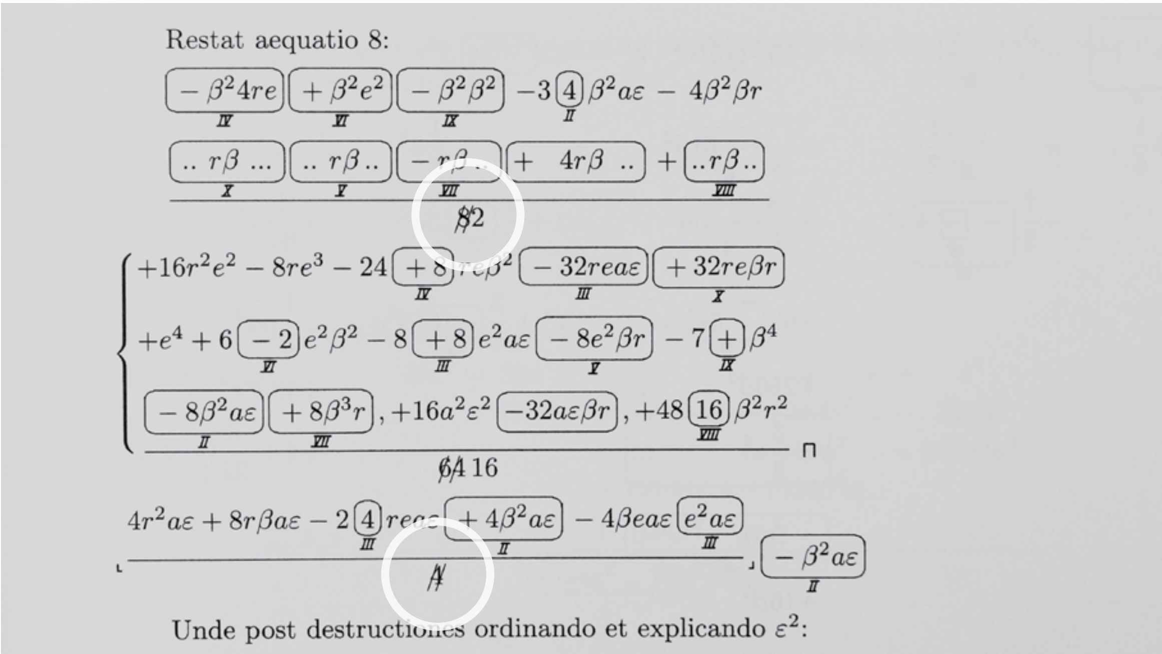
-31λ (41) $\quad -278\mu a$ (319) $\quad +72a^2\xi$ (8) $\quad +66ma^3$

Unde $10g \sqcap ②8b25s^2 + 10\gamma\beta a - ②31\lambda a5s, \smile ②a5s$, et $41h \sqcap ②5s64b^2 - ②5s\beta^2 + \beta a8b - ②278\mu a^2, \smile ②a^2$.

$⑧a^3125s^3 \frown 1000g^3 \sqcap ⑧\llcorner 8b\lrcorner^3 25^3s^6 + ⑫64b^2 25^2s^4 10\gamma\beta a - ㉔64b^2 25^2 s^4 31\lambda a5s,$

$+ ⑥8b25s^2 100\gamma^2\beta^2a^2 + ㉔8b25s^2 31^2\lambda^2 a^2 25s^2$

DOUBLE SLASHED DIGITS (2, 5, 8).
LAA VII-3 p. 630


Restat aequatio 8:

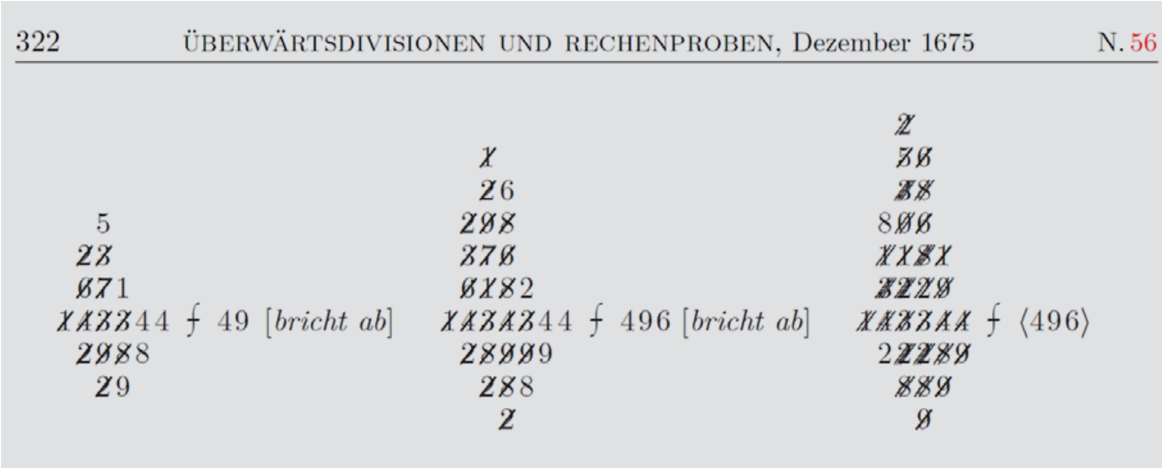
$\frac{-\beta^2 4re + \beta^2e^2 - \beta^2\beta^2 - 3④\beta^2 a\varepsilon - 4\beta^2\beta r \ldots r\beta \ldots r\beta \ldots - r\beta \ldots + 4r\beta \ldots + \ldots r\beta \ldots}{82}$

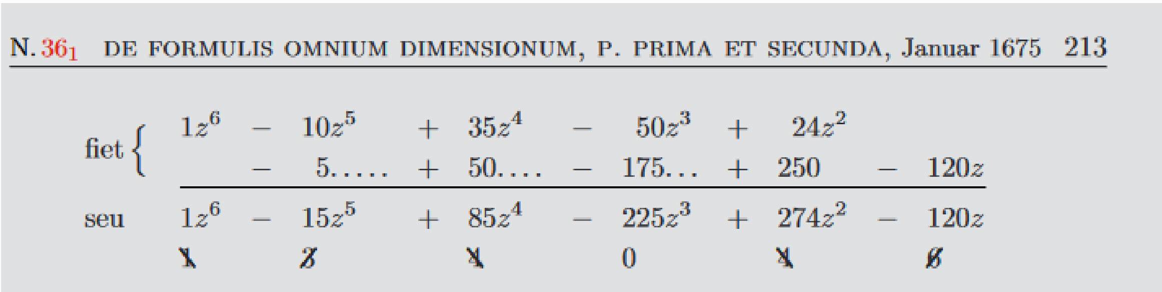
$\frac{+16r^2e^2 - 8re^3 - 24(+8)re\beta^2 - 32rea\varepsilon + 32re\beta r + e^4 + 6(-2)e^2\beta^2 - 8(+8)e^2a\varepsilon - 8e^2\beta r - 7(+)\beta^4 - 8\beta^2 a\varepsilon + 8\beta^3 r, +16a^2\varepsilon^2 - 32a\varepsilon\beta r, +48(16)\beta^2r^2}{64\ 16} \sqcap$

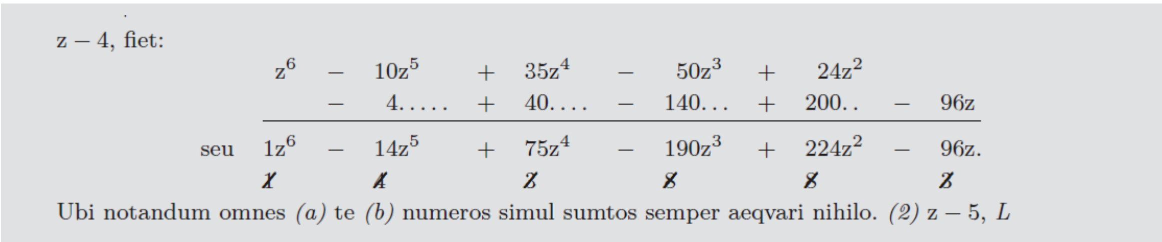
$\llcorner \frac{4r^2a\varepsilon + 8r\beta a\varepsilon - 2④rea\varepsilon + 4\beta^2 a\varepsilon - 4\beta ea\varepsilon\ e^2a\varepsilon}{4} \lrcorner - \beta^2 a\varepsilon$

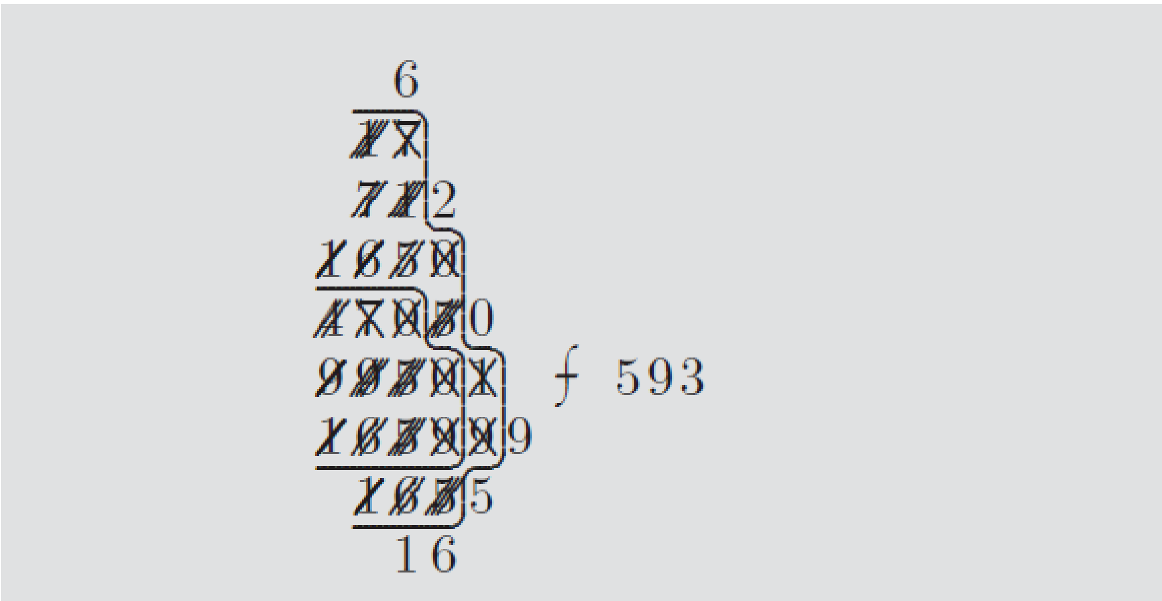
Unde post destructiones ordinando et explicando ε^2:

DOUBLE SLASHED DIGITS (4, 8).
LAA VII-3 p. 657

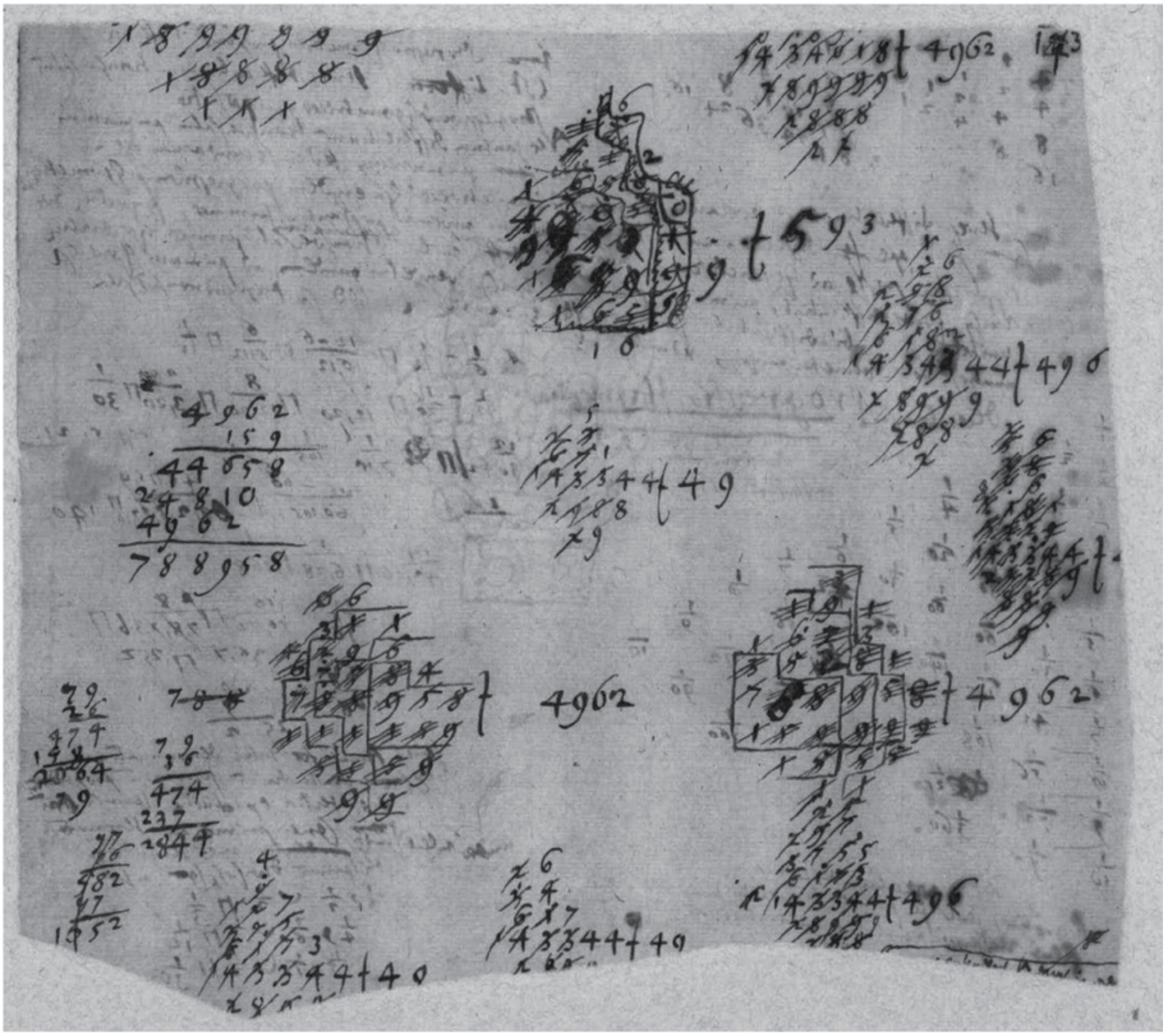




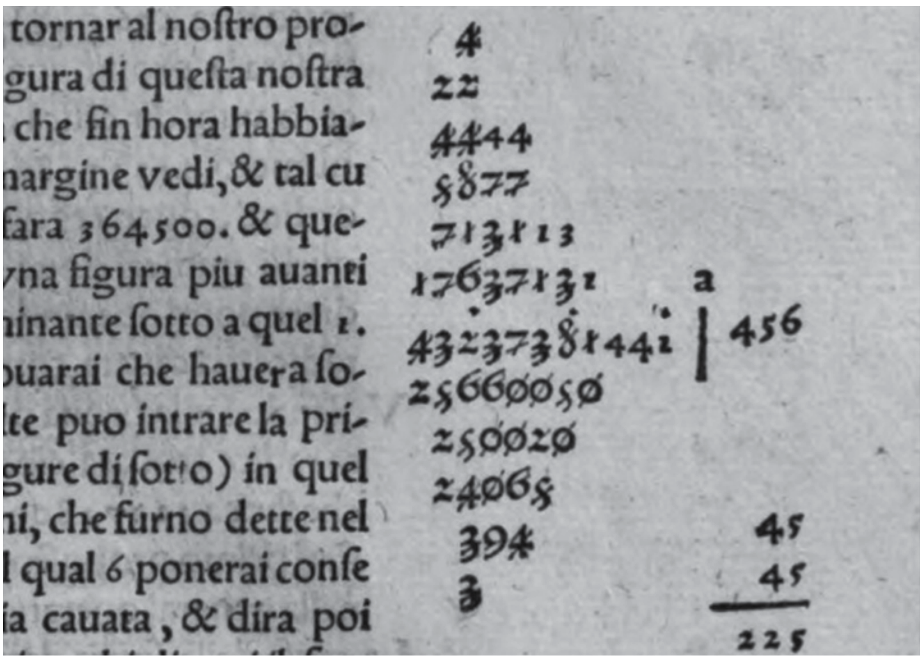




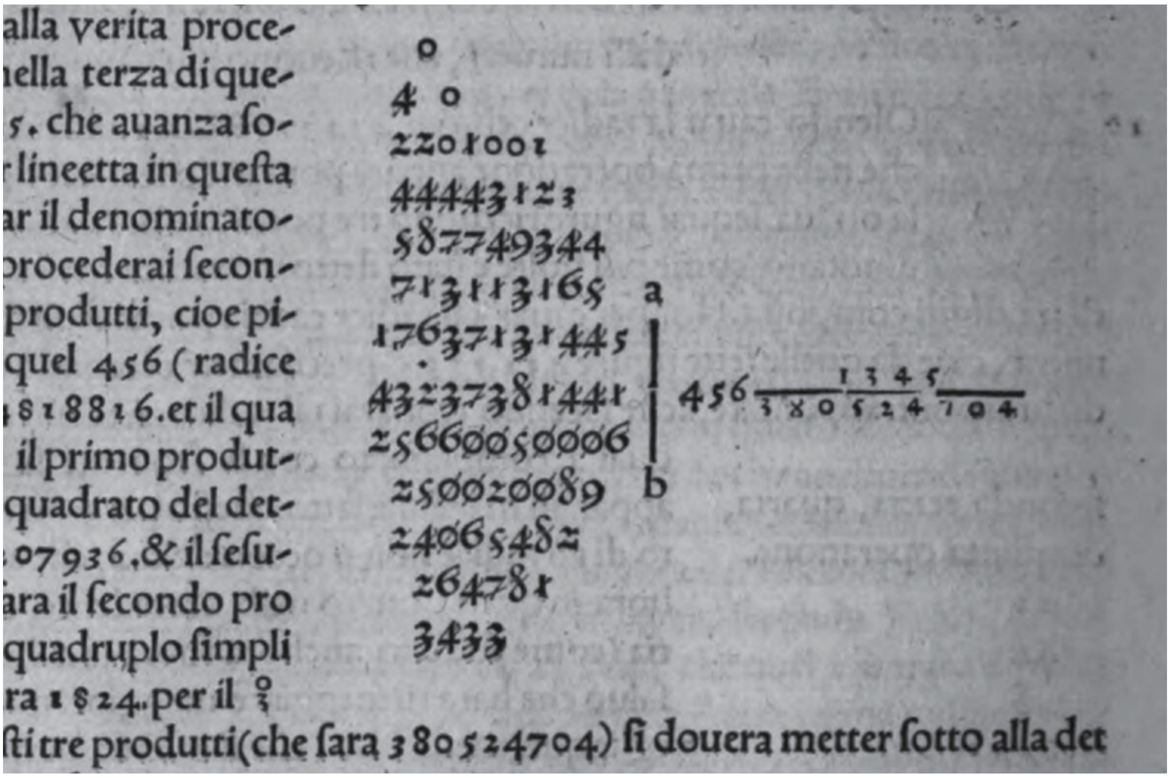
Various examples of SLASHED DIGITS, DOUBLE SLASHED DIGITS, BACKSLASHED DIGITS, TRIPLE SLASHED DIGITS and CROSSED DIGITS.
LAA VII-8 (preliminary edition).



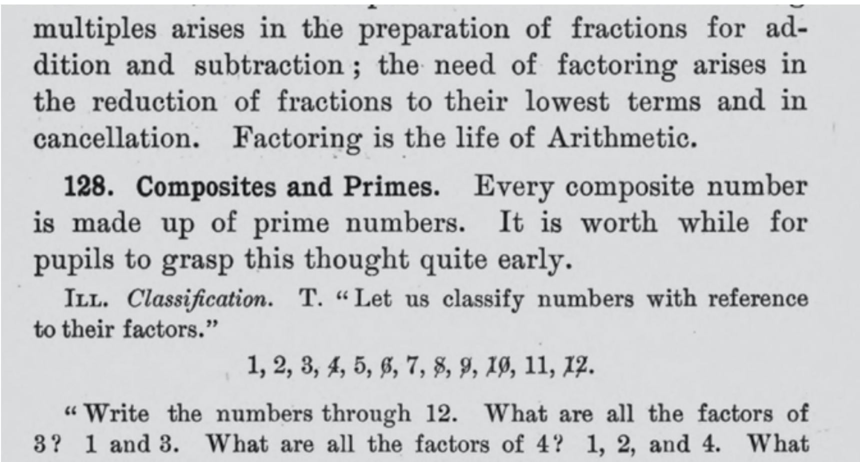
Example of the use of digits in various strike modes; in a Leibniz manuscript.
This sheet shows also the use of ⨍ FACIT SIGN.
LH 12 I fol. 250 v


tornar al noſtro pro-
gura di queſta noſtra
che ſin hora habbia-
nargine vedi, & tal cu
ſara 364500. & que-
vna figura piu auanti
ninante ſotto a quel 1.
ouarai che hauera ſo-
lte puo intrare la pri-
gure di ſotto) in quel
ni, che furno dette nel
l qual 6 ponerai conſe
ia cauata, & dira poi

Use of slashed digits in: La seconda Parte Del General Trattato Di Nvmeri, Et Misvre Di Nicolo Tartaglia (1556), fol. 37r


alla verita proce-
nella terza di que-
5. che auanza so-
r lineetta in questa
ar il denominato-
procederai secon-
produtti, cioe pi-
quel 456 (radice
181 88 16. et il qua
il primo produt-
quadrato del det-
07936, & il sesu-
ara il secondo pro
quadruplo simpli
ra 1 8 24. per il
sti tre produtti (che sara 380524704) si douera metter sotto alla det

Use of slashed digits in:
La seconda Parte Del General Trattato Di Nvmeri, Et Misvre Di Nicolo Tartaglia (1556), fol. 37v


multiples arises in the preparation of fractions for addition and subtraction; the need of factoring arises in the reduction of fractions to their lowest terms and in cancellation. Factoring is the life of Arithmetic.

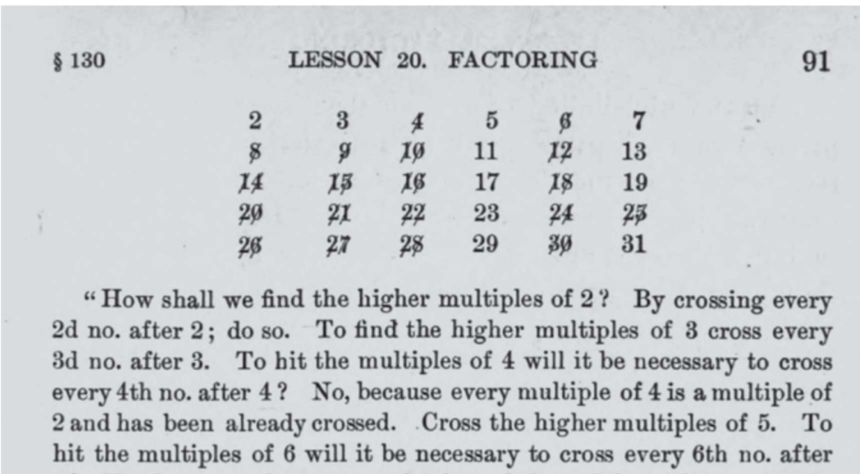
128. Composites and Primes. Every composite number is made up of prime numbers. It is worth while for pupils to grasp this thought quite early.

Ill. *Classification.* T. "Let us classify numbers with reference to their factors."

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.

"Write the numbers through 12. What are all the factors of 3? 1 and 3. What are all the factors of 4? 1, 2, and 4. What

Examples of slashed digits in Bailey (1913).


§ 130 LESSON 20. FACTORING 91

2	3	4	5	6	7
8	9	10	11	12	13
14	15	16	17	18	19
20	21	22	23	24	25
26	27	28	29	30	31

"How shall we find the higher multiples of 2? By crossing every 2d no. after 2; do so. To find the higher multiples of 3 cross every 3d no. after 3. To hit the multiples of 4 will it be necessary to cross every 4th no. after 4? No, because every multiple of 4 is a multiple of 2 and has been already crossed. Cross the higher multiples of 5. To hit the multiples of 6 will it be necessary to cross every 6th no. after

156 DIVISION

Further Work in Averages. A sheep raiser finds that ten of his sheep together weigh 1813 lb., and he wishes to find their average weight. What is this average weight?

If 10 sheep together weigh 1813 lb., their average weight is 1813 lb. ÷ 10.

We divide 1813 lb. by 10 in the manner here shown.

10̸)1813̸ lb.
$181\frac{3}{10}$ lb.

The teacher should explain at the board that we may divide 1810 by 10 by simply cutting off the last figure (0), as has already been shown on page 153. Since 1813 is 3 more than 1810, we have 181 for the whole number in the quotient, with a remainder 3 still to be divided; so the complete quotient is $181\frac{3}{10}$. The abbreviation may be used or not in the computation. In practice, it usually is not written.

Examples of slashed digits in Wentworth & Smith (1919).

192 MULTIPLICATION AND DIVISION

Divisor ending in Zeros. There are 2000 lb. in a ton. A dealer sells coal to-day in small quantities amounting in all to 24,000 lb. How many tons does he sell? How many tons would he sell if there were 24,357 lb.? 25,357 lb.?

We wish to know how many 2000's there are in each of these numbers, and we divide as follows:

20̸0̸0̸)240̸0̸0̸ 2000̸)243̸5̸7̸ 2000̸)253̸5̸7̸
12 $12\frac{357}{2000}$ $12\frac{1357}{2000}$

That is, we *cancel* (cross out) the zeros at the right of the divisor and cancel as many figures at the right of the dividend as we cancel zeros of the divisor, writing the complete remainder over the divisor.

148

4.3 DAS SUBTRAKTIONSVERFAHREN IN WICHTIGEN GASTARBEITERLÄNDERN

Auf der Grundlage der im Abschnitt 4.1.3 zusammenfassend dargestellten Typisierung können wir die Subtraktionsverfahren im Ausland, insbesondere in wichtigen *Gastarbeiterherkunftsländern*, rasch und knapp beschreiben (vgl. auch Ottmann (1982)):

Italien, Jugoslavien (z.T.), Portugal, Spanien, Türkei:

Abziehverfahren kombiniert mit der *Borgetechnik*

Das Entbündeln wird häufig *überhaupt nicht* kenntlich gemacht. In der *Türkei* wird (abweichend von der in 4.1.2.1 vorgestellten Schreibweise) das Entbündeln *folgendermaßen* schriftlich festgehalten:

$$\begin{array}{r} \scriptstyle 3\,5 \\ \not{4}\not{6}2 \\ -\,178 \\ \hline 284 \end{array}$$

Griechenland:

Abziehverfahren kombiniert mit der *Erweiterungstechnik*

Slashed digits in: Padberg 1986.

durchgestrichen und der nächstkleinere Zehner (bzw. Hunderter) hingeschrieben werden. Für manche Kinder macht diese Schreibform das Verfahren des Wechselns noch deutlicher, weil hier der verbliebene Zehner aufgeschrieben wird, so dass sie bei der nächsten Teilberechnung nicht erneut überlegen müssen, wie viele Zehner noch da sind. Diese Schreibweise sieht in der Zwischen- und Endform folgendermaßen aus:

Zwischenform	Endform
$\begin{array}{rrr} 7 & \not{5}^{4} & 2^{10} \\ -\,4 & 3 & 8 \\ \hline 3 & 1 & 4 \end{array}$	$\begin{array}{rrr} 7 & \not{5} & 2 \\ -\,4 & 3 & 8 \\ \hline 3 & 1 & 4 \end{array}$

Die schriftlich
die Kinder als
Schwierigkeit
- Fehler beir
- Fehler bei
- Fehler bei
- Fehler dur

Auch zur sch
(auch in Abele
als Kopiervorl
nen Überblick
se zu bekomm

Tabelle der Schwierigkeitsmerkmale beim diagno

Slashed digits in: Radatz 1999.

Unicode Character Properties

24-02.19. AS

a) Historical mathematical operators

```
A001;LEIBNIZIAN DIVISION SIGN;Sm;0;ON;;;;;N;;;;;
A002;LEIBNIZIAN PRODUCT SIGN;Sm;0;ON;;;;;N;;;;;
A003;LEIBNIZIAN DIVISION-PRODUCT SIGN;Sm;0;ON;;;;;N;;;;;
A004;LEIBNIZIAN DIVISION STAFF SIGN 1;Sm;0;ON;;;;;N;;;;;
A005;LEIBNIZIAN DIVISION STAFF SIGN 2;Sm;0;ON;;;;;N;;;;;
```

b) Historical mathematical relations

```
B001;LEIBNIZIAN EQUAL SIGN;Sm;0;ON;;;;;N;;;;;
B002;LEIBNIZIAN DOUBLE EQUAL SIGN;Sm;0;ON;;;;;N;;;;;
B003;LEIBNIZIAN EQUALITY WITH S SIGN;Sm;0;ON;;;;;N;;;;;
B004;LEIBNIZIAN GREATER;Sm;0;ON;;;;;N;;;;;
B005;LEIBNIZIAN LESS;Sm;0;ON;;;;;N;;;;;
B006;BERNOULLIAN GREATER;Sm;0;ON;;;;;N;;;;;
B007;BERNOULLIAN LESS;Sm;0;ON;;;;;N;;;;;
B008;LEIBNIZIAN GREATER WITH P;Sm;0;ON;;;;;N;;;;;
B009;LEIBNIZIAN LESS WITH P;Sm;0;ON;;;;;N;;;;;
B010;LEIBNIZIAN GREATER-LESS SIGN;Sm;0;ON;;;;;N;;;;;
B011;GREATER 2;Sm;0;ON;;;;;N;;;;;
B012;LESS 2;Sm;0;ON;;;;;N;;;;;
B013;PARALLEL GREATEREQUAL;Sm;0;ON;;;;;N;;;;;
B014;PARALLEL LESSEQUAL;Sm;0;ON;;;;;N;;;;;
B015;FACIT SIGN;Ll;0;L;<font> 0066;;;;N;;;;;
B016;CARTESIAN EQUAL SIGN;Sm;0;ON;;;;;N;;;;;
B017;TSCHIRNHAUS EQUAL SIGN;Sm;0;ON;;;;;N;;;;;
B018;CONGRUENCE SIGN 1;Sm;0;ON;;;;;N;;;;;
B019;CONGRUENCE SIGN 2;Sm;0;ON;;;;;N;;;;;
B020;SIMILARITY SIGN;Sm;0;ON;;;;;N;;;;;
B021;COINCIDENCE SIGN;Sm;0;ON;;;;;N;;;;;
B022;LEIBNIZIAN SIMILARITY SIGN 1;Sm;0;ON;;;;;N;;;;;
B023;LEIBNIZIAN SIMILARITY SIGN 2;Sm;0;ON;;;;;N;;;;;
```

c) Leibnizian ambiguity signs

```
C001;AMBIGUITY SIGN A-01;Sm;0;ON;;;;;N;;;;;
C002;AMBIGUITY SIGN A-02;Sm;0;ON;;;;;N;;;;;
C003;AMBIGUITY SIGN A-03;Sm;0;ON;;;;;N;;;;;
C004;AMBIGUITY SIGN A-04;Sm;0;ON;;;;;N;;;;;
C005;AMBIGUITY SIGN A-05;Sm;0;ON;;;;;N;;;;;
C006;AMBIGUITY SIGN A-06;Sm;0;ON;;;;;N;;;;;
C007;AMBIGUITY SIGN A-07;Sm;0;ON;;;;;N;;;;;
C008;AMBIGUITY SIGN A-08;Sm;0;ON;;;;;N;;;;;
C009;AMBIGUITY SIGN B-01;Sm;0;ON;;;;;N;;;;;
C010;AMBIGUITY SIGN B-02;Sm;0;ON;;;;;N;;;;;
C011;AMBIGUITY SIGN B-03;Sm;0;ON;;;;;N;;;;;
C012;AMBIGUITY SIGN B-04;Sm;0;ON;;;;;N;;;;;
C013;AMBIGUITY SIGN B-05;Sm;0;ON;;;;;N;;;;;
C014;AMBIGUITY SIGN B-06;Sm;0;ON;;;;;N;;;;;
C015;AMBIGUITY SIGN B-07;Sm;0;ON;;;;;N;;;;;
C016;AMBIGUITY SIGN B-08;Sm;0;ON;;;;;N;;;;;
C017;AMBIGUITY SIGN B-09;Sm;0;ON;;;;;N;;;;;
C018;AMBIGUITY SIGN B-10;Sm;0;ON;;;;;N;;;;;
C019;AMBIGUITY SIGN B-11;Sm;0;ON;;;;;N;;;;;
C020;AMBIGUITY SIGN B-12;Sm;0;ON;;;;;N;;;;;
C021;AMBIGUITY SIGN B-13;Sm;0;ON;;;;;N;;;;;
C022;AMBIGUITY SIGN B-14;Sm;0;ON;;;;;N;;;;;
C023;AMBIGUITY SIGN B-15;Sm;0;ON;;;;;N;;;;;
C024;AMBIGUITY SIGN B-16;Sm;0;ON;;;;;N;;;;;
C025;AMBIGUITY SIGN B-17;Sm;0;ON;;;;;N;;;;;
C026;AMBIGUITY SIGN B-18;Sm;0;ON;;;;;N;;;;;
C027;AMBIGUITY SIGN C-01;Sm;0;ON;;;;;N;;;;;
C028;AMBIGUITY SIGN C-02;Sm;0;ON;;;;;N;;;;;
C029;AMBIGUITY SIGN C-03;Sm;0;ON;;;;;N;;;;;
C030;AMBIGUITY SIGN C-04;Sm;0;ON;;;;;N;;;;;
C031;AMBIGUITY SIGN C-05;Sm;0;ON;;;;;N;;;;;
C032;AMBIGUITY SIGN C-06;Sm;0;ON;;;;;N;;;;;
C033;AMBIGUITY SIGN C-07;Sm;0;ON;;;;;N;;;;;
C034;AMBIGUITY SIGN C-08;Sm;0;ON;;;;;N;;;;;
C035;AMBIGUITY SIGN C-09;Sm;0;ON;;;;;N;;;;;
C036;AMBIGUITY SIGN C-10;Sm;0;ON;;;;;N;;;;;
C037;AMBIGUITY SIGN C-11;Sm;0;ON;;;;;N;;;;;
C038;AMBIGUITY SIGN C-12;Sm;0;ON;;;;;N;;;;;
```

C039;AMBIGUITY SIGN C-13;Sm;0;ON;;;;;N;;;;;
C040;AMBIGUITY SIGN C-14;Sm;0;ON;;;;;N;;;;;
C041;AMBIGUITY SIGN C-15;Sm;0;ON;;;;;N;;;;;
C042;AMBIGUITY SIGN C-16;Sm;0;ON;;;;;N;;;;;
C043;AMBIGUITY SIGN C-17;Sm;0;ON;;;;;N;;;;;
C044;AMBIGUITY SIGN C-18;Sm;0;ON;;;;;N;;;;;
C045;AMBIGUITY SIGN C-19;Sm;0;ON;;;;;N;;;;;
C046;AMBIGUITY SIGN C-20;Sm;0;ON;;;;;N;;;;;
C047;AMBIGUITY SIGN C-21;Sm;0;ON;;;;;N;;;;;
C048;AMBIGUITY SIGN C-22;Sm;0;ON;;;;;N;;;;;
C049;AMBIGUITY SIGN C-23;Sm;0;ON;;;;;N;;;;;
C050;AMBIGUITY SIGN C-24;Sm;0;ON;;;;;N;;;;;
C051;AMBIGUITY SIGN C-25;Sm;0;ON;;;;;N;;;;;
C052;AMBIGUITY SIGN C-26;Sm;0;ON;;;;;N;;;;;
C053;AMBIGUITY SIGN C-27;Sm;0;ON;;;;;N;;;;;
C054;AMBIGUITY SIGN C-28;Sm;0;ON;;;;;N;;;;;
C055;AMBIGUITY SIGN C-29;Sm;0;ON;;;;;N;;;;;
C056;AMBIGUITY SIGN C-30;Sm;0;ON;;;;;N;;;;;
C057;AMBIGUITY SIGN C-31;Sm;0;ON;;;;;N;;;;;
C058;LEFT VIRGULA PARANTHESIS;Sm;0;ON;;;;;N;;;;;
C059;RIGHT VIRGULA PARANTHESIS;Sm;0;ON;;;;;N;;;;;
C060;PLUSMINUS SIGN;Sm;0;ON;;;;;N;;;;;
C061;MINUSPLUS SIGN;Sm;0;ON;;;;;N;;;;;

d) Geometrical signs
D001;DOUBLE CIRCLE WITH DOT;So;0;ON;;;;;N;;;;;
D002;CIRCLE WITH DOUBLE VERTICAL LINE;So;0;ON;;;;;N;;;;;
D003;CIRCLE WITH DOUBLE VERTICAL AND HORIZONTAL LINE;So;0;ON;;;;;N;;;;;
D004;DOUBLE CIRCLE WITH DOUBLE HORIZONTAL LINE;So;0;ON;;;;;N;;;;;
D005;CIRCLE WITH HALF MOON OBLIQUE;So;0;ON;;;;;N;;;;;
D006;HALF RIGHTHAND CIRCLE WITH DIAMETER;So;0;ON;;;;;N;;;;;
D007;SMALL SECTOR WITH CHORD;So;0;ON;;;;;N;;;;;
D008;SMALL SECTOR,So;0;ON;;;;;N;;;;;
D009;SMALL SECTOR WITH DOUBLE ARC;So;0;ON;;;;;N;;;;;
D010;SMALL SECTOR TRIANGLE;So;0;ON;;;;;N;;;;;
D011;SMALL SEGMENT;So;0;ON;;;;;N;;;;;
D012;RIGHT TRIANGLE POINTING RIGHT;So;0;ON;;;;;N;;;;;
D013;KITE SIGN;So;0;ON;;;;;N;;;;;
D014;ANGLE 1;So;0;ON;;;;;N;;;;;
D015;ANGLE 2;So;0;ON;;;;;N;;;;;
D016;ANGLE 3;So;0;ON;;;;;N;;;;;
D017;ANGLE 4;So;0;ON;;;;;N;;;;;
D018;ANGLE VERTICAL;So;0;ON;;;;;N;;;;;
D019;CUBUS 1;So;0;ON;;;;;N;;;;;
D020;CUBUS 2;So;0;ON;;;;;N;;;;;
D021;HORIZONTAL DOUBLE SQUARE;So;0;ON;;;;;N;;;;;
D022;VERTICAL DOUBLE SQUARE;So;0;ON;;;;;N;;;;;
D023;THREE-PART BIG SQUARE 1;So;0;ON;;;;;N;;;;;
D024;THREE-PART BIG SQUARE 2;So;0;ON;;;;;N;;;;;
D025;FOUR-PART BIG SQUARE;So;0;ON;;;;;N;;;;;
D026;HYPERBOLE;So;0;ON;;;;;N;;;;;

e) Alchemical symbols
E001;ALCHEMICAL SYMBOL FOR ALUMEN-PISCES;So;0;ON;;;;;N;;;;;
E002;ALCHEMICAL SYMBOL FOR OIL BOILED;So;0;ON;;;;;N;;;;;
E003;ALCHEMICAL SYMBOL FOR MOON-JUPITER;So;0;ON;;;;;N;;;;;
E004;ALCHEMICAL SYMBOL FOR TARTAR-SALT;So;0;ON;;;;;N;;;;;
E005;ALCHEMICAL SYMBOL ENCLOSED SUN;So;0;ON;;;;;;N;;;;;
E006;ALCHEMICAL SYMBOL ENCLOSED MOON;So;0;ON;;;;N;;;;;
E007;ALCHEMICAL SYMBOL FOR REALGAR 3;So;0;ON;;;;;N;;;;;
E008;ALCHEMICAL SYMBOL FOR HORA 2;So;0;ON;;;;;;N;;;;;
E009;ALCHEMICAL SYMBOL FOR RETORT 2;So;0;ON;;;;N;;;;;

f) Miscellaneous scientific signs
F001;CASTING-OUT-NINES;Sm;0;ON;;;;;N;;;;;
F002;LUNATE ENCIRCLED FIGURE ONE;So;0;ON;;;;;N;;;;;
F003;PROPORTION 1;So;0;ON;;;;;N;;;;;
F004;PROPORTION 2;So;0;ON;;;;;N;;;;;
F005;RIGHTHAND RELATION SIGN;So;0;ON;;;;;N;;;;;
F006;LEFTHAND RELATION SIGN;So;0;ON;;;;;N;;;;;
F007;CLOVERLEAF SIGN;So;0;ON;;;;;N;;;;;
F008;INFINITY SIGN WITH DOTS;Sm;0;ON;;;;;N;;;;;
F009;INVOLVED SIGN;Sm;0;ON;;;;;N;;;;;

F010;LEIBNIZIAN ENCIRCLED V SIGN;Sm;0;ON;;;;;N;;;;;
F011;LEIBNIZIAN BOXED ENCIRCLED V SIGN;Sm;0;ON;;;;;N;;;;;
F012;BROKEN EMDASH;So;0;ON;;;;;N;;;;;
F013;CROSSED EMDASH;So;0;ON;;;;;N;;;;;
F014;BOLD PERIOD;Po;0;ON;;;;;N;;;;;
F015;RADIX SIGN 1;Sm;0;ON;;;;;N;;;;;
F016;RADIX SIGN 2;Sm;0;ON;;;;;N;;;;;
F017;RADIX SIGN 3;Sm;0;ON;;;;;N;;;;;
F018;COMBINING BOMBELLI POWER MARK;Mn;220;NSM;;;;;N;;;;;
F019;COMBINING DOUBLE-WIDE SLASH;Mn;1;NSM;;;;;N;;;;;
F020;COMBINING HALF CIRCLE BELOW;Mn;220;NSM;;;;;N;;;;;
F021;COMBINING ENCLOSING SPIRAL MARK;Me;1;NSM;;;;;N;;;;;
F022;COMBINING DOUBLE-WIDE ENCLOSING SPIRAL MARK;Me;1;NSM;;;;;N;;;;;
F023;COMBINING FACTOR MARK;Mn;1;NSM;;;;;N;;;;;
F024;COMBINING OVERLINE WITH TERMINALS;Mn;230;NSM;;;;;N;;;;;
F025;COMBINING DOUBLE-WIDE OVERLINE WITH TERMINALS;Mn;230;NSM;;;;;N;;;;;
F026;COMBINING HORIZONTAL PARANTHESIS;Mn;230;NSM;;;;;N;;;;;

g) Superscript characters
G001;SUPERSCRIPT ENCLOSED SMALL G SIGN;Sm;0;ON;;;;;N;;;;;
G002;SUPERSCRIPT ENCLOSED SMALL N SIGN;Sm;0;ON;;;;;N;;;;;
G003;SUPERSCRIPT ENCLOSED SMALL T SIGN;Sm;0;ON;;;;;N;;;;;
G004;SUPERSCRIPT ENCLOSED SMALL X SIGN;Sm;0;ON;;;;;N;;;;;
G005;SUPERSCRIPT ENCLOSED SMALL Z SIGN;Sm;0;ON;;;;;N;;;;;
G006;SUPERSCRIPT ENCIRCLED SMALL Z SIGN;Sm;0;ON;;;;;N;;;;;
G007;SUPERSCRIPT WAVE;Sm;0;ON;;;;;N;;;;;
G008;SUPERSCRIPT WAVE WITH TOP LINE;Sm;0;ON;;;;;N;;;;;

h) Letterlike symbols
H001;BERNOULLIAN ALPHA-X SIGN;So;0;ON;;;;;N;;;;;
H002;LATIN CAPITAL D WITH TOP BAR AND CROSSBAR;Sm;0;ON;;;;;N;;;;;
H003;LATIN CAPITAL REVERSED L;Lu;0;L;;;;;N;;;;H004;
H004;LATIN LOWERCASE REVERSED L;Ll;0;L;;;;;N;;;H003;;H003
H005;LOWERCASE P WITH DOUBLE CROSSBAR;So;0;ON;;;;;N;;;;;
H006;LOWERCASE KURRENT X SIGN;Sm;0;ON;;;;;N;;;;;
H007;LATIN CAPITAL DOUBLE X;Lu;0;L;;;;;N;;;;H008
H008;LATIN LOWERCASE DOUBLE X;Ll;0;L;;;;;N;;;H007;;H007
H009;SIGMA-SIGMA SIGN;Sm;0;ON;;;;;N;;;;;
H010;GREEK CAPITAL OMICRON-UPSILON;Lu;0;L;;;;;N;;;;H011;
H011;GREEK LOWERCASE OMICRON-UPSILON;Ll;0;L;;;;;N;;;H010;;H010

i) Coss symbols
I001;LOWERCASE C WITH SMALL SLASH;So;0;ON;;;;;N;;;;;
I002;LOWERCASE C WITH DESCENDER;So;0;ON;;;;;N;;;;;
I003;LOWERCASE C WITH RIGHT LOOP;So;0;ON;;;;;N;;;;;
I004;LOWERCASE D ROTUNDA WITH CROSSING LOOP;So;0;ON;;;;;N;;;;;
I005;SMALL CAPITAL R WITH SLASH;So;0;ON;;;;;N;;;;;
I006;LOWERCASE R ROTUNDA WITH LOOP;So;0;ON;;;;;N;;;;;
I007;DOUBLE S ABBREVIATION SIGN;So;0;ON;;;;;N;;;;;
I008;LOWERCASE LONG S WITH TOP LOOP;So;0;ON;;;;;N;;;;;
I009;LOWERCASE KURRENT Z SIGN;So;0;ON;;;;;N;;;;;

k) Digit characters
K000;SLASHED DIGIT ZERO;Nd;0;EN;;0;0;0;;;;;
K001;SLASHED DIGIT ONE;Nd;0;EN;;1;1;1;;;;;
K002;SLASHED DIGIT TWO;Nd;0;EN;;2;2;2;;;;;
K003;SLASHED DIGIT THREE;Nd;0;EN;;3;3;3;;;;;
K004;SLASHED DIGIT FOUR;Nd;0;EN;;4;4;4;;;;;
K005;SLASHED DIGIT FIVE;Nd;0;EN;;5;5;5;;;;;
K006;SLASHED DIGIT SIX;Nd;0;EN;;6;6;6;;;;;
K007;SLASHED DIGIT SEVEN;Nd;0;EN;;7;7;7;;;;;
K008;SLASHED DIGIT EIGHT;Nd;0;EN;;8;8;8;;;;;
K009;SLASHED DIGIT NINE;Nd;0;EN;;9;9;9;;;;;
K010;DOUBLE SLASHED DIGIT ZERO;Nd;0;EN;;0;0;0;;;;;
K011;DOUBLE SLASHED DIGIT ONE;Nd;0;EN;;1;1;1;;;;;
K012;DOUBLE SLASHED DIGIT TWO;Nd;0;EN;;2;2;2;;;;;
K013;DOUBLE SLASHED DIGIT THREE;Nd;0;EN;;3;3;3;;;;;
K014;DOUBLE SLASHED DIGIT FOUR;Nd;0;EN;;4;4;4;;;;;
K015;DOUBLE SLASHED DIGIT FIVE;Nd;0;EN;;5;5;5;;;;;
K016;DOUBLE SLASHED DIGIT SIX;Nd;0;EN;;6;6;6;;;;;
K017;DOUBLE SLASHED DIGIT SEVEN;Nd;0;EN;;7;7;7;;;;;
K018;DOUBLE SLASHED DIGIT EIGHT;Nd;0;EN;;8;8;8;;;;;
K019;DOUBLE SLASHED DIGIT NINE;Nd;0;EN;;9;9;9;;;;;

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K020;TRIPLE SLASHED DIGIT ZERO;Nd;0;EN;;0;0;0;;;;;
K021;TRIPLE SLASHED DIGIT ONE;Nd;0;EN;;1;1;1;;;;;
K022;TRIPLE SLASHED DIGIT TWO;Nd;0;EN;;2;2;2;;;;;
K023;TRIPLE SLASHED DIGIT THREE;Nd;0;EN;;3;3;3;;;;;
K024;TRIPLE SLASHED DIGIT FOUR;Nd;0;EN;;4;4;4;;;;;
K025;TRIPLE SLASHED DIGIT FIVE;Nd;0;EN;;5;5;5;;;;;
K026;TRIPLE SLASHED DIGIT SIX;Nd;0;EN;;6;6;6;;;;;
K027;TRIPLE SLASHED DIGIT SEVEN;Nd;0;EN;;7;7;7;;;;;
K028;TRIPLE SLASHED DIGIT EIGHT;Nd;0;EN;;8;8;8;;;;;
K029;TRIPLE SLASHED DIGIT NINE;Nd;0;EN;;9;9;9;;;;;
K030;BACKSLASHED DIGIT ZERO;Nd;0;EN;;0;0;0;;;;;
K031;BACKSLASHED DIGIT ONE;Nd;0;EN;;1;1;1;;;;;
K032;BACKSLASHED DIGIT TWO;Nd;0;EN;;2;2;2;;;;;
K033;BACKSLASHED DIGIT THREE;Nd;0;EN;;3;3;3;;;;;
K034;BACKSLASHED DIGIT FOUR;Nd;0;EN;;4;4;4;;;;;
K035;BACKSLASHED DIGIT FIVE;Nd;0;EN;;5;5;5;;;;;
K036;BACKSLASHED DIGIT SIX;Nd;0;EN;;6;6;6;;;;;
K037;BACKSLASHED DIGIT SEVEN;Nd;0;EN;;7;7;7;;;;;
K038;BACKSLASHED DIGIT EIGHT;Nd;0;EN;;8;8;8;;;;;
K039;BACKSLASHED DIGIT NINE;Nd;0;EN;;9;9;9;;;;;
K040;CROSSED DIGIT ZERO;Nd;0;EN;;0;0;0;;;;;
K041;CROSSED DIGIT ONE;Nd;0;EN;;1;1;1;;;;;
K042;CROSSED DIGIT TWO;Nd;0;EN;;2;2;2;;;;;
K043;CROSSED DIGIT THREE;Nd;0;EN;;3;3;3;;;;;
K044;CROSSED DIGIT FOUR;Nd;0;EN;;4;4;4;;;;;
K045;CROSSED DIGIT FIVE;Nd;0;EN;;5;5;5;;;;;
K046;CROSSED DIGIT SIX;Nd;0;EN;;6;6;6;;;;;
K047;CROSSED DIGIT SEVEN;Nd;0;EN;;7;7;7;;;;;
K048;CROSSED DIGIT EIGHT;Nd;0;EN;;8;8;8;;;;;
K049;CROSSED DIGIT NINE;Nd;0;EN;;9;9;9;;;;;
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LBr – *refers to:* Leibniz's original correspondence papers, GWLB Hanover
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ISO/IEC JTC 1/SC 2/WG 2
PROPOSAL SUMMARY FORM TO ACCOMPANY SUBMISSIONS
FOR ADDITIONS TO THE REPERTOIRE OF ISO/IEC 10646[1]
Please fill all the sections A, B and C below.
Please read Principles and Procedures Document (P & P) from http://std.dkuug.dk/JTC1/SC2/WG2/docs/principles.html for guidelines and details before filling this form.
Please ensure you are using the latest Form from http://std.dkuug.dk/JTC1/SC2/WG2/docs/summaryform.html.
See also http://std.dkuug.dk/JTC1/SC2/WG2/docs/roadmaps.html for latest *Roadmaps*.

A. Administrative

1. **Title:** **Proposal to add historic scientific characters to the UCS**
2. Requester's name: Uwe Mayer, Siegmund Probst, David Rabouin, Elisabeth Rinner, Andreas Stötzner, Achim Trunk, Charlotte Wahl
3. Requester type (Member body/Liaison/Individual contribution): Individual (work group)
4. Submission date: 2024-02.19.
5. Requester's reference (if applicable): LUCP L-2402
6. Choose one of the following:
   This is a complete proposal: Yes
   (or) More information will be provided later:

B. Technical – General

1. Choose one of the following:
   a. This proposal is for a new script (set of characters): Yes
      Proposed name of script: Historic scientific characters
   b. The proposal is for addition of character(s) to an existing block: Yes
      Name of the existing block: Greek and Coptic 0370
2. Number of characters in proposal: 228
3. Proposed category (select one from below - see section 2.2 of P&P document):

A-Contemporary		B.1-Specialized (small collection)		B.2-Specialized (large collection)	Yes
C-Major extinct		D-Attested extinct		E-Minor extinct	
F-Archaic Hieroglyphic or Ideographic				G-Obscure or questionable usage symbols	

4. Is a repertoire including character names provided? Yes
   a. If YES, are the names in accordance with the "character naming guidelines" in Annex L of P&P document? Yes
   b. Are the character shapes attached in a legible form suitable for review? Yes
5. Fonts related:
   a. Who will provide the appropriate computerized font to the Project Editor of 10646 for publishing the standard?
      Andreas Stötzner
   b. Identify the party granting a license for use of the font by the editors (include address, e-mail, ftp-site, etc.):
      Andreas Stötzner Gestaltung, Klauflügelweg 21, 88400 Biberach/R., Germany, as@signographie.de
6. References:
   a. Are references (to other character sets, dictionaries, descriptive texts etc.) provided? Yes
   b. Are published examples of use (such as samples from newspapers, magazines, or other sources) of proposed characters attached? Yes
7. Special encoding issues:
   Does the proposal address other aspects of character data processing (if applicable) such as input, presentation, sorting, searching, indexing, transliteration etc. (if yes please enclose information)? No

8. Additional Information:
Submitters are invited to provide any additional information about Properties of the proposed Character(s) or Script that will assist in correct understanding of and correct linguistic processing of the proposed character(s) or script. Examples of such properties are: Casing information, Numeric information, Currency information, Display behaviour information such as line breaks, widths etc., Combining behaviour, Spacing behaviour, Directional behaviour, Default Collation behaviour, relevance in Mark Up contexts, Compatibility equivalence and other Unicode normalization related information. See the Unicode standard at http://www.unicode.org for such information on other scripts. Also see Unicode Character Database (http://www.unicode.org/reports/tr44/) and associated Unicode Technical Reports for information needed for consideration by the Unicode Technical Committee for inclusion in the Unicode Standard.

[1] Form number: N4502-F (Original 1994-10-14; Revised 1995-01, 1995-04, 1996-04, 1996-08, 1999-03, 2001-05, 2001-09, 2003-11, 2005-01, 2005-09, 2005-10, 2007-03, 2008-05, 2009-11, 2011-03, 2012-01)

C. Technical - Justification

1. Has this proposal for addition of character(s) been submitted before? No

   If YES explain

2. Has contact been made to members of the user community (for example: National Body, user groups of the script or characters, other experts, etc.)? Yes

   If YES, with whom? Leibniz-Archiv, Forschungsstelle der Leibniz-Edition, Niedersächsische Landesbibliothek (GWLB), Hanover, Göttingen Academy of Science and Humanities in Lower Saxony (DE), Philiumm research group of CNRS (UMR 7219, laboratoire SPHERE) / Université de Paris VII; general: scholars, researchers, authors and editors working in the field of science history and upon editions of historic text corpora (e.g. of G. W. Leibniz, but also many others)

   If YES, available relevant documents: L-2403, L-2404, L-2405, L-2409, L-2410

3. Information on the user community for the proposed characters (for example: size, demographics, information technology use, or publishing use) is included? Yes

   Reference:

4. The context of use for the proposed characters (type of use; common or rare) Common

   Reference: mainly specialist usage, scholarly, worldwide

5. Are the proposed characters in current use by the user community? Yes

   If YES, where? Reference: mainly Germany, France; other countries

6. After giving due considerations to the principles in the P&P document must the proposed characters be entirely in the BMP? No

   If YES, is a rationale provided?

   If YES, reference:

7. Should the proposed characters be kept together in a contiguous range (rather than being scattered)? No

8. Can any of the proposed characters be considered a presentation form of an existing character or character sequence? No

   If YES, is a rationale for its inclusion provided?

   If YES, reference:

9. Can any of the proposed characters be encoded using a composed character sequence of either existing characters or other proposed characters? No

   If YES, is a rationale for its inclusion provided?

   If YES, reference:

10. Can any of the proposed character(s) be considered to be similar (in appearance or function) to, or could be confused with, an existing character? No

    If YES, is a rationale for its inclusion provided?

    If YES, reference:

11. Does the proposal include use of combining characters and/or use of composite sequences? Yes

    If YES, is a rationale for such use provided?

    If YES, reference: a few combining characters, see under f)

    Is a list of composite sequences and their corresponding glyph images (graphic symbols) provided? No

    If YES, reference:

12. Does the proposal contain characters with any special properties such as control function or similar semantics? No

    If YES, describe in detail (include attachment if necessary)

13. Does the proposal contain any Ideographic compatibility characters? No

    If YES, are the equivalent corresponding unified ideographic characters identified?

    If YES, reference: